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Improving RBS estimates – effects of the auxiliary variable, stratification of the crown, and deletion of segments on the precision of estimates

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ABSTRACT: Randomized Branch Sampling (RBS) is a multistage sampling procedure using natural branching in order to select samples for the estimation of tree characteristics. The existing variants of the RBS method use unequal selection probabilities based on an appropriate auxiliary variable, and selection with or without replacement. In the present study, the effects of the choice of the auxiliary variable, of the deletion of segments, and of the stratification of the tree crown on the sampling error were analyzed. In the analysis, trees of three species with complete crown data were used: Norway spruce (*Picea abies* [L.] Karst.), European mountain ash (*Sorbus aucuparia* L.) and Monterey pine (*Pinus radiata* D. Don). The results clearly indicate that the choice of the auxiliary variable affects both the precision of the estimate and the distribution of the samples within the crown. The smallest variances were achieved with the diameter of the segments to the power of 2.0 (Norway spruce) up to 2.55 (European mountain ash) as an auxiliary variable. Deletion of great sized segments yielded higher precision in almost all cases. Stratification of the crown was not generally successful in terms of a reduction of sampling errors. Only in combination with deletion of stem segments, a clear improvement in the precision of the estimate could be observed, depending on species, tree, target variable, and definition and number of strata on the tree. For the trees divided into two strata, the decrease in the coefficient of variation of the estimate lies between 10% (European mountain ash) and 80% (old pine) compared with that for unstratified trees. For three strata, the decrease varied between 50% (European mountain ash) and 85% (old pine).

Keywords: randomized branch sampling; multistage sampling; unequal selection probabilities; auxiliary variables; pps-sampling

Randomized branch sampling (RBS) was developed by Jessen (1955) to estimate the number of fruits on a tree. Since then, this procedure of random sampling has been used for estimating discrete and continuous parameters of individual trees of different species. With the application of RBS, estimates of foliar biomass (Valentine et al. 1994; Raulier et al. 1999; Good et al. 2001), foliar surface (Mundson et al. 1999; Xiao et al. 2000) and even the entire biomass above ground (Valentine et al. 1984; Williams 1989) were obtained.

The application of the method requires the definition of *nodes* (a point at which a branch or a part of a branch branches out to form two or more sub-branches) at certain branching points and *segments* (a part of a branch between two successive nodes). The series of successive segments between the first node and the final segment, i.e. the segment at the end of which no more node is present, is called a *path*. For the selection of the segments of a path, we can define an auxiliary variable which can be measured or estimated at the segments of each node. Each selected path yields an estimate of the target parameter of the tree.

The RBS procedure can be designed in many different ways. Both, the artificial tree structure depending on the definition of nodes and segments and the auxiliary variable must be defined in advance. Not every natural branching point has to be an RBS node, and also the choice of the appropriate auxiliary variable can vary depending on the target variable. Jessen (1955) recommended, for example, the branch cross-sectional area as the auxiliary variable for estimating the number of fruits – a recommendation which agrees with the theory of Shinozaki et al. (1964a,b). This theory suggests that the amount of leaves on a tree should be closely correlated with the branch and stem cross-sectional areas. Valentine and Hilton (1977) estimated the number of leaf clusters of *Quercus* spp. They used the RBS procedure within all main branches, which were considered as strata. Each path was terminated when a single leaf cluster occurred and the visually estimated leaf biomass was defined as the auxiliary variable. Valentine et al. (1984) estimated the total (foliar plus woody) fresh weight in a mixed oak stand. They used the procedure for individual trees and defined the product of the squared diameter and the length of the branch beginning at the base of the segment, a proxy of the volume of that part of the branch, as the auxiliary variable. Each path was terminated when a diameter of 5 cm or less was encountered. The same auxiliary variable was defined by Williams (1989) in order to estimate the entire biomass above ground for loblolly pine (*Pinus taeda* L.). Only the whorls along the stem were considered as nodes and he terminated each path as soon as a branch was selected. Whenever the path selection continued along the stem, it was terminated when a stem diameter of 5 cm or less was encountered. Valentine et al. (1994) stratified the crown into thirds and used the RBS procedure within some branches in order to estimate the foliar biomass of loblolly pine. They used the squared diameter of the segment as the auxiliary variable.

RBS has been used without modifications for more than 40 years (see e.g. Gregoire et al. 1995; Parresol 1999; Good et al. 2001; Snowdon et al. 2001). During this period, there have been only smaller conceptional contributions, such as the introduction of the terms *conditional* and *unconditional* probabilities (Valentine et al. 1984). These authors also introduced an elegant mathematical nomenclature. Further, the application of stratification was suggested – a well-known strategy for variance reduction. Valentine et al. (1994) stratified the crown into three strata of constant length along the stem. Later, Gaffrey and Saborowski (1999) recommended crown sections of variable length in order to achieve smaller variances of needle biomass. It can also be meaningful to stratify the crown into a light and a shade crown (see Raulier et al. 1999).

A further suggestion for variance reduction was made by Saborowski and Gaffrey (1999) and Cancino and Saborowski (2005), respectively. They proposed the selection without replacement (SWOR) of segments at the first or second node, resulting in two modified procedures. The approach is based on the well-known fact that, with simple random samples, SWOR is more efficient than selection with replacement (SWR) (see Cochran 1977). Sampford's method (Sampford 1967) is used for sample selection.

In the publications quoted above, the authors make an ad hoc use of different auxiliary variables, the stratification of the crown and the deletion of segments. In the present study, the effects of the choice of the auxiliary variable and of the created crown structure (segments and nodes, strata) on the variance of the estimate are analyzed in more detail. Theoretical considerations for improving the precision of the RBS procedure are made and the results of an analysis using real data are presented. The analysis of the effects of the crown structure concentrates on the stratification of the crown and on the deletion of greater segments (e.g. the stem) by using the classical RBS.

Statistical foundation of the RBS procedure

The RBS procedure uses the natural branching within the tree in order to gradually select one or more series of segments (paths). The selection of a path begins at the first node by selecting one of the segments emanating from it. Then one follows the selected segment and repeats the selection if a further node exists at the end of this segment. The sequential selection is finished when no further node exists at the end of the selected segment (Fig. 1a).

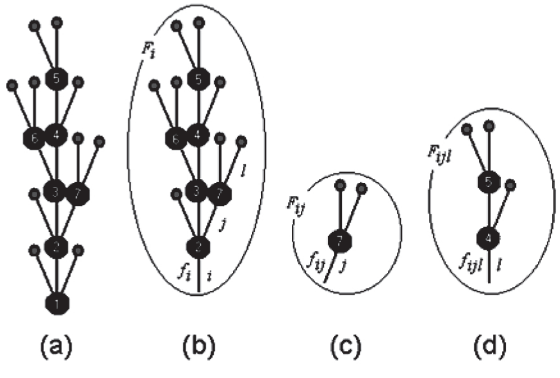


Fig. 1. (a) Scheme of a tree with 7 nodes and 16 segments. Nodes 1 to 5 form the stem. (b), (c) and (d) represent 3 levels of crown compartments, primary (*i*), secondary (*ij*) and tertiary (*ijl*) compartments, with the values of the target variable (f_i, f_{ij}, f_{ijl}) at the segments and the cumulated values (F_i, F_{ij}, F_{ijl})

RBS procedures use probabilities of selection proportional to an auxiliary variable which can be measured or estimated at the segments of a node. Thus, the (conditional) selection probability of the i^{th} segment at a certain node with N segments is given by

$$q_i = x_i / \sum_{i=1}^{N} x_i$$

where: x_i – auxiliary variable of the i^{th} segment.

Each selected path yields an estimate of the total F of the target variable, which is calculated based on the values of that variable at each segment $s = 1, ..., R$ of the path and the unconditional probability Q_s of the segment. If, for example, f_r is measured at the r^{th} segment of the path, then f_r/Q_r is the contribution of this segment to the estimate of the total of the target variable over all segments of stage r, where $Q_r = \Pi^{r}_{s=1} q_s$ and q_s are the unconditional and conditional selection probabilities, respectively, of the r^{th} and s^{th} segments of the path. The estimate of the total from a path with R segments which begins at the first node of the tree is thus

$$\hat{F} = f + \sum_{s=1}^{R} \frac{f_s}{Q_s} \tag{1}$$

since the segment before the first node with the value f is selected with probability 1.

If one randomly selects n paths with replacement, the unbiased estimate $\bar{\hat{F}}$

$$\bar{\hat{F}} = \frac{1}{n} \sum_{p=1}^{n} \hat{F}_p$$

is obtained ($\hat{F}_p$ according to equation [1]). Its variance and unbiased variance estimate are

$$Var\, \bar{\hat{F}} = \frac{1}{n} \sum_{p=1}^{N_{path}} Q_{R_p} (\hat{F}_p - F)^2 \text{ with } Q_{R_p} = \prod_{s=1}^{R_p} q_s \tag{2}$$

and

$$V = \frac{1}{n(n-1)} \sum_{p=1}^{n} (\hat{F}_p - \bar{\hat{F}})^2 \tag{3}$$

respectively,

where: R_p – number of segments of path p,
N_{path} – number of all possible paths at the tree.

As Saborowski and Gaffrey (1999) point out, the RBS procedure is a multistage random sampling procedure. The segments of a path can be assigned to subsequent stages. The segments branching from the first node correspond to the primary units and those from the second node to the second stage etc. So, a node is a transition point from a segment to the segments of the next stage and the path is a sequence of sampling units of different stages (Fig. 1a).

The classical RBS draws n primary branch segments with replacement (SWR) at the first stage and only one segment at all following stages. A clear difference compared with the general multistage procedures of random sampling is the composition of the target variable. Here, not only the units on the last stage but also the units of all superordinate stages can contribute to the target variable (see eq. [1]).

THEORETICAL CONSIDERATIONS FOR THE EFFICIENCY OF THE RBS ESTIMATE

Relationship between auxiliary and target variable

In the general context of selection with unequal probabilities, a suitable auxiliary variable is to be defined which determines the selection probability of each unit. The auxiliary variable should be easy or economical to measure or estimate and be highly correlated with the target variable. In the case of one-stage samples, using SWR as well as SWOR, the best auxiliary variable is that one which is proportional to the value of the target variable; if exact proportionality exists, the variance of the estimate equals zero and the sampling procedure is optimal (Horvitz, Thompson 1952; Hartley, Rao 1962; Cochran 1977).

The preceding statement can easily be transferred to multistage samples (Cancino 2003). (In the following, we write q_i instead of q_1, q_{ij} instead of q_2, and so on, in order to indicate the units selected on each stage: unit i on stage 1, unit j on stage 2 within the primary unit i of stage 1, etc.) It can be shown that, with RBS samples, an auxiliary variable should be used which generates strong proportional relationships between q_i and F_i, q_{ij} and F_{ij}, q_{ijl} and F_{ijl} etc. (Fig. 1); i.e., between the conditional selection probability of a segment and the cumulated values of the target variable f beyond the segment. In a three-stage selection, e.g., F_i and F_{ij} are given by

$$F_i = \sum_{j=1}^{M_i} F_{ij} \quad F_{ij} = f_{ij} + \sum_{l=1}^{K_{ij}} f_{ijl}$$

where: M_i, K_{ij} – total number of segments at the second node and the third node, respectively.

For each node, a diagram of such a strong relationship will produce a straight line through the origin based on the segments of that node. The usually large number of these diagrams is difficult to analyze in order to compare different auxiliary variables on the basis of fully measured trees. A useful approximate

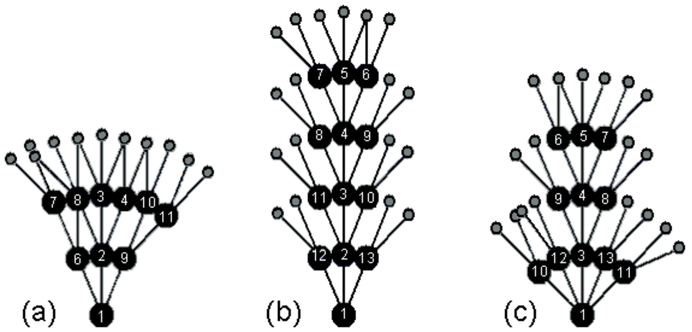


Fig. 2. Two-dimensional representation of two different crown structures: (a) regular, with paths of three segments and (b) irregular with paths of different lengths (two to five segments). (c) Deletion of the middle segment of node 1 of the tree in 2(b). (d), (e) Formation of two strata from the tree in (b). The stratification homogenizes the length of the paths. Both strata (d, e) comprise paths with only 2 or 3 segments

solution is the analysis of the relationship between the unconditional selection probabilities Q_r of all segments and the associated cumulated variable beyond each segment; i.e., between the q_i, q_iq_{ij}, etc., and the F_i, F_{ij}, etc. A stronger relationship between these variables results in estimates with high precision. Precision can be influenced by the choice of the auxiliary variable, by deleting segments, and by stratifying the crown (see the next chapter).

Crown structure, deletion of segments and stratification

The estimate from RBS samples depends both on the cumulated value of the target variable beyond a certain stage or segment and the conditional (and concomitantly, on the unconditional) selection probability of the segments of the paths. Thus, path length variability (number of segments of each path), which depends on the structure of the crown, could play a significant role for the variance of the estimate; i.e., we can reduce the variance of the estimate by appropriately changing one of these variables. In this chapter, we analyze factors that influence both the formal crown structure and the selection probability of the segments.

A rough distinction could be made between regular and irregular crowns. A regular crown consists of paths with equal lengths (Fig. 2a) and can be expected to give RBS estimates with lower variance. An irregular crown consists of paths with unequal lengths (Fig. 2b), which can cause a large variance of the estimate, because of the highly different unconditional selection probabilities of the paths. For this type of crown, it might be helpful to delete large segments, which often belong to longer paths along the stem or to stratify the crown and thereby homogenize the path lengths and hopefully reduce the variance of the estimate.

"Deletion" of large segments means that segments with high selection probabilities are selected with a probability of 1. Thus, on the one hand, these segments are measured in any case; on the other hand, it changes the structure of the crown and the categorization of segments within the crown. Secondary segments can become primary segments and tertiary segments secondary segments, etc.

When a segment of a node is deleted, the node at the end of the deleted segment is dissolved and all

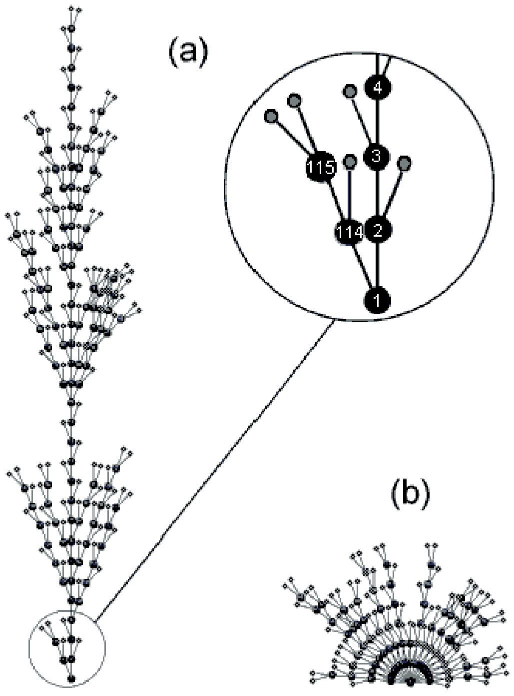


Fig. 3. (a) Two-dimensional representation of spruce 4 with and (b) without stem

Table 1. Characteristics of the measured trees

Species	Tree	Age (years)	dbh (cm)	Height (m)	Biomass	Number of nodes on stem	Number of segments	Number of paths
Norway spruce	1	14	–	0.4	16.6[a]	11	598	337
	2	16	–	–	41.6	29	623	318
	3	12	–	–	99.6	50	901	456
	4	11	–	–	11.2	34	233	119
Young Monterey pine	1	14	25.5	14.4	186.6[b]	27	164	138
	2	14	18.6	14.2	81.8	23	114	92
	3	14	14.8	16.4	25.9	7	52	46
	4	14	14.2	14.4	31.5	13	84	72
Old Monterey pine	1	29	51.5	37.6	249.8[b]	45	184	140
	2	29	51.2	33.2	1,035.9	56	198	143
	3	29	40.7	37.9	146.6	31	147	117
	4	29	36.8	40.2	277.7	53	235	183
European mountain ash	1	16	2.3	4.5	106.9[c]	23	54	28
	2	16	4.0	4.7	351.3	32	156	79
	3	26	4.5	6.9	234.8	25	114	58
	4	19	7.8	7.8	386.4	32	274	138

[a]Dry weight of needles (g), [b]fresh branch biomass (kg), [c]dry weight of leaves (g), – not available

of its segments are integrated in the preceding node. So, the number of segments at that node is increased and thereby their conditional selection probabilities changed and the paths containing the deleted segment are shortened (Fig. 2c). Moreover, the unconditional selection probabilities of all segments in the subordinated stages change. The deletion of the thickest segments, which are usually located in the lower part of the crown, affects the unconditional selection probability of all subordinated segments of the tree.

Also the stratification of the crown along the stem seems to be an efficient aid to variance reduction. It reduces the length of longer paths and changes the unconditional probabilities of all paths in all strata except the first stratum in the lower part of the crown. If we divide, for example, the crown in Fig. 2b into two strata we have to expect, at the top of the crown, a correlation between the unconditional selection probability and the cumulated value of the target variable as in the unstratified tree (Fig. 2e) because the unconditional probabilities of that stratum and those of the unstratified crown differ by the constant factor q_1q_2. In contrast, in the lower stratum, the cumulated target variable above the central segment will be remarkably reduced and consequently the interesting correlation, too (Fig. 2d). The deletion of larger segments can be an appropriate remedy.

MATERIAL

Data on complete trees of three different species were available for the analysis: spruce (*Picea abies* [L.] Karst.), European mountain ash (*Sorbus aucuparia* L.), and Monterey pine (*Pinus radiata* D. Don) (Table 1, Fig. 3a).

The data for the young spruce trees were collected in the Solling mountains (Lower Saxony, Germany). One tree was completely measured and the other trees only sampled. The missing values of the target variable "needle biomass" were estimated by regression. The base diameter of each segment is available.

The eight pine trees come from two pure, even-aged (14 and 29-years old) stands in Cholguán (VIII Región, Chile). For each tree, the position of the branch (height above ground), its length and base diameter, as well as the total weight of each fifth branch were measured. The missing weights were determined by regression, and branches located between two whorls were assigned to the nearest whorl or to an additional node.

The data for the young European mountain ashes were collected in Bärenfels (Sachsen, Germany). Diameter and leaf biomass were measured for each segment of the tree.

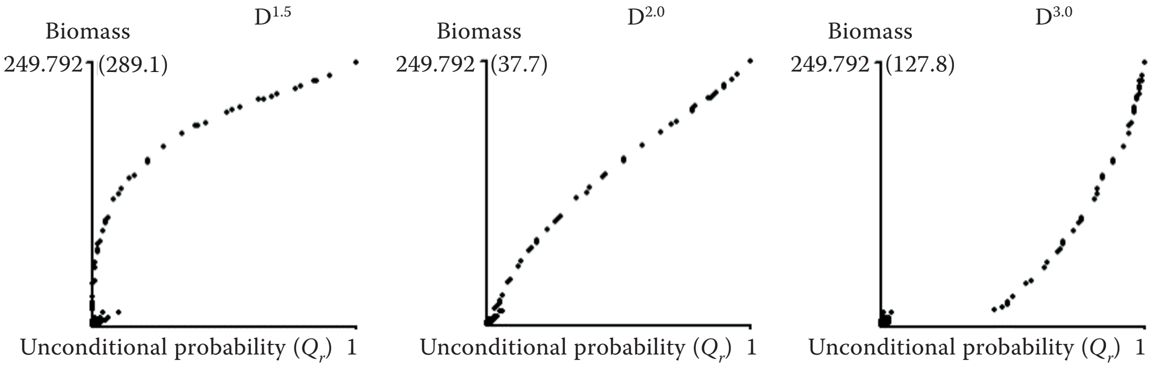


Fig. 4. Relationship between the target variable and the unconditional probabilities of the segments for different functions of the diameter (D) of the segments as the auxiliary variable for an old pine (auxiliary variable: $D^{Exponent}$). The coefficient of variation (n = 1) of the target variable (%) is given in parentheses

RESULTS AND DISCUSSION

All analyses and simulations presented in this chapter were done with the program BRANCH (Cancino et al. 2002; Cancino 2003). The analyses consider the entire population of paths of each tree (eq. [1]) and the true totals and variances of the target variables and the estimates of the totals, respectively.

Choice of the auxiliary variable and variance of the estimate

As discussed above, the relationship between the unconditional selection probabilities Q_r and the cumulated target variable beyond each segment is a helpful indicator of the precision of an RBS procedure. For the first old pine in Table 1, the relation-

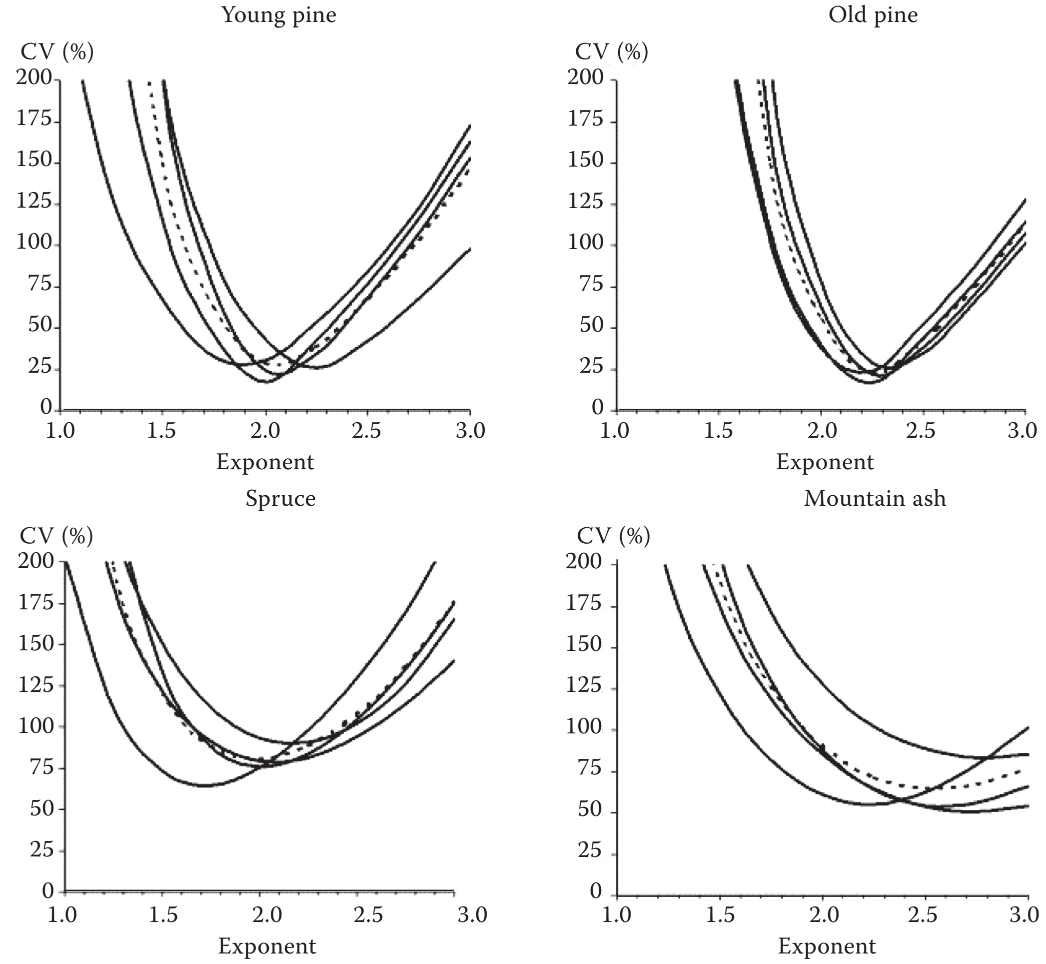


Fig. 5. Coefficient of variation (n = 1) of the estimates for different functions of the diameter (D) of the segments as the auxiliary variable (auxiliary variable: $D^{Exponent}$). Each continuous line represents a tree; the broken line represents the average of these trees

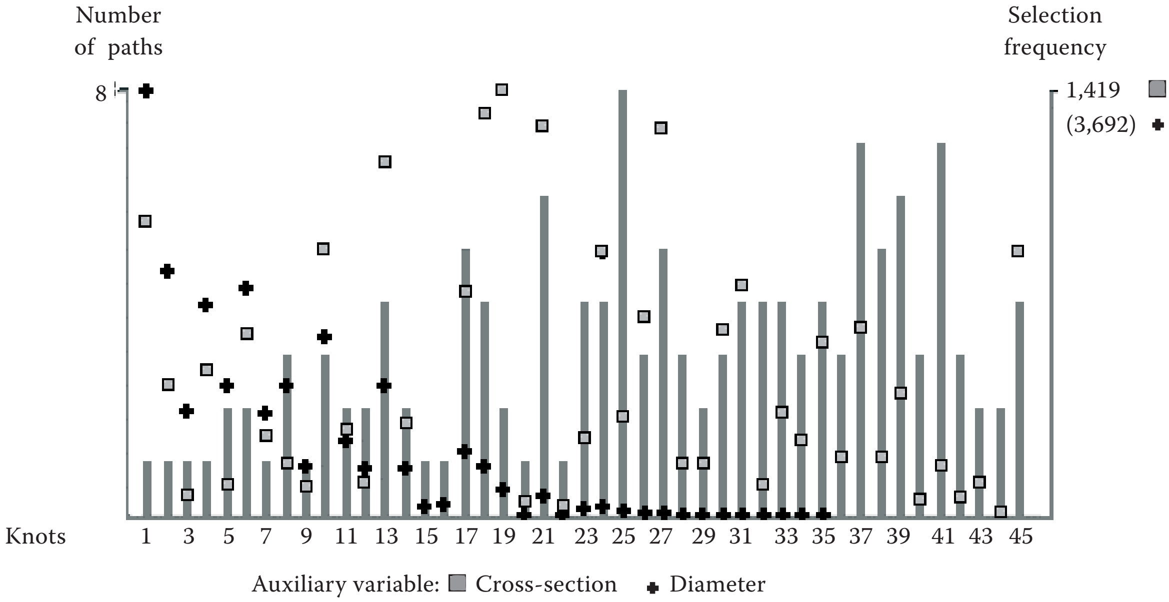


Fig. 6. Distribution of the selected paths along the stem (last node of the path on stem) of an old pine (tree 1) for two different auxiliary variables (classical RBS: 10,000 samples of size 2)

ship between Q_r and branch biomass is depicted in Fig. 4 for three functions of the diameter at the segment base as the auxiliary variable and without modifications of the crown structure. Obviously, the coefficient of variation (CV) is the lowest (37.7%) for the exponent 2.0, which yields the strongest relationship. The highest CV (289.1%) occurs for the exponent 1.5 yielding the weakest relationship.

For the old pines in general, the most precise estimates are obtained with an exponent between 2 and 2.5 (Fig. 5). The precision of the estimates shows a high variability depending on the exponents of the diameter and the tree species. The best results are obtained with an exponent of approximately 2.05 for the young pine trees, 2.25 for the old ones, 2.0 for the spruce trees and 2.55 for the European mountain ashes (Fig. 5). So, for the old pines and the ashes, the cross sectional area of the segments is clearly a suboptimal choice of the auxiliary variable.

The greatest curvature in the relationship between the coefficients of variation of the branch biomass and the exponent of the diameter was observed for

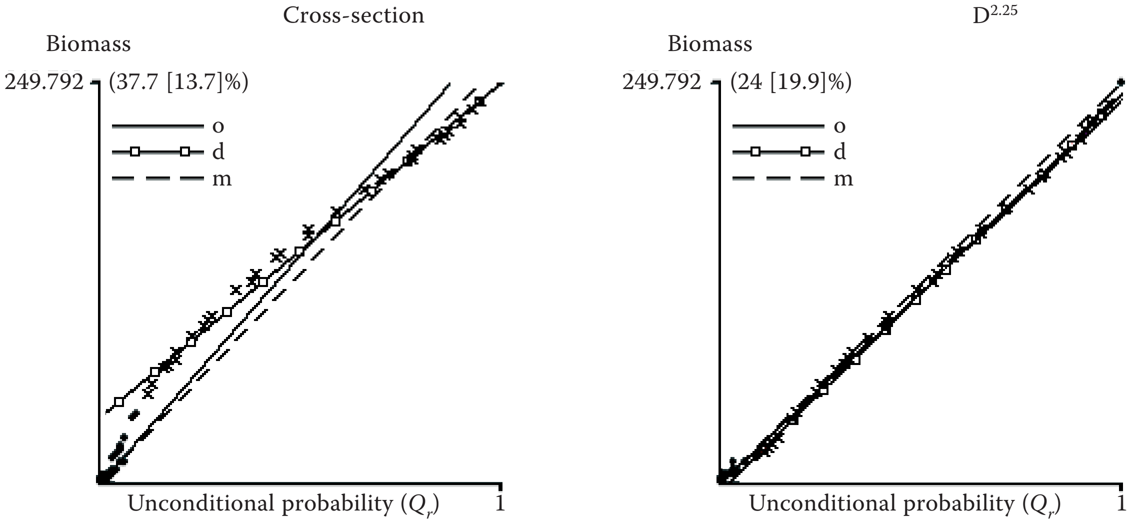


Fig. 7. The deletion of segments (*x*) based on two different auxiliary variables for an old pine (deletion for $Q_r \geq 0.1$). The lines represent the slope of the relationship between the target variable and the probability (o – original tree; d – deleted segments; m – modified tree). The coefficients of variation (n = 1) for the natural and for the modified tree, respectively, are given in parentheses

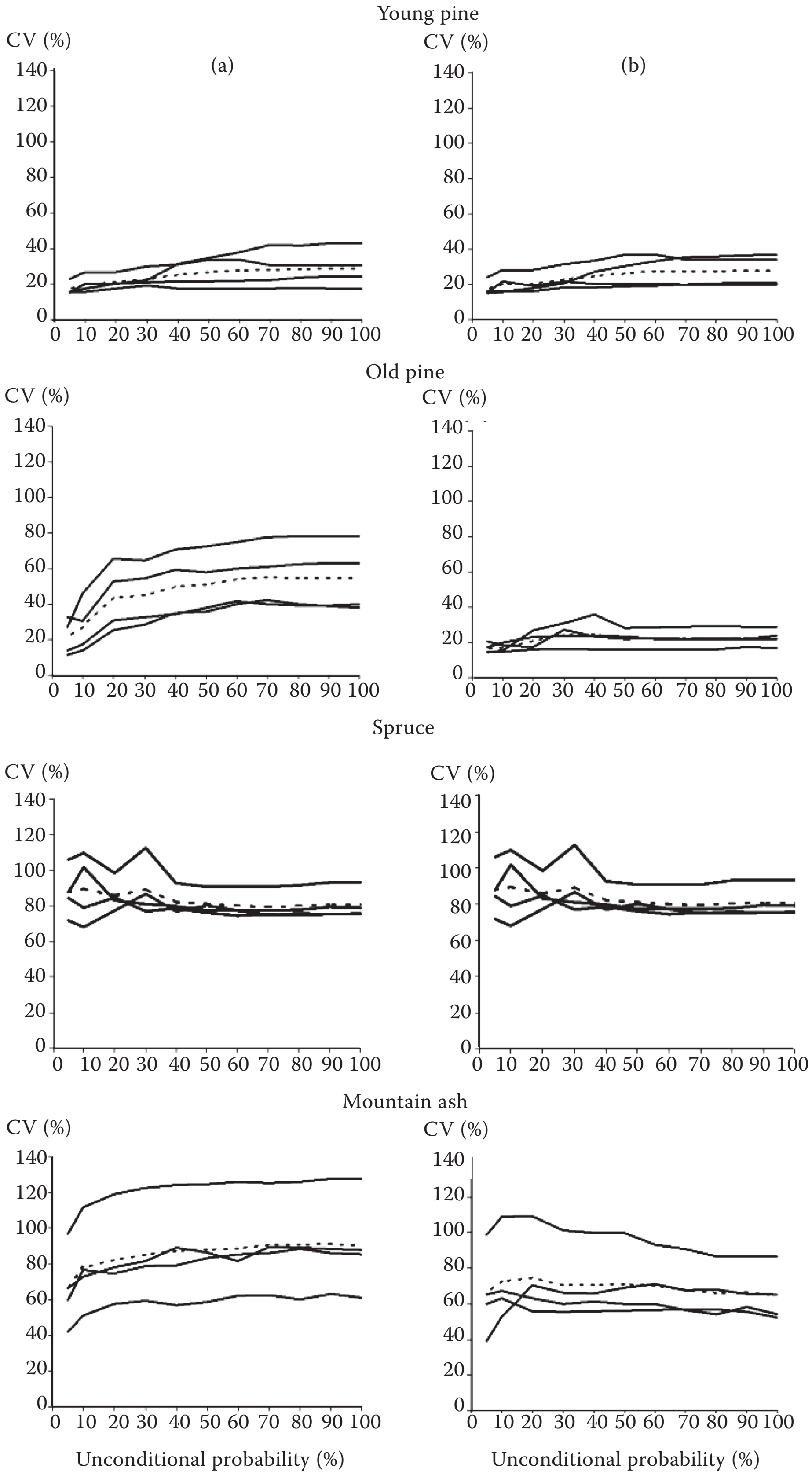


Fig. 8. Coefficient of variation ($n = 1$) of the target variable after the deletion of larger segments (auxiliary variable: (a) Cross section, (b) Diameter$^{\text{Exponent}}$; exponent: young pine, 2.05; old pine, 2.25; spruce, 2.0; European mountain ash, 2.55). Each continuous line represents a tree; the broken line represents the average of these trees

the old pines. This means that a deviation from the optimal exponent causes a bigger decrease in precision than for every other species.

The choice of the auxiliary variable also affects the distribution of the samples within the crown. According to Fig. 6, the cross section, an auxiliary variable closely related to the fresh branch biomass (Fig. 4), causes a more homogeneous distribution of the samples along the whole stem of old pine 1 than the diameter, which is only weakly related to the target variable. The diameter as auxiliary variable distributes the samples predominantly in the lower range of the stem (Fig. 6).

Deletion of larger segments

The deletion of segments changes the structure of the crown and causes a set of effects which can be explained by the altered selection probabilities of the segments. For the pine of Fig. 7, using the cross section as the auxiliary variable, the segments with a larger unconditional selection probability (i.e. mainly the segments of the stem) do not exhibit the same relationship between the target variable and the unconditional selection probability as the smaller segments. $D^{2.25}$ as auxiliary variable produces a strong linear relationship and yields more precise estimates (CV = 24% instead of 37.7%). However, after deletion of segments with $Q_r \geq 0.1$ the cross section is a more effective auxiliary variable (CV = 13.7% instead of 19.9%). Thus, the deletion of segments can even affect the choice of the optimal auxiliary variable.

The increased precision by deletion of larger segments is a direct result of the changed probability distribution of the estimator. In the example of the old pine the distribution is changed from a u-shaped to a unimodal distribution. Particularly for the longest paths along the stem, which generally yield the highest estimates because of their low selection probabilities Q_R (many segments!), deletion increases these selection probabilities more than for shorter paths, where only few of the lower stem segments are deleted. Therefore, the number of extremely large estimates tends to be reduced. These changes clearly lead to a smaller variance of the estimate.

The effect of the deletion of segments depends both on the species and on the auxiliary variable. When the cross section is used as the auxiliary variable, the CV decreases with increasing deletion intensity beginning at the upper end of selection probabilities for all trees except the spruces (Fig. 8a). The CV was usually smaller when using an approximately optimal auxiliary variable instead of the cross section as the auxiliary variable. This occurs independently of the degree of deletion of segments (compare Figs. 8a,b) but with some exceptions, such as the old pine 1 (Fig. 7). When the optimal exponent was used, the coefficient of variation for the pines was only slightly reduced by the deletion of segments; there is no clear decrease for spruces and mountain ashes.

The higher the intensity of deletion (e.g. deletion with $Q_r \geq 0.05$), the smaller the differences between the coefficients of variation of the target variable (Figs. 8a,b). For the highest deletion intensity, the differences between the CVs using cross section and optimal auxiliary variable vanish.

All effects of the deletion of larger segments described above can be referred as positive or at least as indifferent. However, there are also negative effects. In practice, the target variable at the deleted segment must be measured and later be added to the estimate if the segment contributes to the target variable (e.g. wood biomass). Therefore, there is a mandatory measurement of the target variable at the deleted segments, which will cause higher expenditure of time. Moreover, more time must be spent in order to capture the auxiliary variable of all segments that form the new larger node. Of course, the drawback represented by that mandatory measurement depends on the target variable and its distribution on the segments of the tree. When, for example, the branch biomass of the old pines is analyzed, the deletion of the stem segments is clearly advantageous.

Stratification of the crown combined with the deletion of larger segments

The stratification of the crown means a formation of at least two strata the size and variability of which are important for the precision of the estimate. The larger the stratum, the greater is the variation among units. Thus, a suitable allocation of the crown is sought which reduces the variance of the estimate. For practical reasons the tree crowns were stratified according to stem sections. All nodes and segments of a stem section and their subordinated nodes and segments form one stratum.

Generally, the following rule applies for non-stratified trees: the longer the path, the larger is the estimate of any target variable. Thus, we can expect larger estimates and higher variability at the upper end of the crown than within its lower parts (Fig. 9a).

Stratification shortens all paths of the upper strata, increases their selection probabilities and decreases the related estimates (Figs. 9b,c) and their variability. All paths of the unstratified tree that ended before the last node of the lowest stratum remain unchanged. Nevertheless, those original paths that

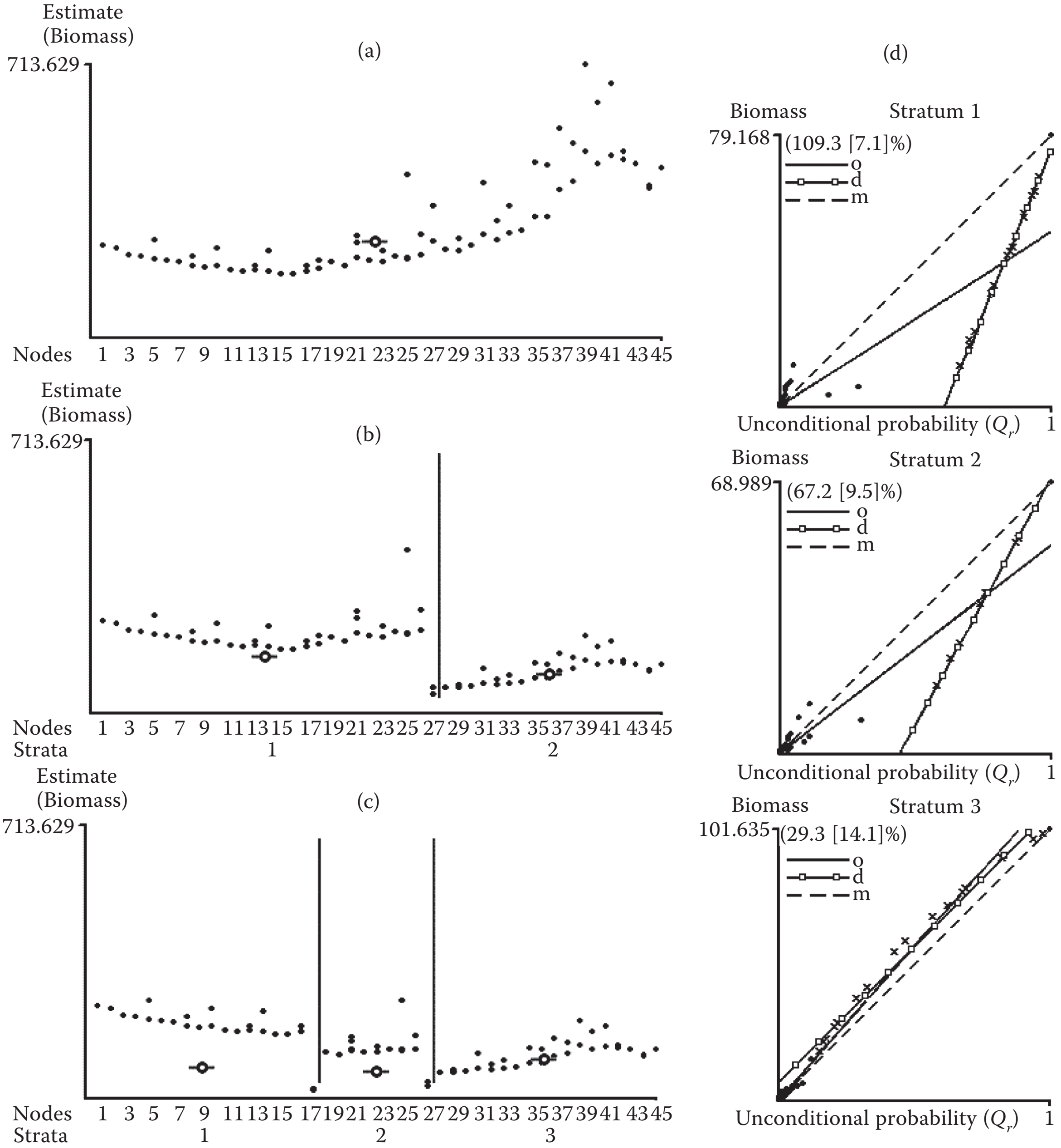


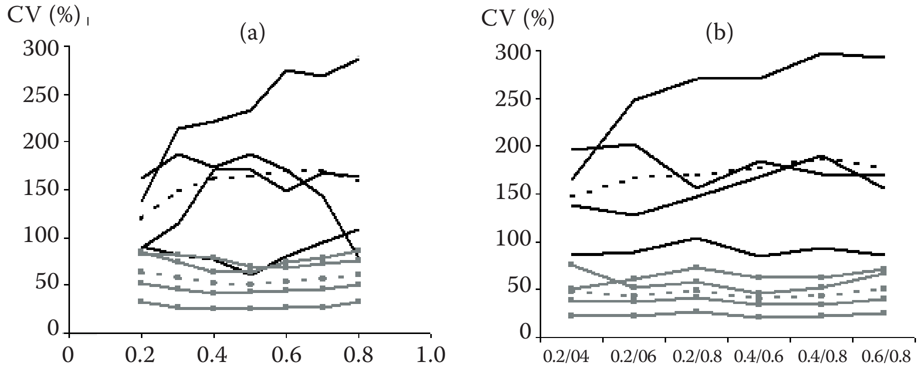
Fig. 9. (a) Estimates along the stem of an old pine and effect of the stratification of the crown into two (b) and three strata (c). The stratification was realized along the stem. The symbol –o– (in a, b and c) represents the current total of the target variable of the tree or stratum. (d) Deletion of segments in the strata of (c). The lines represent the slope of the relationship between the target variable and the unconditional selection probability (o – original tree; d – deleted segments; m – modified tree). The coefficients of variation (n = 1) for the natural and modified tree are located in parentheses (auxiliary variable: cross section)

ended further above are now cut at the last node of the lowest stem section. Now they have less segments and therefore higher selection probabilities as well as lower cumulated values of the target variable and can easily be recognized in Figs. 9b,c at the nodes 27 (b), and 18 and 27 (c).

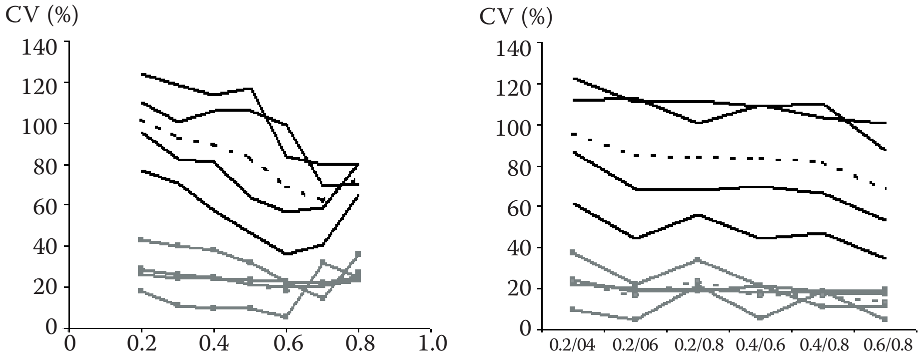
Within the strata, the relationship between the unconditional selection probability and the cumulated target variable is completely altered, in particular for the lower strata (compare Figs. 7 and 9d) where it is far from being optimal. In the upper stratum (stratum 3), both the strength of the relationship and the CV of the estimate (29.3%) are comparable to the unstratified tree. The CV of the overall estimation increases from 37.7% (unstratified) to 41.1% (stratified into three strata). Without a close look at the key relationships in Fig. 9d, this would have been a surprising result because usually stratification is expected to yield lower sampling errors.

Deletion of stem segments can be suggested to solve this drawback. According to Fig. 9d, the CVs within the strata are reduced to 7.1% (stratum 1), 9.5% (stratum 2) and 14.1% (stratum 3); CV of the

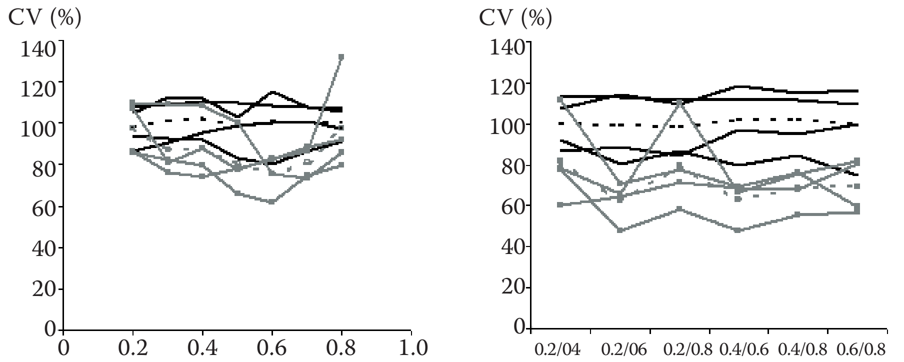
Young pine



Old pine



Spruce



Mountain ash

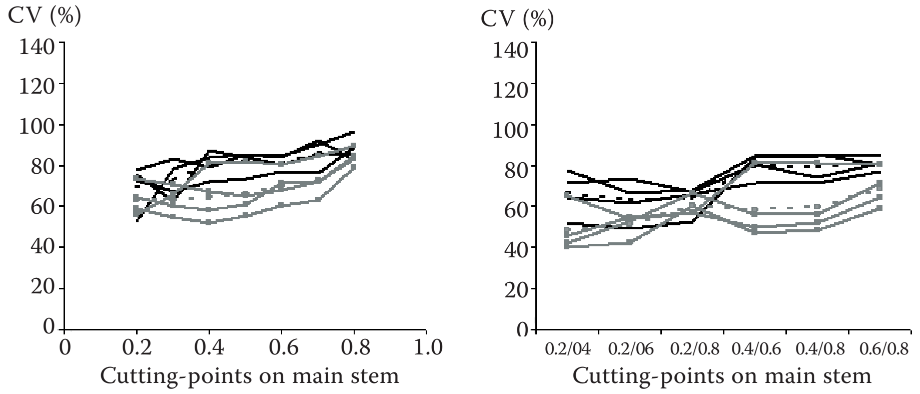


Fig. 10. Coefficient of variation ($n = 1$) of the target variable for the tree with (black) and without a stem (grey lines) after the division of the crown into (a) two and (b) three strata (compared to the coefficient of variation without stratification). Each continuous line represents a tree; the broken line represents the average of these trees. The coefficient of variation of the target variable of the complete tree was considered as 100% (auxiliary variable: cross section)

total fresh branch biomass reduces to 6.7% after deletion of the stem segments.

After this closer look at the old pine, the effect of stratification and deletion of the stem segments on the precision of estimates is to be studied for all trees of the database. The effect varies broadly between the species when the crown of tree is cut into two (Fig. 10a) or three strata (Fig. 10b). When using RBS sampling, the effect of the stratification can be positive, negative or indifferent, as a function of species, tree and cutting point at the stem.

For trees divided into two strata and without deleting the stem segments, more precise estimates are observed for European mountain ashes and old pines. For the young pines, the estimate for three of the stratified trees was worse than for the respective unstratified trees. The CV is minimized when the lowest 20% of the nodes at the stem are assigned to the first stratum, but still 20% higher than for the unstratified trees. The relationship between the selection probability and the target variable is weak within the first stratum. For the spruces, the stratification produced nearly the same coefficient of variation as for the non-stratified trees. Here, the variability of the estimate is independent of the cutting point.

The stratification decreases the coefficient of variation of the branch biomass for the old pines. For these long-crown trees, the coefficient of variation is reduced by nearly 40% when the crown is split at 70% of the number of nodes. For the ashes, the coefficient of variation decreased between 10% and 30%. The greatest reduction was achieved when the crown was split at 20% of the number of nodes along main stem.

The deletion of stem segments for the stratified trees increased the precision of estimates. The estimate of the target variable without the stem was always more precise than with the stem for all species. Optimal cutting points for the ashes are at the lower end of the stem, for spruces and old pines at the higher end, and for young pines in the middle of stem.

For three strata, the same tendencies can be observed as for two strata (Fig. 10b). Again, compared to the unstratified trees, more precise estimates with three strata for all species can be obtained only if the stem segments are deleted. The best combination of the two cutting points was indifferent for the young pines, at 60% and 80% of the stem height for the old pines, at 20% and 60% or at 40% and 60% for the spruces and at 20% and 40% for the mountain ashes. Generally, stratification with three strata yields slightly better results than using two strata.

CONCLUSION

The relationship between the unconditional selection probability of segments and the cumulated values of the target variable beyond the segments was shown to be a helpful diagnostic tool for a rapid comparison of different RBS designs. This tool, among others, is offered by Branch, a Delphi programme that can be downloaded together with instructions and two tree data sets[1].

The detailed analyses of trees of different species revealed that stratification of tree crowns does not necessarily increase the precision of estimates of crown parameters if RBS with varying selection probabilities is used. This is a result of the new crown structure after stratification, which affects the relationship between selection probabilities and target values in the unstratified tree. A clear positive effect of stratification on the precision of estimates could only be obtained by an additional deletion of stem segments, which usually have a higher selection probability than the branch segments at a node.

For the target variables considered in this study, fresh branch biomass and dry weight of needles and leaves, the squared diameter performed well as an auxiliary variable, particularly after deletion of larger segments. Other powers of the diameter can be assessed in practice by a preliminary analysis of sample trees; this can be carried out using the Branch software. This is also valid for the number and sizes of strata and the deletion of larger segments. At least for trees with long crowns such as the old pines, stratification with two or more strata together with a deletion of stem segments seems to be essential to reduce the variation of target variables.

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Zlepšení odhadů metodou RBS – vliv přídavné proměnné, stratifikace koruny a vynechání segmentů na přesnost odhadu

ABSTRAKT: Randomized Branch Sampling (RBS) je vícestupňová výběrová metoda používající přirozené větvení ke stanovení výběrového souboru použitelného k odhadu stromových charakteristik. Existující varianty RBS používají nestejné výběrové pravděpodobnosti, založené na vhodné přídavné proměnné, a je používán výběr s opakováním nebo bez opakování. Článek analyzuje vliv výběru přídavné proměnné, odstranění segmentů a stratifikace koruny na velikost výběrové chyby. Pro analýzu byly využity stromy tří dřevin, u kterých byly známé kompletní údaje o koruně: smrk ztepilý (*Picea abies* [L.] Karst.), jeřáb ptačí (*Sorbus aucuparia* L.) a borovice montereyská (*Pinus radiata* D. Don). Výsledky jasně indikují, že výběr doprovodné proměnné ovlivňuje jak přesnost odhadu, tak i rozložení vzorků v rámci koruny. Nejmenšího rozptylu bylo dosaženo při použití tloušťky segmentů (D) jako přídavné proměnné při použití mocniny od hodnoty 2,0 (smrk ztepilý) až do hodnoty 2,55 (jeřáb ptačí). Odstranění velkých segmentů vedlo téměř ve všech případech k vyšší přesnosti. Naopak není možné konstatovat, že by stratifikace koruny obecně vedla ke snížení chyby; jasného zlepšení přesnosti lze dosáhnout pouze kombinací s odstraněním segmentů kmene, přičemž záleží na dřevině, stromu, cílové proměnné a na definici a počtu strat (oblastí) v koruně. Pro stromy s korunou rozdělenou do dvou oblastí (strat) se pokles variačního koeficientu pohybuje od 10 % (jeřáb) do 80 % (stará borovice) ve srovnání s nestratifikovanými korunami. Pro stromy s korunami dělenými do tří oblastí se pokles pohybuje mezi 50 % (jeřáb) a 85 % (stará borovice).

Klíčová slova: randomized branch sampling; vícestupňový výběr; nestejné výběrové pravděpodobnosti; přídavná proměnná; pps-sampling

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