

Crop plan optimization under risk on a farm level in the Czech Republic

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Abstract: The precise agricultural operations research models claiming complex solutions and specialized software tools may appear too complicated to truly support the decision making process on the farm level. The specific balance between the simplifications made and the correspondence to the real problem must be found when building decision support for a local farmer. In this paper, a decision support model for agricultural production planning in the Czech Republic is developed covering both the randomness of parameters entering the problem and the complex crop succession requirements. Crucially, the model can be approached using software tools commonly available to farmers. The structure of the simplified model is described in detail, its validity proved by the recently suggested simulation procedures and the general applicability of the model structuring for a local agriculture decision support is discussed.

Key words: crop plan, crop rotation constraints, optimization, stochastic programming.

The operations research (OR) models began to be applied for agriculture decision making in the early 1950s. At first, the linear programming was proposed to establish the least cost combinations of feeding and livestock ratios and later to determine the optimum crop rotations on a farm (Bjorndal et al. 2012). Even though the linear programming has been the most common OR technique in agriculture, many other approaches have already been applied to the agribusiness problems. From the modelling perspective, the OR models in agriculture can be classified as deterministic and stochastic, according to the certainty of the value of the parameters used. Where the parameters are assumed to be deterministic, apart from the linear programming, also the dynamic programming, the mixed integer programming and the goal programming are frequently used, otherwise the stochastic modelling approaches are employed, these including mainly the stochastic programming, the stochastic dynamic programming, the simulation and risk programming (Maatman et al. 2002; Lowe and Preckel 2004; Torkamani 2005; Benjamin et al. 2009; Bohle et al. 2009).

Currently, the main stream of operations research (OR) focuses on developing sophisticated models for

the decision making support that reflect the reality as precisely as possible. However, these realistic models claim complex solution procedures and advanced software tools. What is acceptable for the decision support of enterprises or policy makers on the national or international level may appear a barrier for the decision support for small enterprises on a local level (Boehlje 1999). No doubt that finding optimal decisions within the board of small enterprises and local problems is as important as finding these for the global problems. This is particularly the case in agriculture business where the sustainable and ecologically sound production of all farms is of the substantial global importance (Cramer et al. 2000). In this paper, we focus on structuring, solution and validation of the typical decision problem of production planning on the farm level in the Czech Republic in a way that it is:

- (1) covering the complex structure of the problem
- (2) user-friendly and solvable in the EXCEL which is commonly available at farms,
- (3) valid,
- (4) possibly transferable for structuring similar production planning problems under the specific local conditions.

Particularly, in this paper we develop a decision support tool for the farm crop planning covering both the stochastic nature of the harvest parameters and the complex crop succession requirements in such a way that the resulting mathematical programming model can be processed by the EXCEL. Such an approach provides the farmers with a user-friendly alternative to the estimative crop plan decision making relentlessly used. Note that including the crop succession requirements into a mathematical programming model is a nontrivial task that is permanently in the focus of the operations researchers (Seppelt 2000; Dogliotti et al. 2003; Klein Haneveld and Stegeman 2005; Bachinger and Zander 2007; Detlefsen and Jensen 2007; Castellazzi et al. 2008; Jatoe et al. 2008; Benjamin et al. 2009; Parsons et al. 2009; dos Santos et al. 2011). Moreover, the combination of the crop succession restrictions and the risk consideration in a single optimization model is very rare (Myers et al. 2008) and leads to a complex optimization problem. None of the results of the current research in the field of the agriculture production planning is immediately applicable to the problems comprising both of the above mentioned features. Hence, the aim of the proposed approach is to provide a farmer with a user-friendly model yet covering the nonstandard complex structure of the real problem. In Materials and methods, the production planning problem in agriculture is formulated in general, the local specifics of Czech farming are summarized and the model structuring, the assumptions and simplifications made are discussed in detail. The crop succession constraint is designed and its construction is explained using a practical example. In Results and discussion, this model is applied and solved for a particular problem of the South Moravian farm, the validation procedure is applied and evaluated and the discussion is provided.

MATERIAL AND METHODS

Let us systematize the general aspects determining the mathematical programming model for the agricultural crop planning problem:

- (1) Behaviour of the farmer
 - the objective of production process
 - risk attitude, sources of uncertainty
- (2) Sources of restrictions
 - available land and capital
 - materials and supplies used in farm production
 - environmental factors – soil conservation, water quality control, manure management, etc.

- legal aspects of production -regulations and laws, quality control, inspection requirements, etc.
- (3) The form of crop plan

(1) Behaviour of the farmer

We consider a representative generic farm in the Czech Republic managing a single compact one-soil type area of land and using its own capital. The farmer's objective is to maximize profit from the future harvest for the given area of arable land. Note that the profit maximization as a goal of the farmer has been doubted by some authors (Bjorndal et al. 2012), but in our study, we follow the farmer's attitudes and preferences for the particular real situation.

The high level of uncertainty is present in the estimation of the parameters of the agriculture production planning models (e.g. yield, profit, demand for products, weather conditions, etc. (Hazell and Norton 1986). The effects of uncertainty are particularly important if the farmers are risk averse, as it has been traditionally assumed in the economic literature (Hardaker al. 1991; Benjamin et al. 2009, Ahumada and Villalobos 2009), and as it is the case also in our problem. In our approach, we will consider the random yields which reflect the uncertainty of weather and natural conditions. In the real situation, the demand for the farmer's products and the selling prices are unknown as well at the moment of the production planning. We deal with these using the farmers' expert estimation of further prices and demand. Hence, we will consider the future prices as the given constant and the farmer sets the margins for each crop to follow the expected demand.

The objective function representing the profit maximization takes the form

$$z^* = \max \sum_{i=1}^n c_i x_i \quad (1)$$

where the decision variables x_i stand for the areas of arable land planted with crop i . In the objective function (1), the parameters c_i are the random variables of the *total profit* per 1 ha of the planted crop i defined as follows:

$$c_i = p_i q_i - n_i \quad (2)$$

q_i being the random variable *yield* of the respective crop-plant i . We consider n_i (*total costs* per 1 ha of arable land planted by crop i) and p_i (*selling price* for 1 ton of crop i), $1 \leq i \leq n$, constants of the known values. Note that in the profit function, we assume zero fixed costs, which, although unrealistic, is ac-

ceptable since the fixed costs do not influence the performance of the model.

The formulation (1) seems to represent a simple linear function but, as the harvests q_i must be seen as random variables, the optimization problem is of the stochastic nature. Note that, through (2), the profits c_i are random as well. We assume that, for the next period, the prices per ton p_i are already given. We will employ the Markowitz criterion to obtain a deterministic equivalent of the objective function. Note that there is a number of approaches to transform a stochastic program into a deterministic one. The Markowitz model was one of the first used in the agriculture production planning optimization under risk (Freund 1956). Markowitz formulated the portfolio problem as a choice of the mean and variance of the portfolio assets. Later, the alternative portfolio theories were suggested (Kraus and Litzenberger 1976; Lee 1977) and also a number of alternative approaches appeared for the agriculture optimization under risk (Hardaker et al. 2004; Lien et al. 2009). Nevertheless, the mean variance theory has remained the cornerstone of the modern portfolio theory. Its persistence is due to the fact that the theory is widely known, well developed and has a great intuitive appeal understandable even by the professionals who never run an optimizer (Elton and Gruber 1997; Rubinstein 2002). Following the Freund approach and denoting

Σ = the covariance matrix of the random vector (c_1, \dots, c_n)

x = the decision variables vector

a = the risk aversion coefficient

γ = the unit profit means vector, i.e.

$$(\gamma_1, \dots, \gamma_n) = (\langle c_1 \rangle, \dots, \langle c_n \rangle)$$

the Markowitz-type objective function takes the form

$$\tilde{z}^* = \min \left(\frac{a}{2} x \Sigma x^T - \gamma x^T \right) \quad (3)$$

where the maximization of the random profit (1) is replaced by the minimization of the difference between the terms representing the variability and the mean value of the total profit.

(2) Sources of restriction

We will assume the associated animal production generating the requirements on a certain level of the feed crops and limiting the manure available. No additional restriction will arise due to the need of the machinery, equipment, materials, etc., since the needs of the production process are consistently

stable and the arm has a sufficient capacity to cover all production plans considered. As for the environmental factors, the manure management will affect the decision making in the form of specific needs for each crop to manure the field. Moreover, the crop succession rules must be obeyed to respect the agronomic regulations. The legal aspects of production cause no further restrictions to our optimization problem. All the restrictions mentioned can be represented by the simple linear inequality constraints up to the more complex crop succession constraints.

Construction of the crop succession constraints

The crop succession constraints ensure that the same crop will not be re-sowed on one piece of land during the relevant number of years and that the prohibited succession of two different crops will not appear. These restrictions can be represented by a system of linear inequalities. Denote n the number of crop types planted by the particular farm, and x_i the area planted by crop i . Then the constraints are generated using the algorithm

for $p = 1$ to n :

for each p -combination $\{i_1, \dots, i_p\}$ from a set $\{1, \dots, n\}$:

$$\sum_{s=1}^p x_{i_s} \leq X - \tilde{x}_{i_1, \dots, i_p} - \tilde{y}_{i_1, \dots, i_p} \quad (4)$$

where X is the total area of arable land available, $\tilde{x}_{i_1, \dots, i_p}$ is the total area cropped with all the crops i_1, \dots, i_p during the $r(i_1), \dots, r(i_p)$ past years, $r(i_s)$ is the number of years after which the crop i_s may be planted on the same area, $\tilde{y}_{i_1, \dots, i_p}$ is the total area not cropped with all the crops i_1, \dots, i_p during the relevant past years, but cropped last year with the crop plants after which none of the crops i_1, \dots, i_p may succeed. Note that the constraint was mentioned (but not constructed) and validated in (Janová 2012), here the thorough construction is provided (for more illustration of the constraint construction, see Appendix.)

(3) The form of a crop plan

The results to be obtained from the model solution are the total areas of land cropped by the particular crop plants. Hence, we do not assign the crops to the particular fields. This is to simplify the optimization model and to provide the farmer with a valid and practical support for making the final production planning decisions.

A number of real aspects of the decision problem have not been directly reflected in our model (e.g. the impact of subsidies and the international environment

or the unknown future prices). The practice of the model design has shown that the more accurate the model, the more complex form of the model (and the more sophisticated solution techniques) is needed. The aim of the model suggested in our contribution is to be as simple as possible while having a high information value.

To attain this objective we employ the specific farmer's information on the parameters (financial and technical) that enters his/her real decision making and in this way, we suppose to cope with the part of the issues not directly reflected by the model. Particularly, the existence of subsidies can be incorporated by the farmer in the unit costs and/or unit prices, and the randomness of the future prices can be covered by the expert judgement of the farmer.

These assumptions must be justified by the validation that for our model has been done through the Monte Carlo simulations. The validation itself is described and its results are discussed at the end of the section Results and Discussion. Note that even though the results of the validation are satisfactory, we do not state our model to provide a final crop plan but recommend it to be used as an additional supporting source of information when making the final decision concerning the crop plan.

The Monte Carlo simulations

In our validation experiments we will use the Monte Carlo simulations as an experimental device that provides information on the model performance. Here we briefly summarize the basis of the method.

In the simulation, the optimal values of decision variables (crop areas) are the inputs and the simulation experiment evaluates the objective function for particular set of values of the random variables. Particularly, in our problem we

- (1) generate the random numbers from the interval $[0,1]$
- (2) assume that each random number is a value of the cumulative distribution function of the natural yield for one crop
- (3) find the particular yield for each crop using the inverse of the cumulative distribution function
- (4) given the yields of the crops, we enumerate the profit of the optimal crop plan
- (5) repeat the simulation.

By this procedure, we obtain a number of the harvests' scenarios, each of them evaluated by the par-

ticular total profit. During the validation, we use these data to evaluate the total profit performance of the model in comparison to the real decision of the farmer.

RESULTS AND DISCUSSION

As a result of the above described construction, we can write down a mathematical program for the particular farm. In our case study, we consider a one year ahead planning in the South Moravian agriculture cooperative farming on the area of 1265 ha. The crop plan decision making is restricted by

- total area of arable land available (X),
- capital (N),
- maximal area fertilized by manure (M),
- restrictions on the minimal resp. maximal area cropped by the particular crop (A_i resp. B_i , $1 \leq i \leq n$, C , D),
- re-sowing of the crops.

The total costs n_i of planting the crop i consists of the costs of seeds, labour and machine time used for planting the particular crop. The constraints on the maximal area fertilized by manure and the thresholds for the areas cropped by the particular crops follow from the livestock breeding potential and the needs of the particular farm. Moreover, the threshold restrictions reflect also the expected demand for the products of the particular farm.

The data were gathered in (Janová and Ambrožová 2009), where a deterministic linear programming problem without crop rotation restrictions was solved. The decision variables x_i are the areas planted by the particular crops (Table 1). Due to different prices of the food and feed crops, each area sowed by one agricultural crop was split into two decision variables. Nevertheless, the harvest characteristics are for both food and feed type of the crop the same. The quadratic

Table 1. Decision variables

x_i	area sowed by	x_8	spring feed barley
x_1	winter food wheat	x_9	triticale
x_2	winter feed wheat	x_{10}	corn
x_3	spring food wheat	x_{11}	corn silage
x_4	spring feed wheat	x_{12}	oilseed rape
x_5	winter food barley	x_{13}	potatoes
x_6	winter feed barley	x_{14}	grass
x_7	spring food barley	x_{15}	grass seed

programming model discussed above takes, for the particular case of a South Moravian farm, the form

$$\tilde{z}^* = \min\left(\frac{a}{2}x\Sigma x^T - \gamma x^T\right) \tag{5}$$

$$\sum_{i=1}^n n_i x_i \leq N \tag{6}$$

$$\sum_{i=1}^n m_i x_i \leq M \tag{7}$$

$$x_i \geq A_i \tag{8}$$

$$x_i \leq B_i \tag{9}$$

$$x_5 + x_5 \leq C \tag{10}$$

$$x_6 + x_7 \leq D \tag{11}$$

for each p -combination $\{i_1, \dots, i_p\}$ from a set $\{1, \dots, n\}$:

$$\sum_{s=1}^p x_{i_s} \leq X - \tilde{x}_{i_1, \dots, i_p} - \tilde{y}_{i_1, \dots, i_p} \tag{12}$$

$$x_i \geq 0, 1 \leq i \leq n \tag{13}$$

Table 2. The parameters of the problem (rounded): γ_i = the unit profit mean, n_i = the total costs per 1 ha, p_i = selling price for 1 ton, μ_i = yield mean, m_i = the manure coefficient

i	γ_i (CZK/ha)	n_i (CZK/ha)	p_i (CZK/t)	μ_i (t/ha)	m_i
1	2 590.0	20 524	3 900	5.93	0
2	1 373.3	17 592	3 200	5.93	0
3	1 764.5	15 428	3 900	4.41	0
4	882.7	13 224	3 200	4.41	0
5	5 513.9	16 088	5 300	4.08	0
6	1 384.3	12 066	3 300	4.08	0
7	5 570.5	17 776	5 300	4.41	0.2
8	1 204.5	13 332	3 300	4.41	0.2
9	433.7	15 457	3 200	4.97	0
10	1 856.0	33 408	4 800	6.19	0.67
11	1 726.5	15 906	600	29.39	0.67
12	3 185.3	24 256	8 800	3.12	0.2
13	1 397.1	64 589	2 740	24.08	1
14	-561.5	12 933	485	25.51	0
15	3 041.3	8 420	26 500	0.43	0

The restrictions on the total capital available are contained in (6), where n_i are the unit costs and N denotes the budget, the constraint (7) reflects the restriction on the total manure. Note that m_i is the *manure coefficient* that plays the role of the ratio of the arable land (to be cropped by the crop i) that will be fertilized by manure. This coefficient reflects the intended procedure at the farm. The conditions (8)–(11) ensure the maximal and minimal areas cropped by the particular crops and (12) are the crop succession constraints developed in the above. The parameters of the constraints and the objective function are listed in Table 2 and Table 3. Note that the elaborated model is a simplification of the one mentioned in (Janová 2012).

Identifying the minimum number of years after which each crop can be re-sowed on the same area and the prohibited succession of crops, the set of the succession constraints (12) is generated:

$$\sum_{k=1}^9 x_k \leq 979 \quad x_{12} \leq 131 \tag{14}$$

$$x_{12} + \sum_{k=1}^9 x_k \leq 1125$$

Note that, for the purpose of generating the constraint, crops 1–9 were clustered together as cereals. The model was solved for the circumstances of the year 2009¹ and for several values of the risk aversion coefficient a covering the low aversion to risk with $a = 0$ up to high risk aversion attitude of the decision

Table 3. Thresholds for the areas sowed by the particular crops

i	x_i /area	A_i (ha)	B_i [ha]	C (ha)	D (ha)
2	x_2	69	–		
4	x_4	17	–		
6	x_6	20	–		
8	x_8	76	–		
9	x_9	–	40		
10	x_{10}	20	–		
12	x_{12}	–	240		
13	x_{13}	–	30		
14	x_{14}	60	100		
15	x_{15}	–	85		
–	$x_5 + x_6$			22	
–	$x_7 + x_8$				330

¹This choice follows from the data availability.

maker with $a = 1 \times 10^{-6}$. The quadratic programming problem (5–13) can be solved using any available optimization software including the Solver in MS Excel. The standard Excel Solver has a limit of 200 decision variables or changing cells. It also imposes a limit on the number of constraints in a situation where the problem is nonlinear (there is a limit of 100 constraints other than the constant bounds on the variables and integer constraints). Since in our quadratic programming problem there are 15 decision variables and about 50 constraints including the constant bounds, the Solver appears to be an appropriate software tool for finding the solution. Note that the covariance matrix and the mean values as the right hand sides of the re-sowing constraints (12) can easily be enumerated with the use of the Excel. The user must be aware of the fact that all the crop history on each field must be entered into the spreadsheet table. In case there is no electronic version of the crop history at the farm, it may appear more simple to determine the left hand sides of the re-sowing constraints by hand.

The results of the optimization model for all choices of the parameter a can be compared to the real decision of the farmer in Table 4. The differences between the model results and the real decision for higher risk aversion ($a \times 10^6 = 0.5–1.0$) are not significant except the area cropped by the oilseed rape and

the winter food wheat. In the model results, the oil seed rape area 131 ha is on the upper bound of the succession constraint (14) while in the real plan, the oil seed rape area is 230 ha. Hence, for the year 2009, the farmer decided to produce oil seeds on a higher area than it is theoretically recommended. The model solution replaces the area of the oil seed rape especially by the winter food wheat. Note that the area of the winter food wheat is increasing in the model solution for a lower risk aversion coefficient, which reflects the high profits of the crop together with its high variability in harvests.

To evaluate the model performance, we will compare the expected profit for the optimal plans obtained by the model with the farmer's decision using the Monte Carlo simulations (the particular validation procedure was suggested in Janová 2012). For the purpose of the harvests simulation, the natural yields (q_i) were described using the Beta distribution and the correlation between the crop yields was considered using the Spearman rank correlation. Using the harvests simulation, the profit characteristics (the 95% confidence interval for mean and standard deviation) were enumerated (the calculations were done in the MATLAB). The results can be seen in Figure 1 where, for the optimal sowing plan suggested by the model and for the real decision of the farmer, the particular confidence intervals are visualized for the

Table 4. The optimization results for different settings of the risk aversion parameter

$a \times 10^6$		0–0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	Decision made (ha)
x_1	winter food wheat	531	511	416	369	340	314	288	251	231	216	188
x_2	winter feed wheat	69	69	69	69	69	69	69	69	69	69	78
x_3	spring food wheat	0	0	0	0	0	10	11	8	0	0	0
x_4	spring feed wheat	17	17	17	17	17	17	17	17	34	46	14
x_5	winter food barley	2	2	2	2	2	2	2	2	2	2	0
x_6	winter feed barley	20	20	20	20	20	20	20	20	20	20	17
x_7	spring food barley	254	254	254	254	254	254	254	254	254	254	260
x_8	spring feed barley	76	76	76	76	76	76	76	76	76	76	74
x_9	triticale	0	0	0	0	0	0	0	34	40	40	0
x_{10}	corn	20	20	20	20	20	20	20	20	20	20	29
x_{11}	corn silage	0	20	115	162	190	206	194	196	199	202	230
x_{12}	oilseed rape	131	131	131	131	131	131	131	131	131	131	231
x_{13}	potatoes	0	0	0	0	0	0	0	1	3	4	0
x_{14}	grass	60	60	60	60	60	60	98	100	100	100	79
x_{15}	grass seed	85	85	85	85	85	85	85	85	85	85	64

different risk aversion coefficients. We can see that, depending on the decision maker's risk attitude, the profit mean and the standard deviation confidence intervals resulting from the modelled optimal crop plan can be above or below those based on the real decision. There are the important conclusions following from the Figure 1.

- (1) The model reflects the risk as expected: the higher the aversion to risk, the lower is the profit mean but also the lower is the profit variance.
- (2) The profit mean performance of the model optimal solution is comparable to that obtained by the real crop plan while the profit variability performance is much better for the model solution (as it can be demonstrated e.g. for $a \times 10^6 = 0.6$ in Figure 1, where the real decision and the model solution have almost the same mean profit interval, while the standard deviation interval is considerably higher for the real decision). This means that the model considering the randomness of the harvests provides a solution generating the profit that is comparable to the one obtained so far, but with a lower risk.

The model solution provides the farmer with important information on the theoretical crop plan that

- satisfies all the theoretical crop rotation rules,
- expects the comparable profits to those obtained so far,
- suggests the combination of the crops areas that as a whole show low variability in the final profit.

In this way, the model may be used as a decision support tool when designing the crop plan. Although the farmer can occasionally decide not to follow the

crop rotation rules (which is unfeasible for the model), the model solution can be used as a help when deciding on the rest of the arable area. Indeed, the model solution aims to reach a high expected profit while keeping its variability low. Hence, adapting the model solution for the real crop plan design may improve the results of the farming by lowering the risk of the decision making.

Note that in (Ambrožová 2013) the validation was provided for three mathematical programming models optimizing the crop plan at Czech farms. The model appearing in (Janová 2012) and elaborated in detail in this article together with the models based on (Santos et al. 2010) and (Klein Haneveld and Stegeman 2005) were adapted to find the theoretical optimal crop plans for the years 2005–2011 at two particular Czech farms. The results of the validation calculation together with the farmers' expert opinion have shown that the model presented in this paper is valid in the sense of representing sufficiently the real decision making process at Czech farms and that its profit performance is very good. In conclusion, (concerning both the validation calculation and the expert opinion) the model was appointed as a helpful information source for the crop plan design at farms.

CONCLUSION

In the crop plan model presented, the harvests' randomness is considered and in addition to the common agribusiness restrictions, also the crop succession requirements are incorporated via the linear constraints. The results obtained from the model for a

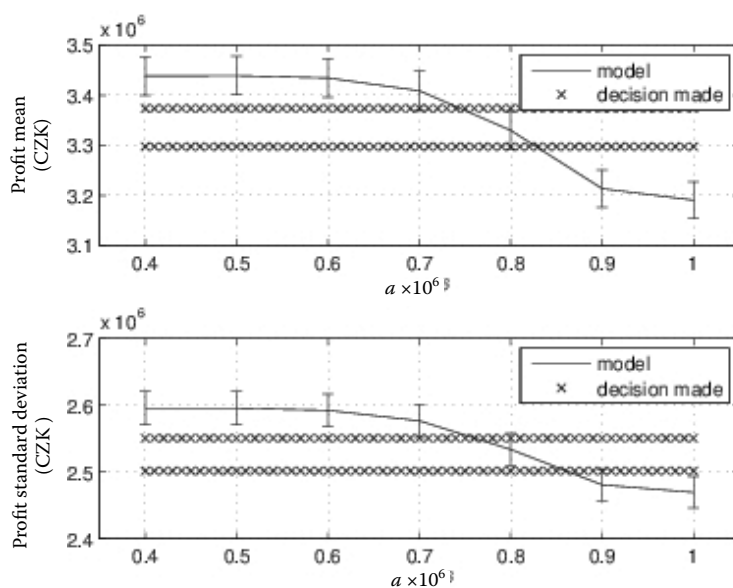


Figure 1. The confidence intervals for the profit mean and the standard deviation according to the decider's risk attitude

representative Czech Republic farm provide the areas of arable land cropped by the particular crop plants. These areas fulfil the fundamental crop rotation rules while performing the expected profit at a sufficiently high level as shown by the Monte Carlo simulation. The model does not determine the “pattern” of the land, hence the farmer himself/herself has to decide where the crops will be planted (it is, however, guaranteed by the model that such a configuration exists). The farmer can use the results of the model as a true decision supporting information enabling him/her to correct or improve the up-coming crop plan in a way that all relevant restrictions are satisfied and a good level of profit can be expected. The possibility of applying the model in the farmer’s practice is increased by the fact that the final model is solvable using an Excel spreadsheet which is commonly available at any farm. The results of this paper can be applied either directly for similar conditions in the form of the suggested crop plan model or, for different local conditions, applying the suggested model structuring supported by the recently developed validation approach may provide the decision maker with the desired valid user friendly decision support.

Appendix

Let us perform the construction of the succession constraint using a small-scale example. Let us have only four types of crops (i.e. $n = 4$). The crop 1 may be re-sowed after 2 years (i.e. $r(1) = 2$) on the same piece of land, while the crop 2 after 1 year (i.e. $r(2) = 1$) and both of them must not directly succeed the crop 3. Other two crop types may be re-sowed without restrictions (i.e. $r(3) = r(4) = 0$). Let us have 4 fields (A, B, C, D), each of them of 1 ha area and suppose there is known a 2 years history of the crop design for each field (Table 5).

For $p = 1$, we get the first set of conditions:

$$x_i \leq X - \tilde{x}_i - \tilde{y}_i \quad (15)$$

where on the left-hand side, there is the total area of land sowed by the crop i_1 , and on the right-hand side, there is the total area of the arable land available less

Table 5. Crop design history

Field	A	B	C	D
2 years ago	1	2	4	2
1 year ago	2	4	2	3

the area, where the crop i_1 was sowed during past $r(i_1)$ years, and less the area not already covered by \tilde{x}_i , where the crop prohibited in preceding the crop i_1 was sowed last year. Hence, taking into account that i_1 gradually takes all the combinations of the indexes 1, 2, 3, 4, the condition (1) represents four constraints, each of them for one of the crops planted in the farm. These constraints can be written down enumerating the values of the prohibited areas \tilde{x}_i , \tilde{y}_i from the crop history (Table 6). The constraints 1 take the form:

$$x_1 \leq 2, x_2 \leq 1, x_3 \leq 4, x_4 \leq 4 \quad (16)$$

If no other constraints entered the problem, the solution $x_1 = 2, x_2 = 1$ is feasible. However, sowing the crop 1 on the fields B and C, there is no feasible field for planting the crop 2. Therefore, another set of constraints must emerge in the problem.

Indeed, for $p = 2$, restriction (4) leads to

$$x_{i_1} + x_{i_2} \leq X - \tilde{x}_{i_1 i_2} - \tilde{y}_{i_1 i_2} \quad (17)$$

On the left-hand side, there is the total area of the arable land sowed by the crop i_1 or crop i_2 . On the right-hand side, there is the total area available less the area where both of the crops occurred during the past relevant numbers of years and less the area not included in $\tilde{x}_{i_1 i_2}$, where the crops prohibited in preceding both the crops i_1 and i_2 were sowed last year. In relation (17), coefficients $i_1 i_2$ range over all the combinations of size 2 from the set $\{1, 2, \dots, n\}$. Hence, (17) forms a set of conditions for all pairs of crops planted:

$$x_1 + x_2 \leq 2 \quad (18)$$

$$x_1 + x_3 \leq 4 \quad (19)$$

The total area cropped by the plants 1 and 2 is decreased in (18) by 1 ha, where both plants were cropped during the relevant number of years, and by another 1 ha on which the plant 3 was planted last year. The other constraints arising for $p = 2$ and all for $p = 3$ need not be written, because the crops 3 and 4 need not meet any succession requirements and all of the possibly arising constraints are included in (20) as obtained for $p = 4$:

Table 6. The values of \tilde{x}_i, \tilde{y}_i for the particular crops

Crop	1	2	3	4
\tilde{x}_i (ha)	1	2	0	0
\tilde{y}_i (ha)	1	1	0	0

$$x_1 + x_2 + x_3 + x_4 \leq 4 \quad (20)$$

In this way, the case of sowing more than one crop at the same time in the same area is prevented. Although it seems that there will be a large number of conditions in the model and handling the model will be complicated, thanks to the low real number of both the crops planted on farms and the number of years during which the crops cannot be re-sowed in the same area, the volume of the model remains reasonable.

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Received: 29th May 2013

Accepted: 19th September 2013

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