Exponential model of the Engel curve: Application within the income elasticity analysis of the Czech households' demand for meat and meat products

Exponenciální model Engelovy křivky: aplikace při analýze příjmové pružnosti poptávky českých domácností po mase a masných výrobcích

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Abstract: The paper is focused on the economic and mathematical analysis of the Engel demand model in the exponential form: $q = a_0 \times e^{a_1/X}$. Properties of this exponential model are studied with respect to its application possibilities in the field of evaluation of the income elasticity of the Czech households' demand for meat and meat products. According to the used database of the consumer behaviour of the average Czech household from 1995 to 2000 (CZSO-HES), the analysed exponential model of the Engel curve attained the following parameters: $Q_t = A_t \times e^{-17336.8908/X_t}$, where $A_t = 44.6019 \times e^{1.1119 \times 10^{-4} \times t^2}$ and $t = 1, 2, ..., 24$. For the analysis of the income-demand elasticity of the developed exponential form, the model offers the static hyperbolic function: $\eta(X_t) = 17\ 336.8908/X_t$. The derived hyperbolic function of the income-demand elasticity falls digressively and the simulated values tend to the zero level. In analysed time period (1995–2000), the income-demand reactions were simulated in the elastic form with the values from 1.3866 to 1.1340. The average level of the analysed income-demand elasticity between the observed years reached the value of 1.2121, thus the 1% rise in the real level of the quarter households' incomes per capita led to the average increase in the average Czech household's demand for meat and meat products, including fish and fish products, of about 1.21%.

Key words: exponential Engel model, explicit dynamic model, income demand elasticity, demand for meat and meat products

Abstrakt: Článek je zaměřen na ekonomicko-matematický rozbor exponenciálního modelu Engelovy křivky vymezeného ve tvaru: $q = a_0 \times e^{a_1/X}$. Vlastnosti této konstrukce Engelova modelu jsou analyzovány s ohledem na možnosti jeho využití při analýze příjmové pružnosti poptávky českých domácností po mase a masných výrobcích. Na základě získaných čtvrtletních údajů ze ČSÚ – Statistika rodinných účtů za roky 1995 až 2000 bylo zjištěno, že pro kvantitativní vyjádření příjmových vztahů v poptávce průměrné české domácnosti po mase a masných výrobcích včetně ryb lze použít explicitně dynamický exponenciální Engelův model ve tvaru: $Q_t = A_t \times e^{-17336,8908/X_t}$, kde $A_t = 44,6019 \times e^{1,1119 \times 10^{-4} \times t^2}$; $t = 1, 2, ..., 24$. Uvedená exponenciální konstrukce modelu s dynamicky vyjádřenou úrovňovou konstantou poskytuje pro analýzu citlivosti příjmově-poptávkových reakcí hyperbolickou funkci pružnosti: $\eta(X_t) = 17\ 336,8908/X_t$. Vydefinovaná funkce příjmové pružnosti poptávky je na rozdíl od výchozího exponenciálního Engelova modelu je ve všech svých parametrech statická. Hodnota příjmové elasticity podle této funkce závisí pouze na velikosti příjmů průměrné české domácnosti. Odvozená funkce pružnosti v aplikačním intervalu: $X \in (0;+\infty)$ degresivně klesá. Konvexně klesající průběh hodnot příjmové elasticity poptávky bude se zvyšujícími se příjmy domácnosti konvergovat k nule. Naopak při velmi nízkých příjmech blížích se k zprava k nule bude získaná funkce pružnosti vracet nekonečně vysoké hodnoty příjmové elasticity dané poptávky. Přesně jednotkové příjmové elasticity poptávky průměrné české domácnosti po mase a masných výrobcích včetně ryb bude dosaženo při úhrnu čtvrtletních příjmů na osobu v této domácnosti ve výši: $X_L = 17\ 336,8908$ Kč. Při čtvrtletních příjmech $X^* = 8\ 668$ Kč/osobu, tj. místo, kde exponenciálně nasimulovaná Engelova křivka přechází z progresivního růstu

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do degresivního, je příjmová elasticita poptávky rovna dvěma. Jestliže si pak rozlišíme v rámci aplikačního intervalu uvažovaného exponenciálního Engelova modelu následující subintervaly: $X \in (0; 8668\rangle$; $X \in (8\,668; 17\,377\rangle$; $X \in (17\,377; +\infty\rangle$ bude v nich odvozená monotónně klesající funkce pružnosti nabývat postupně těchto hodnot: $\eta(X_t) \in (+2; +\infty)$; $\eta(X_t) \in (+1; +2)$; $\eta(X_t) \in (0;+1)$. Ve sledovaném období 1995 až 2000, kdy reálné čtvrtletní příjmy na osobu v průměrné české domácnosti dosahovaly hodnot z intervalu $\langle 12\,503; 15\,288\rangle$, byly prostřednictvím sestaveného exponenciálního Engelova modelu nasimulovány příjmově pružné reakce v poptávce českých domácností po mase a masných výrobcích. Hodnoty takto nasimulované příjmové pružnosti se pak pohybovaly v rozmezí od 1,3866 do 1,1340. Průměrná úroveň příjmové elasticity byla ve vymezeném období rovna 1,2121. Mezi zkoumanými roky 1995 až 2000 lze tudíž zkonstatovat, že zvýšení reálného příjmu na osobu v průměrné české domácnosti o 1% se odrazilo v nárůstu nakupovaného masa a masných výrobků včetně ryb průměrně o 1,21%.

Klíčová slova: exponenciální Engelův model, explicitně dynamický model, příjmová elasticita poptávky, poptávka po mase a masných produktech

INTRODUCTION AND THE AIM OF THE PAPER

The sensitivity of income-demand reactions is frequently evaluated through the elasticity coefficients. These coefficients record the relationship (ratio) between the quantity changes of the studied demand and the income changes of the consumer subject or subjects and their values are interpreted in percentage terms, see Maurice, Phillips (1992). The percentage (thereby dimensionless) definition of the income elasticity coefficients enables comparison of the income-demand reactions within different categories of the consumption. These dimensionless coefficients may also be used for comparison of subject behaviour within different consumer groups, see Banks et al. (1996), Mc Dowell et al. (1997), or Syrovátka (2001).

The evaluation of the income elasticity of consumer demand is a multi-aspect problem. If the elasticity coefficients are based on the estimated model of the Engel curve, then the mathematical construction of the used model plays a very important role. This fact is not sufficiently respected in a lot of econometrical analyses of income-demand relationships. Unfortunately, we can find the problem in other quantitative analyses from other economic fields too.

The paper is focused on the analysis of theoretical properties of the exponential Engel model with a fractional exponent and on the possibilities of its use within the simulations of consumer income-demand relationships. A particular emphasis of the paper was laid on the use of the exponential construction of the Engel model for the evaluation of income-demand elasticity. The properties of the exponential Engel model were investigated in the parametric way so that the achieved findings would be valid in general. The application of the exponential form of the Engel demand model with the fractional exponent was examined in the fields of the Czech households' demand for meat and meat products.

PROPERTIES OF THE EXPONENTIAL ENGEL MODEL AND INCOME ELASTICITY FUNCTION

Let us suppose that the income-demand model is given in the exponential form with the fractional exponent:

$$Q = a \times e^{b/X} \quad (1)$$

Dependent variable Q in Model (1) represents the quantity of consumer demand. The income level of the consumer subject is independent variable X in this model. Parameters in Model (1) are described as a and b. For the estimations of parameters based on OLS technique, the introduced exponential model of the Engel curve must be transformed into the additive form:

$$\ln Q = \ln a + b/X \quad (2)$$

The exponential Engel model in form (1), or more precisely in form (2), gives the hyperbolic function of the income elasticity (3):

$$\eta(X) = -b/X \quad (3)$$

In relation to the expected use of the studied exponential construction, we can reduce the next analysis only to the cases of Model (1), where $a > 0$. If the exponential Engel model (1) has simultaneously also $b > 0$, then we can simulate the income-demand reactions in the field of buying of inferior goods. The exponential model of Engel curve in term (1), where the parameters are $a > 0 \wedge b > 0$, is showed in Figure 1. In this figure, the behaviour of the derived function of the income elasticity is depicted as well.

Figure 1 illustrates that the exponential construction (1) by parameters $a > 0 \wedge b > 0$ in the application interval: $X \in (0;+\infty)$ fits for the simulation of Engel curves in the sphere of inferior goods. In this interval of incomes, the investigated exponential form (1) gives back the positive levels of the shaped demand, i.e. $Q(X) > 0$. If the exponential Engel model (1) is to be used in practice, the low endpoint of the application interval must be adjusted, because the supposed exponential function is not defined at the zero level of incomes. The demanded quantity at the incomes tending to the zero level ($X \to 0^+$) may be, under introducing of Model (1) by parameters $a > 0 \wedge b > 0$, implied by Limit (4):

$$\lim_{X \to 0^+} \left[a \times e^{b/X} \right] = +\infty \qquad (4)$$

With regard to the abovementioned findings, we will redefine the application interval for the exponential Engel model (1) onto the interval of incomes: $X \in X_s;+\infty)$, where $X_s > 0$ and the demanded quantity at this income level, $Q(X_s)$, is reasonable for the given sphere of consumer purchases. In the income interval: $X \in (X_s;+\infty)$, under the Exponential model (1) by parameters $a > 0 \wedge b > 0$, we achieve the descending convex Engel curve tending to the absolute member of the given model (1):

$$\lim_{X \to +\infty} \left[a \times e^{b/X} \right] = a \qquad (5)$$

Thus the exponential Engel model in Shape (1) simulates saturation of the consumer demand for inferior goods in the asymptotic way, Limit (5). The exponential construction (1) by parameters $a > 0 \wedge b > 0$ is not able to simulate the initiate stage of consumer demand, but it is not a relevant restriction for the simulation of purchasing of inferior goods.

When using the exponential Engel model (1) by the parameters: $a > 0 \wedge b > 0$ within the income interval: $X \in (X_s;+\infty)$, we will get negative values of the income-demand elasticity, ranging from $\eta_s = \eta(X_s)$; $\eta_s < 0$ to $\eta(X) = 0$. The zero level of the income elasticity of the simulated demand is theoretical only, because we attain this elasticity level at the infinite consumer income, see the following limit:

$$\lim_{X \to +\infty} \left[-b/X \right] = 0 \qquad (6)$$

In the relation to mentioned Limits (5) and (6), we could define that the zero income elasticity of the simulated demand is obtained at the consumer saturation with related goods. The income elasticity function (3) within interval: $X \in (X_s;+\infty)$ rises concavely and tends to the value of a parameter.

In view of the practical introduction of the exponential Engel model in Form (1), it is necessary to examine the second specification of the given model, but with parameters $a > 0$ and $b < 0$. This combination of parameters of the studied Model (1) is showed in Figure 2. In Figure 2, the behaviour of the derived income-elasticity function is in parallel displayed too.

The exponential construction of the Engel demand model (1) under parameters $a > 0 \wedge b < 0$ is theoretically suitable for the simulation of income-demand reactions more or less within all categories of normal goods, i.e. necessary, relatively necessary, luxury goods. These possibilities are based on the potential values of the income elasticity function (3), see next findings. In view

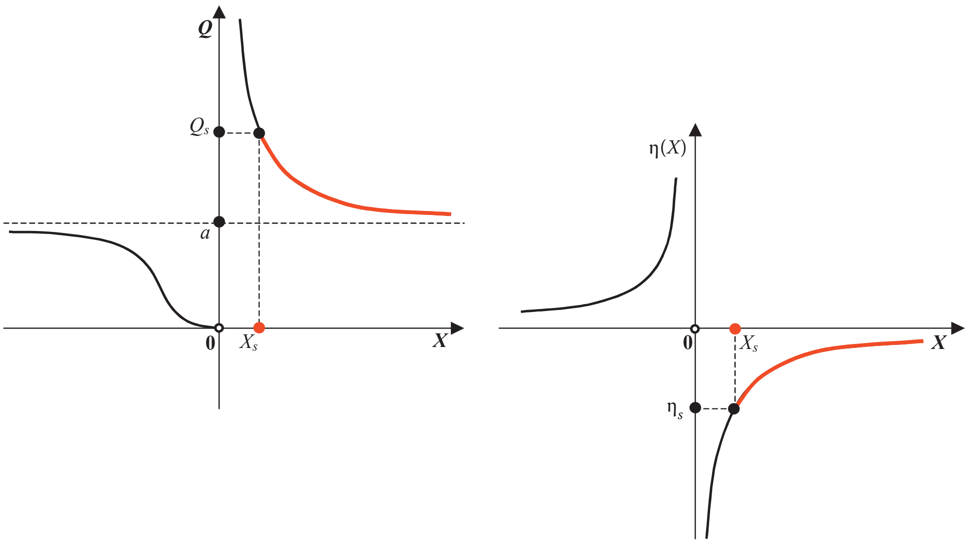

Figure 1. Exponential model (1) by parameters $a > 0 \wedge b > 0$ and behaviour of the income elasticity function (3)

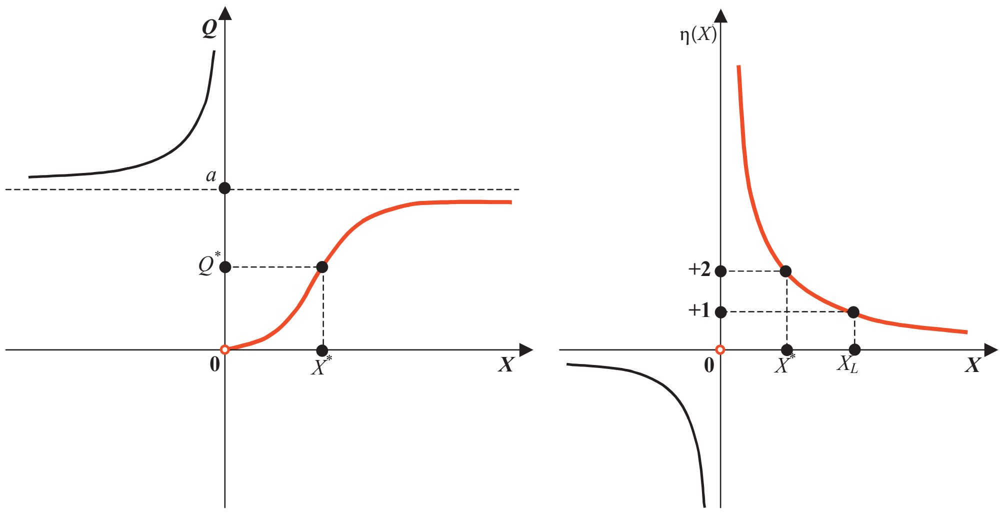

Figure 2. Exponential model (1) by parameters $a > 0 \wedge b > 0$ and behaviour of income elasticity function (3)

of economic applications, the exponential Engel model (1) by parameters $a > 0 \wedge b < 0$ has the almost unlimited application interval. Only zero income presents the exception in this aspect, because Exponential model (1) is not defined at the zero income level. The limit of the right and the left is different:

$$\lim_{X \to 0^+} \left[a \times e^{b/X}\right] = 0 \wedge \lim_{X \to 0^-} \left[a \times e^{b/X}\right] = +\infty$$

The application interval of the exponential Engel model (1) under parameters $a > 0 \wedge b < 0$ may be specified as the income interval: $X \in (0;+\infty)$, thus analogically as in the first studied case of Model (1). The exponential construction of the Engel model simulates the positive levels of consumer demand within the given income interval. In the application interval, the exponential Engel model (1) under parameters $a > 0 \wedge b < 0$ is partially restricted for simulations in the sphere of consumer demand for luxury or some relatively necessary goods, because the zero stage of demand is not available under the given specification of parameters. To eliminate the model's shortcoming, we can introduce the artificial initial level of income (X_s). The interpretation of X_s level is analogical as under the Exponential model (1) by parameters $a > 0 \wedge b > 0$. The investigated exponential model of the Engel curve simulates the saturation of consumer demand asymptotically, see Limit (4) under parameters $a > 0 \wedge b < 0$. According to Limit (4), the demand saturation is achieved at the infinite level of consumer income and the quantity of saturation corresponds to the absolute parameter a in Model (1). Within the simulation of the normal income-demand reaction, it is important that the shaped Engel curve has two distinguishable stages. Firstly, the simulated Engel curve increases progressively and then the curve increases digressively. We can detect the change in the increment of the Engel curve at the X^* income. From the mathematical view, the simulated Engel curve comes from the convex-growth stage to the concave-growth stage. Thus, the X^* level is a horizontal coordinate of the inflexion point of the shaped Engel curve. The value of X^* may be determined through the second-order derivative rule: $\partial^2 q / \partial X = 0$. In the case of the studied exponential model (1), we achieved the following equation:

$$\frac{\left(a \times b \times e^{b/X}\right) \times \left(b + 2 \times X\right)}{X^4} = 0 \tag{7}$$

Equation (7) is held for the income level:

$$X^* = -b/2 \tag{8}$$

After the behavioural analysis of the Engel curve given by the exponential model in form (1) by parameters $a > 0 \wedge b < 0$, we can study the properties and behaviour of the related function of income-demand elasticity (3). The derived elasticity function (3) by parameter $b < 0$ convex falls within the application interval: $X \in (0;+\infty)$. The hyperbolic decreasing of the simulated income-demand elasticity tends to the zero level:

$$\lim_{X \to +\infty} \left[-b/X\right] = 0 \tag{9}$$

The second theoretical outer value of the income-demand elasticity may be defined by means of the next limit of the Elasticity function (3). If $b < 0$, then we obtain:

$$\lim_{X \to 0^+} \left[-b/X\right] = +\infty \tag{10}$$

In the application interval: $X \in (0;+\infty)$, the exponential construction of the Engel model (1) by parameters $a > 0 \wedge b < 0$ provides positive values of the income elasticity, ranging from $+\infty$ to 0. According to the identified range of the income elasticity values, it is evident that the exponential Engel model (1) by parameters $a > 0 \wedge b < 0$ can simulate the elastic income-demand reactions: $\eta(X) > 1$ as well as inelastic income-demand reactions: $\eta(X) < 1$. From this point of view, it is useful to determine the income level (X_L) at which the value of income-demand elasticity is equal to plus one: $\eta(X_L) = +1$. This brake income is specified for the studied construction of the Engel model (1) by term:

$$X_L = -b \tag{11}$$

Thus, under the application of the exponential Engel model (1) by parameters $a > 0 \wedge b < 0$, we will be able to attain within the income interval: $X \in (0;X_L)$ the values of income-demand elasticity greater than +1. On the contrary, within the interval of incomes: $X \in \langle X_L;+\infty)$, under Model (1), we will be able to simulate the income elasticity +1 or lower.

In addition to the values of the income elasticity within the specified intervals, it is possible to study the values of the income-demand elasticity in the progressive and digressive phases of growth of the simulated exponential Engel curve. We can use the X^* income (7) for that purpose. By the following calculation, we will determine that the value of the income-demand elasticity at the X^* is +2:

$$\eta(X^*) = -b \times \frac{1}{X^*} = -b \times \left(-\frac{2}{b}\right) = +2 \tag{12}$$

According to Calculation (12) and the determined behaviour of Function (3), we can say that the values of the income-demand elasticity are greater than +2 in the progressive (convex) stage of the growth of the shaped Engel curve. In the digressive (concave) increasing stage of the simulated Engel curve, the values range from 0 to +2. The income-demand elasticity is just equal to +2 at the X^* income (inflexion point), see (12). At the close of the parametrical analysis of the exponential Engel model in Form (1), where $a > 0 \wedge b < 0$, it is useful to summarise the achievable values of the income-demand elasticity within the defined parts of the application interval. The obtained values of the income elasticity are displayed in Table 1.

Table 1. Income elasticity levels within specified parts of application interval

Income interval	Value of income elasticity function [a)]
$X \in (0; X^*\rangle$	$\eta(X) \in (+2; +\infty)$
$X \in (X^*; X_L\rangle$	$\eta(X) \in (+1; +2)$
$X \in (X_L; +\infty\rangle$	$\eta(X) \in (0; +1)$

[a)] The elasticity function falls monotonously

EXPONENTIAL ENGEL MODEL OF THE CZECH HOUSEHOLD DEMAND FOR MEAT AND MEAT PRODUCTS

The determined relationships and defined findings were verified on the developed exponential Engel model of the average Czech households' demand for meat and meat products. The parameters of the given exponential model were estimated from the evidence of consumer behaviour on the market for meat and meat products in years 1995 and 2000. These records are available in the databases of the Czech Statistical Office – Household Expenditure Surveys. There are records of the quarterly purchases of meat and meat products, including fish and fish products, by the average Czech household in kilograms per capita. For the same years, the quarterly nominal incomes of the average Czech household in CZK per capita were obtained from the database of Household Expenditure Survey too. The achieved purchases and household incomes are displayed in Table 2.

Naturally, the nominal households' incomes of the average Czech household had to be transformed into their real levels so that the distorted estimations of the parameters in the developed exponential Engel model associated with the price-demand effects could be reduced (Maurice, Phillips 1992). Quarterly geometric means of the month total index numbers[1] of the consumer price (CPI) with the fixed base were used for these transformations. The values of the month basic CPI were determined from the related chain forms of the indexes, which are available in the CZSO database within the Prices Evidences[2]. However, the time series of CPI was immediately suitable for the given transformations only between 1995 and 2000, because before and after the period, the definitions of the CPI weights are different so that they would correspond with the actual structure of the consumer

[1] The total index numbers of consumer prices is the index of the relative price changes within the whole consumer bundle.

[2] Publication number 71, Consumer Prices.

bundle. January 1995 was chosen as the basic period (100%). That fact determined the total range of the used time series of meat purchases and incomes of the average Czech household. The quarterly geometric means of the month total index numbers[3] of the CPI and the real incomes of the average Czech household in the investigated period are depicted in Table 3.

In relation to the character of the received database (time series) and thus the possibility to identify the distorted regressions, the dynamic forms of the Engel demand model were used, Seger et al (1998). Time factor in the shaped income-demand relationships were explicitly expressed so that the time variable would be declared as:

The first quarter of 1995 $t = 1$
The second quarter of 1995 $t = 2$
....
The fourth quarter of 2000 $t = 24$

Within the exponential specification (1) of the Engel demand model, the following explicit-dynamic constructions were examined:

$$Q_t = a \times e^{b/X_t + c_1 \times t} \quad Q_t = a \times e^{b/X_t + c_2 \times t^2} \quad Q = a \times e^{b/X + c_3 \times t^3}$$

$$Q = a \times e^{b/X + c_1 \times t + c_2 \times t^2} \quad Q = a \times e^{b/X + c_2 \times t^2 + c_3 \times t^3}$$

$$Q_t = a \times e^{b/X_t + c_1 \times t + c_2 \times t^2 + c_3 \times t^3}$$

The suggested explicit-dynamic forms of models can simulate the trend in the income-demand relationships but they cannot take into account the eventual periodical oscillations. For the proposed dynamic forms of the exponential Engel model, the index of determination (I^2), the index of correlation (I) and the adjusted index of determination ($\bar{I}^2$) were determined. The statistical verification of the models was proceeding with F-test of and T-tests of the individual parameters: a, b, c_1, c_2, c_3. The first-order autocorrelations within the constructed Engel models were investigated by means of DW-tests. From the investigated explicit-dynamic forms of the exponential Engel model, the construction:

$$Q_t = a \times e^{b/X_t + c_2 \times t^2} \tag{13}$$

Table 2. Quarterly purchases of meat and meat products including fish and fish products (kg per capita) and quarterly nominal incomes of the average Czech Household (CZK per capita)

Year	First quarter		Second quarter		Third quarter		Fourth quarter	
	purchases	nominal incomes	purchases	nominal incomes	purchases	nominal incomes	purchases	nominal incomes
1995	11.31	12 582	12.40	13 422	12.21	14 129	13.77	14 129
1996	12.12	14 665	13.05	16 196	12.86	15 798	14.65	15 798
1997	12.56	16 368	13.18	17 732	12.94	17 573	15.00	17 573
1998	12.59	18 466	13.47	19 200	13.18	19 397	16.63	19 397
1999	13.74	19 002	13.47	20 322	14.07	20 229	16.08	20 229
2000	13.47	18 750	13.74	20 817	13.17	20 346	15.09	20 346

Source: CZSO-HES, 30 Labour, Social statistics – Living Standard

Table 3. Quarterly geometric means of CPI (%) and real incomes of the average Czech household (CZK per capita)

Year	First quarter		Second quarter		Third quarter		Fourth quarter	
	CPI	real income	CPI	real income	CPI	real income	CPI	real income
1995	100.63	12 503	102.76	13 062	103.96	13 591	105.88	14 553
1996	109.51	13 392	111.53	14 522	113.73	13 891	115.18	15 196
1997	117.42	13 940	118.87	14 917	124.88	14 072	126.72	14 953
1998	132.98	13 887	133.95	14 333	136.68	14 191	136.13	15 286
1999	136.86	13 885	137.08	14 824	138.37	14 620	138.73	15 284
2000	141.85	13 219	142.27	14 632	143.98	14 131	144.70	15 288

Source: Author's calculations

[3] The total index numbers of consumer prices is the index of the relative price changes within the whole consumer basket.

was acceptable without reserve in view of the given statistical verification.

The model's construction (13) may be rewritten into the following logarithmical shape:

$$\ln Q_t = \ln a + b/X_t + c_2 \times t^2 \qquad (14)$$

The explicit-dynamic Engel model in Form (13), or more precisely in Form (14), satisfied all tested statistical criteria at more than the 90% level of significance, in contrast to other proposed models. The values of parameters in the dynamic Engel model (13) or (14), which were estimated under ordinary least square technique, are depicted in Table 4. This table also contains the results of the performed statistical verification for the given model.

With regard to the immediate application of the derived terms and the practical verification of the theoretically supposed properties of the exponential Engel curve, the model of the average Czech household's demand for meat and meat products, including fish (13), was transformed into the construction with the parametrically specified absolute parameter[4]:

$$Q_t = A_t \times e^{b/X_t} \quad A_t = a \times e^{c_2 \times t^2} \quad (t = 1, 2, \ldots, 24) \quad (15)$$

Under Rearranging (15), we obtained 24 single models for the individual quarters of 1995–2000 from the original dynamic form (13) of the exponential Engel model. The achieved partial models for the individual quarters of the studied time period have a different level of the absolute term (A_t). The determined values of A_t for Engel demand model (15) are displayed in Table 5.

Table 4 and Table 5 show that the average Czech household's demand for meat and meat products, including fish and fish products, can simulate, within the studied quarters of 1995–2000, under the exponential Engel model in Form (1), where the absolute parameter has the positive value and the parameter in exponent has the negative value, see case two in the theoretical parts of this paper. Under the parameters:

Table 4. Explicit-dynamic model of the exponential Engel curve of the average Czech household for meat and meat products: parameters and statistical verification

$Q_t = a \times e^{b/X_1 + c_2 \times t^2}$		
$\ln a = +3.798$; $a = +44.6019$	$\lvert T_{\ln a} \rvert = 18.3335$	$\alpha \lvert T_{\ln a} \rvert = 2.1204 \times 10^{-14}$
$b = -17\,336.8908$	$\lvert T_b \rvert = 6.0399$	$\alpha \lvert T_b \rvert = 5.4005 \times 10^{-6}$
$c_2 = +1.1119 \times 10^{-4}$	$\lvert T_{c_2} \rvert = 1.8415$	$\alpha \lvert T_{c_2} \rvert = 7.9721 \times 10^{-2}$
$I_2 = 0.7295$	$\bar{I}^2 = 0.7037$	$I = 0.8541$
$F(2; 21) = 28.3166$	$\alpha(F) = 1.0910 \times 10^{-6}$	
$DW = 1.8218$		

Source: Author's calculations

Table 5. Values of the absolute parameter in individual quarters of 1995–2000

Year	First quarter		Second quarter		Third quarter		Fourth quarter	
	t	A_t	t	A_t	t	A_t	t	A_t
1995	1	44.6068	2	44.6217	3	44.6465	4	44.6813
1996	5	44.7260	6	44.7807	7	44.8455	8	44.9204
1997	9	45.0054	10	45.1005	11	45.2060	12	45.3217
1998	13	45.4479	14	45.5845	15	45.7317	16	45.8896
1999	17	46.0583	18	46.2379	19	46.4285	20	46.6303
2000	21	46.8434	22	47.0678	23	47.3039	24	47.5518

Source: Author's calculations

[4] Parametrical definition of the absolute term presents a certain possibility of interpretation or practical use of the explicit-dynamic models. The changes of intercept value just simulate the dynamics of studied relationships – the static projection of dynamic model for a particular time period.

$A_t > 0 \wedge b < 0$, the simulated exponential Engel curve is ascending within the determined application interval: $X \in (0;+\infty)$. We could differentiate two growth stages of the simulated Engel curve (see Figure 2). In the income interval, i.e. $X \in (0;8\,668)$, the shaped exponential Engel curve increases progressively. In the following income interval, i.e. $X \in (8\,668;+\infty)$, the simulated exponential Engel curve increases digressively. The growth of the simulated Engel curves is broken at the income X^*; $X^* = 8\,668$ CZK. We determine the break income in accordance with Term (8). The assembled Engel model in Form (15) simulates the saturation of the average Czech household's demand for meat and meat products, including fish and fish products, at the infinite incomes of the given household. In the case of Model (15), the demand saturation has the variable level for the individual quarters, because $A_t = a \times e^{c_2 \times t^2}$ and $t = 1, 2, \ldots, 24$. The simulated values of the demand saturation are shown in the previous table (Table 5). In the observed period, the simulated saturation of the average Czech household's demand for meat and meat products, including fish and fish products, attains the average level of 45.64 kg per capita and per quarter, i.e. 0.50 kg per capita and per day.

RESULTS AND DISCUSSION – INCOME ELASTICITY OF THE DEMAND FOR MEAT AND MEAT PRODUCTS

Under the developed exponential Engel model of the average Czech household's demand for meat and meat products, including fish and fish products:

$$Q_t = A_t \times e^{-17336.8908/X_t} \quad A_t = 44.6019 \times e^{1.1119 \times 10^{-4} \times t^2}$$
$$(t = 1, 2, \ldots, 24) \qquad (16)$$

it is possible to define the income elasticity function in the hyperbolic form:

$$\eta(X_t) = 17\,336.8908/X_t \qquad (17)$$

The derived hyperbolic function of the income-demand elasticity in Form (17) is static in all of parameters, in contrast to the basic exponential Engel model (16). The attained levels of the income-demand elasticity depend only on the size of the households' incomes. In the application interval of Model (16): $X \in (0; +\infty)$, the achieved elasticity function decreases digressively. Convexly decreasing values of the income-demand elasticity tend to the zero level; see Limit (9). Limit (10) implies that Function (17) gives the infinity level of the income-demand elasticity at the incomes near by zero ($X \to 0^+$). The unit income elasticity of the average Czech household's demand for meat and meat products, including fish and fish products, is obtained at the quarter sum of the average Czech household's incomes: $X_L = 17\,336.8908$ CZK. At the income level: $X^* = 8\,668$ CZK, where the simulated Engel curve by Model (16) changes the behaviour of its increments, the value of the income-demand elasticity is just equal to +2; see Term (12). The summary of riches values of the income-demand elasticity in the partial parts[5] of the application interval of incomes is shown in the following table (Table 6).

With respect to the range of the real incomes in the observed time period (see Table 3): $X \in (12\,503; 15\,288)$, under the exponential Engle model in Form (16), we can simulate only the values of the income-demand elasticity from +1 to +2. Naturally, the income elasticity level of the average Czech household's demand for meat and meat products, including fish and fish products, will fall in the given income interval, see the determined behaviour of Function (17). The list of the achieved values of the income-demand elasticity in the individual quarters of 1995–2000 by Function (17) is depicted in Table 7.

In Table 7, we can see that the highest income elasticity of the average Czech household's demand for meat and meat products, including fish and fish products (1.3866), was attained in the first quarter of 1995, where the real level of the household incomes was 12 503 CZK per capita, thus the lowest in the observed time period. On the contrary, the minimal income elasticity of the given demand (1.1340) was obtained in the fourth quarter of 2000. There was the maximal real level of the quarter households' incomes per capita in the time period (15 288 CZK per capita). The average level of the simulated income-demand elasticity between 1995 and 2000 attained the value of 1.2121. Thus in the observed years, the 1% rise in the real level of quarter households' incomes per capita led to the average increase in the average Czech

Table 6. Level of the income elasticity within the partial income interval

Income interval	Values of income-elasticity
$X \in \langle 0; 8\,668 \rangle$	$\eta(X) \in \langle +2; +\infty)$
$X \in \langle 8\,668; 17\,337 \rangle$	$\eta(X) \in \langle +1; +2)$
$X \in \langle 17\,337; +\infty \rangle$	$\eta(X) \in \langle 0; +1)$

Source: Author's calculations

[5] The partial income intervals were defined in relation to the values of X_L and X^*.

Table 7. Income elasticity of the average Czech household's demand for meat and meat products, including fish and fish products, in observed quarters, η(X)

Year	First quarter	Second quarter	Third quarter	Fourth quarter
1995	1.3866	1.3273	1.2756	1.1913
1996	1.2946	1.1938	1.2481	1.1409
1997	1.2437	1.1622	1.2320	1.1594
1998	1.2485	1.2096	1.2216	1.1342
1999	1.2486	1.1695	1.1858	1.1343
2000	1.3116	1.1849	1.2269	1.1340

Source: Author's calculations

household's demand for meat and meat products, including fish and fish products, of about 1.21%.

With respect to the minimal and maximal determined values of the income-demand elasticity and with respect to the behaviour of the derived elasticity function (17), it is evident that in the observed period, we will be able to simulate only the income elastic reactions in the average Czech household's demand for meat and meat products, including fish and fish products. Of course, within the investigated income interval the elasticity of the income-demand reactions does not exceed the level of +2, because the minimal real size of the quarter incomes of the average household (12 503 CZK) is greater than X^* (8 667 CZK). According to the determined values of the income elasticity coefficients, the average Czech household purchased meat and meat products, including fish and fish products, as luxury goods in the studied period. Naturally, the achieved higher levels of the income-demand elasticity can induce a discussion about the adequacy of the exponential Engel model (1) for the simulation of the average Czech household's demand for meat and meat products, including fish and fish products. For comparison, see the values of analogical coefficients in Table 8.

Table 8. Income elasticity of consumer demand for meat and meat products

Specification of goods	Income elasticity[1]	
Meat and meat products	0.65	
	short-term[2]	long-term[2]
Pork	0.27	0.18
Beef	0.51	0.45
Poultry	0.49	1.06

Source: [1]Tvrdoň (1999), [2]Wohlegenant, Hann (1982)

Table 9. Income elasticity of the average Czech household's demand for meat and meat products, different mathematical construction of the models

Specification of Engel model	Income elasticity
Linear model	0.8164
Hyperbolic model	0.9296
Power model	0.4374
Exponential model	0.9352

Source: Syrovátka (2002)

On the other hand, the author's previous analyses in the given field of consumer behaviour, Syrovátka (2002), determined higher levels of the income-demand elasticity, too. The values of the income-demand elasticity from the author's previous studies are presented in Table 9 and they illustrate relatively unambiguously other positions of meat and meat products within the consumer bundle of Czech households. In Table 9, we can see the elasticity coefficients under the different mathematical specifications of the Engel model. These previous analyses of the consumer demand for meat and meat products were also based on the CZSO databases – Household Expenditure Surveys – and the explicit-dynamic form of Engel models were used too.

In relation with the higher values of the simulated demand elasticity, it is naturally useful to remind that seasonal oscillations were not considered within the dynamic construction of exponential model of the Engel curve. Just the seasonal oscillations can affect the distortion of the analysed demand elasticity.

CONCLUSION

According to the CZSO-HES database about the size of meat purchases and the level of the real incomes of the average Czech household in the observed period (1995–2000), the studied exponential form of the Engel model reached the following values of parameters:

$$Q_t = A_t \times \mathrm{e}^{-17336.8908/X_t} \qquad A_t = 44.6019 \times \mathrm{e}^{1.1119 \times 10^{-4} \times t^2}$$

and t = 1, 2, ..., 24

The developed exponential model with the dynamic specification of the absolute term gives the income-demand elasticity function in the static hyperbolic form:

$$\eta(X_t) = 17\,336.8908/X_t$$

Thus the level of the income-demand elasticity depends only on the real sizes of households' incomes. The derived elasticity function decreases digressively. Convexly decreasing values of the income elasticity tend to the zero level. Contrariwise, the elasticity function simulates the infinity level of the income-demand elasticity at the real incomes near by zero. The unit income elasticity of the average Czech household's demand for meat and meat products, including fish and fish products, is attained at the quarter sum of the average Czech household's incomes X_L, where X_L = 17 336.8908 CZK per capita. At the income level of X^* = 8 668 CZK per capita, where the simulated exponential Engel curve changes the behaviour of its increments, the income-demand elasticity is just equal to two. If we divide the complete application interval into three parts, i.e. the income interval: $X \in (0;8\,668\rangle$, the income interval: $X \in (8\,668;17\,337\rangle$ and the income interval: $X \in (17\,337;+\infty\rangle$, then we could determine the following intervals of income elasticity within the simulated demand: $\eta(X) \in (+2;+\infty)$; $\eta(X) \in (+1;+2)$; $\eta(X) \in (0;+1)$.

In the analysed time period (1995–2000), where the real incomes of the average Czech household moved from 12 503 CZK to 15 288 CZK, the income reactions in the given demand were simulated as elastic. The income elasticity of the average Czech household's demand for meat and meat products, including fish and fish products, was reaching the values from 1.3866 to 1.1340. The average level of the simulated income-demand elasticity between the observed years attained the value of 1.2121, thus the 1% rise in the real level of quarter households' incomes per capita led to the average increase in the average Czech household's demand for meat and meat products, including fish and fish products, of about the 1.21%.

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