The Structural Porosity in Soil Hydraulic Functions – a Review

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Abstract: Products of biological processes are the dominant factor of soil structure formation in A horizons, while in B horizons their role is less expressed. Soil structure influences dominantly the structural domain of the pore system in bimodal soils thus affecting soil hydraulic functions. The form of soil hydraulic functions depends upon the pore size distribution and generally upon configuration of the soil pore system. We used the functions derived for the lognormal pore size distribution and modified them to bi-modal soils. The derived equations were tested by experimental data of catalogued soils. The procedure leads to the separation of two mutually different domains of structural and matrix pores. The value of the pressure head (potential) separating the two domains is not constant and varies in a broad range. For each domain we obtained its water retention function and unsaturated hydraulic conductivity function. The separation of hydraulic functions for the two domains is a key problem in the solution of preferential flow and in controlling lateral flow between the structural and matrix domains. Water retention function is fully physically based while the conductivity function still keeps fitting parameters, too. Their simple relationship to tortuosity and pores connectivity was not confirmed. Since they differ substantially for matrix and structural domains, we suppose that there exists a great difference in configuration of porous systems in structural and matrix domains. The use of uniform fitting conductivity parameters for the whole range of pores is not justifiable.

Keywords: soil structure; bi-modal soils; soil water retention; unsaturated conductivity; pore size distribution; structural pore domain; matrix pore domain

Soil water retention function θ(h) and unsaturated hydraulic conductivity $K(h)$ or $K(\theta)$ are hydraulic functions important for the solution of transport processes in soils as e.g. the flow of water and solutes, or transport of suspended particles, including bacteria. θ is the volumetric soil water content (L^3/L^3), h is the soil water pressure head (L) and the unsaturated hydraulic conductivity K has the dimension (L/T). Both soil hydraulic functions are mutually closely related. It is convenient to use the relative unsaturated conductivity $K_R = K/K_S$ where K_S is the saturated hydraulic conductivity with the dimension (L/T). Pore size distribution as well as the shape and configuration of pores are determinant factors of the soil hydraulic functions. The relationship between the relative unsaturated conductivity and the pore size distribution function $g(r)$ was gradually improved (Childs & Collis George 1950; Fatt & Dijkstra 1951; Burdine 1953; Mualem 1976) up to the recent general form:

$$K_R = S^{\alpha}\left[\frac{\int_0^r r^{\beta} g(r)dr}{\int_0^{\infty} r^{\beta} g(r)dr}\right]^{\gamma} \tag{1}$$

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where:

r – radius of soil pores,

$g(r)$ – pore size distribution function, usually taken as the soil water retention function,

α, β, γ – are parameters characterizing the soil porous system, α, β are usually interpreted as the tortuosity of the flow path and γ is assumed to be the pores connectivity (Mualem 1976).

The relative saturation of soil pores by water S is modified to

$$S = \frac{\theta - \theta_R}{\theta_S - \theta_R} \quad (2)$$

It takes into account the residual soil water content θ_R. Eq. (1) is valid provided that the function $g(r)$ and parameters α, β, γ are the same in the whole range of the studied K, i.e. that the characteristics of the configuration of the soil porous system do not change with the relative saturation by water. The pore size distribution function $g(r)$ in Eq. (1) is approximated by the soil water retention function and Eq. (1) is modified in accordance to the form of retention function. The most frequently used expression of the soil water retention function is the equation of van Genuchten (1980). It describes the sigmoidal form of a smooth curve fitted by three parameters to the measured $\theta(h)$ data. Since it lacks a linkage to the soil porous system, we are allowed to denote it as an empirical equation. When it is inserted into Eq. (1), equation $K_R(h)$ is obtained. It was applied by many authors in numerous simulation models (Šimůnek *et al.* 2003). The empirical equation enters here into a physically based relationship of $K(h)/K_S$. The combination of empirical and physical approaches could be a source of imperfectness in the resulting forms. This drawback is compensated by an introduction of further fitting parameters. In order to eliminate this imbalance, we performed a research for a physical expression of the soil water retention curve. It was accompanied by the expectation on a more exact description of the unsaturated conductivity function. In addition to this theoretically based intention, we assume that the physical form of the water retention function can be well related to micromorphological studies of the soil porous systems. Brutsaert (1966) studied four models of pore size distribution, among them the lognormal distribution in relation to soil water retention curve. We conclude from his research that the lognormal distribution looks as an acceptable approximation. Kosugi (1994) formulated the lognormal pore size distribution function

$$g(r) = d\theta/dr \quad (3a)$$

$$g(r) = \frac{\theta_S - \theta_R}{\sigma r\sqrt{2\pi}} \exp\left\{-\frac{[\ln(r/r_m)]^2}{2\sigma^2}\right\} \quad (3b)$$

where:

r_m – geometric mean radius,

σ – standard deviation of log transformed pore radius.

Kosugi combined (3a) with (3b) and (2). Finally with

$$f(h) = d\theta/dh \quad (3c)$$

and after rearrangement and integration he derived the soil water retention function in the form

$$S = \frac{1}{2} erfc\left[\frac{\ln(h/h_m)}{\sigma\ \sqrt{2}}\right] \quad (4)$$

with a known relationship between pressure head and pore radius $h = a/r$ where a is the coefficient dependent upon the geometry of pore section we use in the model, h_m is the pressure head related to r_m, and *erfc* is the complementary error function. For a cylindrical pore of radius r (μm), contact angle = 0 (full wetting), h (cm) and for water at 20°C is $a = 1490$ (L^2). Pachepsky *et al.* (1992) summarized his earlier studies on "pF-curves" into a similar equation to (4) with a fitting parameter m which he later interpreted as $m = 1/\sigma(s\ \sqrt{2})$ (Pachepsky *et al.* 1995) and the resulting equation was finally identical with Eq. (4). Kosugi (1999) introduced $g(r)$ from Eq. (3b) into Eq. (1) to get the equation of the relative unsaturated hydraulic conductivity $K_R(h)$. We obtain after rearrangement of his equations (Kutílek 2004)

$$K_R = S^\alpha \left\{\frac{1}{2} erfc\left[(\ln\frac{h}{h_m})\frac{1}{\sigma\sqrt{2}} + \frac{\beta\sigma}{\sqrt{2}}\right]\right\}^\gamma \quad (5)$$

or, if we transcribe Eq. (4) to the form with argument M

$$S = \frac{1}{2}\ erfc(M) \quad (6)$$

then Eq. (5) is

$$K_R = S^\alpha \left\{\frac{1}{2} erfc\left[M + \frac{\beta\sigma}{\sqrt{2}}\right]\right\}^\gamma \quad (7)$$

and Eq. (7) shows a strong dependence of K_R upon the soil water retention function. We find

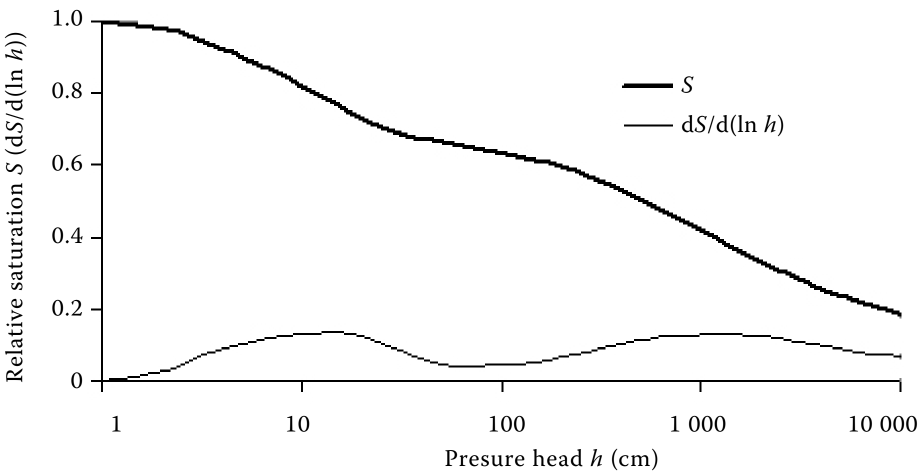

Figure 1. Derivative curve dS/d(ln h) to soil water retention curve $S(h)$ in soil SO15. It has the minimum at h_A = – 55 cm. The structural domain is in the range $0 > h \geq -55$ cm, the matrix domain is for $h < -55$ cm

frequently (Othmer *et al.* 1991; Durner 1992) two or even three peaks on the derivative curve to the soil water retention curve (Figure 1) and then we speak on bi-modal or tri-modal pore size distribution, or briefly on bi- or tri-modal soils. The direct experimental proof on existence of soil bi-modality is in publications on micromorphology (Pagliai & Vignozzi 2002) and in evaluation of the pore size distribution in water filled pores by nuclear magnetic resonance (Bird *et al.* 2005). The separation of individual domains is a crucial point not only theoretically, but also in the solution of the preferential flow in soils. The objective of this paper is to apply the theory on hydraulic functions in lognormal pore size distribution systems to bi-modal soils where the bi-modality is mainly influenced by the soil structure. The aggregate stability is dominantly fixed by the products of biotransformation of organic matter (humic substances) in A-horizons. The fixing role of humic substances is less important in B-horizons while in C-horizons the biofactors are without its importance upon the formation of structural pores, while the volumetric ratio of structural pores is decreased. We modify the Kosugi-Pachepsky's model of soil water retention curve and the Kosugi's unsaturated hydraulic conductivity function to soils with bi-modal porosity. The knowledge on hydraulic functions of the structural domain may contribute to quantification of the physical quality of soil structure.

Theory

We start with the assumption that the derivative of the soil water retention curve is related to the pore size distribution. It follows from this that if two peaks on the pore size distribution function appear then two porous systems exist within the domain of capillary pores (Kutílek & Nielsen 1994):

– Matrix (intra-aggregate, intra-pedal, textural) pores within soil aggregates or soil blocks. The arrangement of the soil skeleton, coating of aggregates, cutans and nodules typical for each soil taxon have main influence upon the soil water hydrostatics and hydrodynamics in the matrix domain.

– Structural (inter-aggregate, inter-pedal) pores between the aggregates, or eventually between the soil blocks. Their morphology and interconnection depends upon the shape, size and stability of aggregates and blocks, or, generally upon the soil genesis and the type of soil use. A certain portion of these pores is formed by the pedo-edaphon, too.

The boundary between the domains of matrix and structural pores is denoted by h_A. Let us note that it is the air entry value of the matrix domain, too. It is determined as the minimum value between two peaks on the derivative curve to the soil water retention curve. An illustrative example is in Figure 1. Eq. (4) of soil water retention curve and Eq. (2) of relative saturation of soil by water have the forms for bi-modal soils (Kutílek 2004)

$$S_i = \frac{1}{2} erfc\left[\frac{\ln(h_i / h_{mi})}{\sigma_i \sqrt{2}}\right] \tag{8}$$

$$S_i = \frac{\theta_i - \theta_{R_i}}{\theta_{Si} - \theta_{Ri}} \tag{9}$$

where:
i = 1 is for matrix pores and
i = 2 for structural pores

Eq. (8) is valid even for n-modal soils, $n > 2$. With the principle of superposition, applied already by Othmer *et al.* (1991), Pachepsky *et al.* (1992) and by Zeiliguer (1992) we define for bi-modal soils

$$\theta = \theta_1 + \theta_2 \quad (10)$$

Since micropores of $r > r(h_A)$ would cause instability of aggregates, we assume that the matrix porous system does not contain micropores above $r(h_A)$. Then

$$\theta s_1 = \theta(h_A) \quad (11)$$

$$\theta s_2 = \theta_{s(MEAS)} - \theta_{s1} \quad (12)$$

where:

$\theta_{s(MEAS)}$ – denotes the measured saturated water content.

For $0 > h \geq h_A$ is

$$\theta_1 = \theta_{S1}, S_1 = 1 = \text{const} \quad (13)$$

$$\theta_2 < \theta_{S2}, S_2 < 1 \quad (14)$$

where S_2 or θ_2 are obtained by optimization.

For $h < h_A$ is

$$\theta_1 < \theta_{S1}, S_1 < 1 \quad (15)$$

$$\theta_2 = 0, S_2 = 0 \quad (16)$$

where S_1 or θ_1 are obtained by optimization. We assumed that θ_{R1} in S_1, Eq. (9), is physically below the wilting point θ_{WP} (h = –15000cm) in the range of hygroscopic water and we approximated $\theta_{R1} = 0.5\ \theta_{WP}$. For structural domain we took $\theta_{R2} = 0$.

Kosugi's unsaturated relative hydraulic conductivity K_R is modified to bi-modal soil in a similar way as the soil water retention function (Kutílek 2004)

$$K_{R_i} = S_i^{\alpha_i}\left\{\frac{1}{2}erfc\left[(\ln\frac{h_i}{h_{mi}})\frac{1}{\sigma_i\sqrt{2}}+\frac{\beta_i\sigma_i}{\sqrt{2}}\right]\right\}^{\gamma_i} \quad (17)$$

The subscripts in parameters α_i, β_i, γ_i, reflect the assumption that values of parameters differ for the two domains. Since parameters h_{mi} and σ_I are known from the evaluation of the water retention curve (Eq. 8), the parameters α_i, β_i, γ_i have to be optimized. With $K_i = K_{Ri}K_{Si}$ and using the principle of superposition we obtain

$$K = K_1 + K_2 \quad (18)$$

For $h < h_A$ is $\theta_2 = 0$ and $S_2 = 0$. Consequently is $K_2 = 0$. The two new parameters, namely K_{S1}, K_{S2} could be optimized independently, but we found that we obtained better results when we optimized only K_{S2} and when K_{S1} was constrained by $K_{S1} = K_{S(MEAS)} - K_{S2}$. We denote the measured saturated conductivity of the whole soil by the symbol $K_{S(MEAS)}$. It was determined on the undisturbed core samples by the falling head method. The procedure allows us to define separately conductivities of the two domains and to separate from $K_{S(MEAS)}$ that portion K_2 which can be considered as preferential conductivity, see Figure 4 as an illustrative example.

MATERIALS AND METHODS

Experimental data sets

The theory was tested on data sets of soil water retention and of unsaturated hydraulic conductivity catalogued in the UNSODA data base (Leij *et al.* 1996; and Nemes *et al.* 1999) and on the data sets published by Othmer *et al.* (1991). We selected soils texturally comparable where the swelling/shrinkage processes are negligible on the macroscale (silt loam and loam) and in addition to them we included into the study sandy soils, too. All soils and their horizons were typical by their bi-modality. Soil characteristics relevant to the studied problem are in Table 1. Soil water retention curves of both sources, the UNSODA data base and of Othmer were determined on undisturbed soil samples in the laboratory. Unsaturated conductivity data were determined in the laboratory for the UNSODA soils. Data in Othmer´s publication were measured in the field by the instantaneous method.

Optimization

In order to find an optimal value for the parameters h_{m1}, h_{m2}, σ_1, σ_2, α_1, α_2, β_1, β_2, γ_1, γ_2 the iterative fitting procedure was applied. We used the Powell method (1977, 1978). The optimized function is evaluated at the minimum sum of fitted relative errors, SFRE

$$\text{SFRE} = \sum_{i=1}^{n}\left|\frac{y_i^m - y_i^f}{y_i^m}\right| \quad (19)$$

where y_i^m is for measured data and y_i^f denotes fitted data. The solution uses a conjugate gradient method to find the minimum of a function $f(x)$ of n vari-

Table 1. Characteristics of soils

Soil	Soil taxon	Depth (cm)	Soil horizon	Soil texture	Soil structure
UNSODA (Leij *et al.* 1996; Nemes *et al.* 1999)					
4040	typic Hapludalf	0–30	Ap	silt loam	nd
4041		30–50	B2t	silt loam	nd
4660	typic Dystrochrepts	15–25	Ah	sand	single grain
4661		30–40	Bv	sand	single grain
4670	typic Hapludalf	20–30	Al	silt	coherent
4671		40–50	Agl	silt loam	coherent to fine
4672		70–85	Bt	silt loam	fine to moderate
Othmer *et al.* (1991)					
SO 15	Aquic Hapludalf	15	Ap	loam	medium subangular
SO 60		60	Btv	loam	medium subangular

nd – not determined

ables, (i.e. fitting parameters). Only function values are required, i.e. functional gradients are calculated numerically. The routine is based on the version of the conjugate gradient algorithm described in Powell (1977, 1978). The main advantage of the conjugate gradient technique is that it provides a fast rate of convergence without the storage of any matrices. Therefore, it is particularly suitable for unconstrained minimization calculations as it is the case of the present problem. The described procedure proves robust and efficient. It converges within few seconds (Jendele & Kutílek 2007).

Optionally, the value of h_A can also be subject of the optimization, however our experience shows that manual setting of h_A from the derivative curve to $S(\theta)$ensures better results. With h_A we optimized first the parameters h_{m1}, h_{m2}, σ_1, σ_2. In the next phase, we carried on similar procedure to optimize the parameters α_1, α_2, β_1, β_2, γ_1, γ_2. However, it was found by Kosugi (1999) that the same quality approximation could be obtained by the assumption $\gamma = 1$ in mono-modal soils. We started therefore with the alternative $\gamma_1 = \gamma_2 = 1$. It simplified the optimization process. In the next step, we optimized all parameters including γ_i.

The conductivity model Eq. (16) with fitted parameters α, β, γ was compared with the same model but with fixed parameters of Mualem (1976), i.e. for $\alpha = 0.5$, $\beta = 1$, $\gamma = 2$.

The quality of fitting procedure is characterized by the model efficiency parameters: Root mean square error, RMSE

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i^f - y_i^m\right)^2}{n}} \qquad (20)$$

Relative square error, RSE

$$\mathrm{RSE} = \frac{\sum_{i=1}^{n}\left(y_i^f - y_i^m\right)^2}{\sum_{i=1}^{n}\left(y_i^m - \bar{y}^m\right)^2} \qquad (21)$$

with $\bar{y}^m = \dfrac{\sum_{i=1}^{n} y_i^m}{n}$

where:

y_i^m, y_i^f – stands for i^{th} measured and fitted curve value.

For strong non-linear model of $K(h)$ is RMSE less appropriate and it characterizes the fitted model efficiency just close to the saturation.

RESULTS AND DISCUSSION

Soil water retention curves

The separation of structural and matrix domains by h_A is in Table 2. The data h_A were read from the graph $\mathrm{d}S/\mathrm{d}(\ln h)$ in all soils except of SO 60b, where we obtained it by optimization in order to demonstrate a comparison to the same soil SO 60a where h_A was read from the graph. All soils are distinctly bi-modal with the exception of sands UNSODA 4660 and 4661 with a feeble bi-modality and with values h_A very low and prob-

Table 2. Separation of matrix (index 1) and structural (index 2) domains at h_A; porosity of the matrix domain is P_1, and porosity of the structural domain is P_2; P_T is the total soil porosity

Soil	h_A (cm)	P_1	P_1/P_T	P_2	P_2/P_T
4040	273	0.302	0.76	0.096	0.24
4041	307	0.257	0.64	0.146	0.36
4660	15	0.318	0.69	0.145	0.31
4661	8	0.325	0.76	0.103	0.24
4670	296	0.336	0.73	0.125	0.27
4671	185	0.338	0.82	0.074	0.18
4672	626	0.307	0.78	0.087	0.22
SO 15	55	0.328	0.71	0.137	0.29
SO 60a	30	0.361	0.84	0.068	0.16
SO 60b[a]	44	0.353	0.82	0.076	0.18

[a]the same soil as 60a, but h_A was estimated by optimization

ably not well guessed. In all remaining silt loams and loams was h_A in a very broad range of –30 to –626 cm, i.e. from 50 μm to 2.4 μm equivalent pore radius. In sands was h_A between –15 to –8 cm or in ranges between –100 to –186 μm equivalent pore radius. We assume that the substantial difference from loamy soils is due to very feeble aggregation of sands. We are confirming the earlier statement (Kutílek 2004) that the poor soil structure has the consequence in decrease of the absolute value of h_A up to the extreme of $h_A = 0$, or to transition of bi-modal to mono-modal soil porous system The data in Table 2 show that a fixed constant boundary between structural and matrix domains does not exist. The separation of effective porosity by Ahuja *et al.* (1984), or the boundaries of macro-, mezzo- and micro-porosity of Luxmoore (1981) are not corresponding to the real pore size distributions in soils.

Structural porosity makes about 25% of the total porosity, or in other words the matrix porosity exceeds the structural porosity, again with the exception of weekly aggregated sands.

Values h_{mi} and standard deviations σ_I for log-normal pore size distribution Eq. (3b) in matrix domain ($i = 1$) and in structural domain ($i = 2$) are in Table 3. The distribution functions are close to standard type ($\sigma = 1$) except of UNSODA 4660 and 4661 (sand) for matrix domain. The parameters h_{mi} and σ_I enter into the water retention function S_i(h), Eq. (8). After using Eq. (8) we obtained separate water retention functions $\theta_i(h_i)$ of matrix

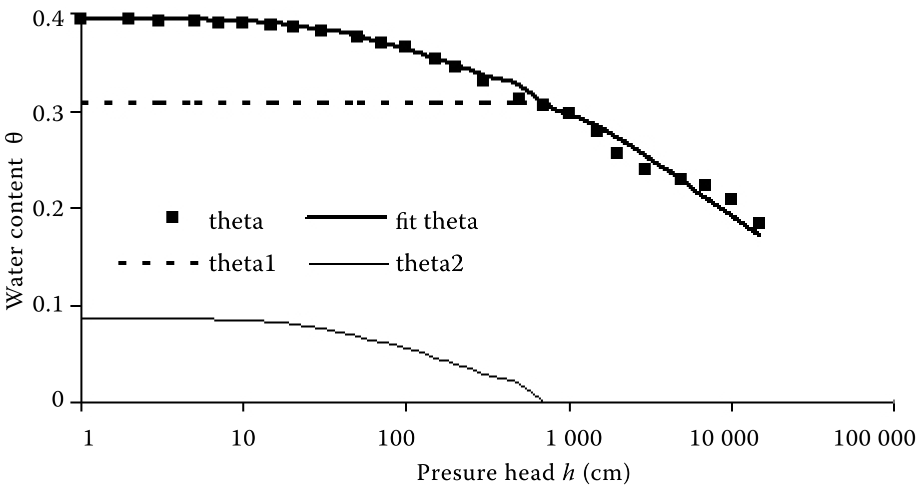

Figure 2. Measured (theta) and fitted (fit theta) soil water retention curves of soil UNSODA 4672. Separated soil water retention curves of matrix (theta1) and structural domains (theta2) were computed for parameters h_{mi} and σ_i in Table 3, Eqs (8) and (10)

Table 3. Evaluation of parameters of lognormal pore size distribution in matrix (index 1) and structural domains (index 2); the mean value of the pressure head in matrix domain is h_{m1}, in structural domain h_{m2}, σ_1 is the standard deviation in matrix domain, σ_2 in the structural domain; characteristics of the model accuracy RMSE (Eq. 20), RSE (Eq. 21) and the sum of the fitted relative errors SFRE (Eq. 19) when the fitted soil water retention curves (Eqs 8, 9 and 10) were related to measured data

Soil	h_{m1}	σ_1	h_{m2}	σ_2	RMSE	RSE	SFRE
4040	1 675	1.41	82.2	1.52	0.016240	0.01684	0.5990
4041	1 951	1.52	90.9	1.99	0.01741	0.01828	0.6499
4660	42.6	4.86	2.7	1.03	0.01982	0.02312	1.7020
4661	11.7	4.71	3.0	1.15	0.04246	0.08989	3.4833
4670	1 644	2.05	47.9	1.69	0.01610	0.01245	1.7147
4671	2 269	1.84	57.3	0.76	0.01510	0.01545	1.5071
4672	10 369	1.88	165.3	1.45	0.00896	0.01189	0.8344
SO 15	2 035	2.09	9.1	0.94	0.00659	0.00283	0.3753
SO 60a	1 382	2.24	8.5	0.93	0.01121	0.00972	0.4800
SO 60b[a]	1 549	2.09	11.2	1.09	0.01183	0.01082	0.4665

[a]the same soil as 60a, but h_A was estimated by optimization

and structural domains (Figure 2). Applying the principle of superposition, we obtained the water retention function of the whole soil in Figure 2. The optimized water retention function can be compared visually to measured data. More objective comparison is offered by criteria for assessment of model efficiency RSME Eq. (20) and RSE Eq. (21) in Table 3. In addition to them, there are the values of SFRE (sum of fitted relative errors, Eq. (19)). The computed values show a moderate to good efficiency except of UNSODA 4661 (B-horizon of sand).

Soil SO 60b is identical to SO 60a with the exception that its h_A was estimated by optimization. The resulting RMSE and RSE in Table 3 were quite close to the criteria of SO 60a and the model efficiency of SO 60b was only slightly worse than was SO 60a. We obtained similar results for all

Table 4. Errors of fitted soil water retention curves; maximum absolute error (MAE) and maximum relative error (MRE) when the measured and fitted data are compared

Soil	MAE	at θ	at *h*	MRE	at θ	at *h*
4040	0.041	0.103	15 000	0.397	0.103	15 000
4041	0.034	0.274	200	0.360	0.092	15 000
4660	0.088	0.294	20	0.617	0.022	50 000
4661	0.124	0.310	10	0.436	0.272	15
4670	0.039	0.151	1 500	0.263	0.134	2000
4671	0.042	0.175	1 500	0.241	0.175	1500
4672	0.025	0.090	70 000	0.347	0.090	70 000
SO 15	0.018	0.124	15 000	0.188	0.062	70 000
SO 60a	0.032	0.147	15 000	0.220	0.147	15 000
SO 60b[a]	0.035	0.147	15 000	0.239	0.147	15 000

[a]the same soil as 60a, but h_A was estimated by optimization

soils, but for the sake of brevity we do not include a detailed table.

The values of maximum absolute error MAE and maximum relative error MRE are in Table 4 in order to demonstrate the weakest parts of the optimized retention functions, when they were compared to the measured data. Maximum relative errors appear in the dry part of the retention curve. The errors are a good and simple illustration of our statement on a moderate to good efficiency of the fitted physical model.

Unsaturated hydraulic conductivity

We checked the results of optimization for three types of parameters:

(1) Parameters α,β were optimized while γ was considered constant, γ = 1 according to the proposal of Kosugi (1999).
(2) All three parameters α, β, γ were optimized.
(3) The values of parameters were taken as fixed according to model of Mualem (1977), α = 0.5, β = 1, γ = 2.

The results of optimization α, β in the first type with γ = 1 are in Table 5. The sum of fitted relative errors SFRE fluctuates in order of magnitude for studied soils and their two domains. They indicate the extent of variation of distance between the computed unsaturated conductivities and the measured data in matrix and in structural domains when the optimized parameters enter into Eq. (19). The UNSODA soils 4660 and 4661 are exceptional in structural domain due to their feeble aggregation and consequent feeble bi-modality, but even if we do not consider them, the variation of SFRE is high. The values of parameters α_I, β_I are very variable in individual soils. When we compare the parameters in two domains we find that α_1, β_1 differ substantially from parameters α_2, β_2 see the correlation coefficients α_1:α_2 and β_1:β_2 (Table 5). We assume therefore that porous systems in matrix and in structural domains are different and that they have to be studied and modelled separately. In addition to it, the negative sign in 22% of instances doubts the interpretation of both parameters as tortuosity dependent.

Characteristics of the accuracy of the optimized $K(h)$ function are in Table 6. While RMSE is relevant to conductivities near to the saturation, RSE

Table 5. Parameters α, β in unsaturated conductivity relationship (Eq. 17) with γ = 1; index 1 is for the matrix domain and index 2 is for the structural domain; sum of fitted relative errors SFRE used in the optimisation process is defined by Eq. (19).

Soil	α_1	β_1	SFRE	α_2	β_2	SFRE
4040	1.29	0.86	1.905	8.15	1.70	0.393
4041	6.74	0.27	2.842	2.78	0.44	3.138
4660	6.77	–0.18	5.948	2.39	–1.00	4.044
4661	2.12	0.54	9.755	1.45	–0.70	1.641
4670	3.31	0.47	0.537	2.69	–0.55	4.048
4671	0.88	1.20	0.358	–2.73	3.28	4.204
4672	3.47	1.03	0.928	3.07	–0.57	2.031
SO 15	–7.68	1.68	1.159	–0.96	1.90	1.554
SO 60a	6.53	1.23	1.989	0.34	1.18	0.379
SO 60b[a]	5.96	1.32	1.193	1.61	–2.57	0.747
Mean	2.94	0.84	2.661	1.88	0.31	2.218
SD	4.32	0.53	2.970	2.73	1.63	1.533
α_1:α_2[b]	0.28					
β_1:β_2[b]		0.36				

[a]the same soil as 60a, but h_A was estimated by optimization
[b]correlation coefficient between parameters of matrix and structural domains

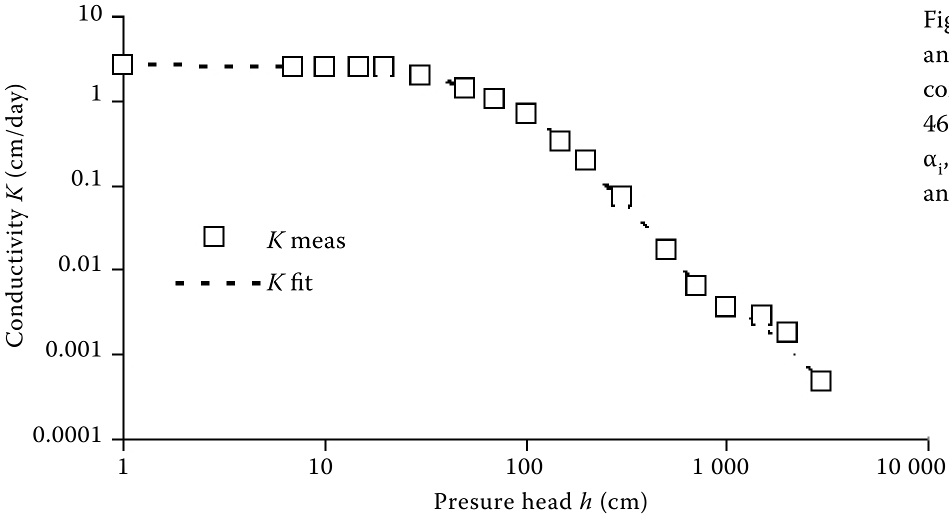

Figure 3. Measured (K meas) and fitted (K fit) unsaturated conductivities in soil UNSODA 4672 obtained for parameters α_i, β_i, γ_i in Table 8, Eqs (17) and (18)

reflects objectively the model efficiency in the whole range of measured $K(h)$ where K changes in several orders of magnitude. The values of RSE show a satisfactory identity of modelled and measured values. It means that a sort of imperfectness in parameters and K_i estimation in domains was reduced after fitting the whole $K(h)$ function. The difference in RSE values of SO 60a and SO 60b shows that the optimization of h_A in SO 60b leads to a lower efficiency of fitted $K(h)$ model. The maximum absolute and relative errors appear mainly close to saturation.

With one exception of sand (UNSODA 4661), the saturated conductivity of the matrix domain is about two orders of magnitude lower than the saturated conductivity of the whole soil, see the Table 7. It is in agreement with the observed rapid fluxes in preferential domains related to very slow fluxes into the matrix quoted in the literature. Let us note that both K_{S1} and K_{S2} were determined on the basis of the optimization, i.e. we optimized K_{S2} and using it we computed $K_{S1} = K_{S(MEAS)} - K_{S2}$.

The results of optimization α, β and γ (the second of the three optimization models) are in Table 8. The aim of optimizing all three parameters was to find out the consequences of the simplification with γ = 1. Even if the sum of fitted relative errors, SFRE fluctuates in order of magnitude like in the first type of optimization, we trace a systematically lower value of SFRE in all soils and their domains.

Table 6. Characteristics of the conductivity model accuracy. RMSE (Eq. 19), RSE (Eq. 20), maximum absolute error MAE and maximum relative error MRE for optimized solution of α, β and with γ = 1 in unsaturated conductivity function, Eqs (17) and (18)

Soil	RMSE	RSE	MAE	at h	MRE	at h
4040	35.78	0.158	172	1	1.64	1
4041	0.167	0.0164	0.728	1	2.35	1846
4660	68.72	0.179	206	2	18.0	30
4661	130.3	0.128	449	2	603.0	20
4670	7.38	0.0584	18.08	30	2.76	70
4671	0.978	0.0694	2.05	5	2.50	50
4672	0.137	0.0146	0.4	20	1.69	2000
OS 15	0.904	0.0908	3.26	1	3.20	75
OS 60a	0.504	0.0156	1.71	1	2.39	39
OS 60b[a]	0.295	0.0476	1.00	6	3.84	70

[a]the same soil as 60a, but h_A was estimated by optimization

Table 7. Saturated conductivity of the matrix domain K_{S1}, its relation to the measured saturated conductivity K_S of the soil; sum of fitted relative errors SFRE (Eq. 19) of the fitted $K(h)$ in Eq. (18)

Soil	K_{S1}	K_{S1}/K_S	SFRE
4040	0.029	7E-5	2.298
4041	0.030	0.005	4.541
4660	40.97	0.065	8.817
4661	271.70	0.24	10.79
4670	0.23	0.003	4.290
4671	0.42	0.03	3.920
4672	0.01	0.004	2.442
SO 15	0.16	0.014	1.962
SO 60a	0.72	0.048	2.239
SO 60b[a]	0.35	0.083	1.722

[a]the same soil as 60a, but h_A was estimated by optimization

Their mean value as well as standard deviation are lower for the optimized γ compared to parameters with fixed γ = 1. It is the first indication that optimization of three parameters α, β, γ results in a higher accuracy of the model. The variation of values of parameters α, β, γ is again great. The fitting of γ did not result in narrowing this variation when compared to model with γ = 1, see nearly the same values of standard deviation of SFRE in both models. The low correlation coefficients α_1:α_2, β_1:β_2 and γ_1:γ_2 confirm our earlier assumption that the porous systems in matrix and in structural domains are substantially different. The characteristics of the model efficiency RMSE and RSE in Table 9 confirm a higher accuracy of the model with fitted parameters α, β, γ. The maximum relative error MRE is in this model lower than in model with the fixed value γ = 1 (Table 6). The good agreement between the measured and fitted data $K(h)$ is demonstrated on the example of UNSODA 4672 in Figure 3. A substantial advantage of the model on conductivity in bi-modal soils with lognormal pore size distribution is in Figure 4 where the conductivities of matrix and structural domains are plotted separately. The separation of unsaturated conductivity in structural domain is important for the solution of preferential flow.

Table 8. Parameters α, β, γ in unsaturated conductivity relationship (Eq. 17); index 1 is for the matrix domain and index 2 is for the structural domain; sum of fitted relative errors SFRE used in the optimization process is defined by Eq. (19).

Soil	α_1	β_1	γ_1	SFRE	α_2	β_2	γ_2	SFRE
4040	4.09	0.28	5.60	1.643	−7.05	0.60	9.08	0.183
4041	9.47	−0.63	−4.93	2.741	4.10	1.60	0.12	3.078
4660	9.12	1.32	−0.15	4.264	−0.44	−1.29	9.96	2.306
4661	10.39	2.24	−0.12	8.996	10.0	0.54	−4.91	0.483
4670	4.41	0.89	0.30	0.482	1.12	−0.86	9.79	3.091
4671	2.18	1.66	0.49	0.141	−0.39	6.13	0.22	2.058
4672	7.80	2.51	0.12	0.711	1.29	−0.61	6.60	1.528
SO 15	−9.16	0.72	5.91	1.083	−0.31	2.54	0.60	1.528
SO 60a	−9.73	0.45	8.56	1.173	−8.57	0.22	9.39	0.329
SO 60b[a]	−9.41	0.61	6.28	1.111	1.80	1.64	−0.10	0.737
Mean	1.92	1.00	2.21	2.234	0.16	1.05	4.08	1.424
SD	7.83	0.90	3.94	2.670	4.96	2.04	5.17	1.142
α_1:α_1[b]	0.49							
β_1:β_1[b]		−0.06						
γ_1:γ_1[b]			0.23					

[a]the same soil as 60a, but h_A was estimated by optimization
[b]correlation coefficient between parameters of matrix and structural domains

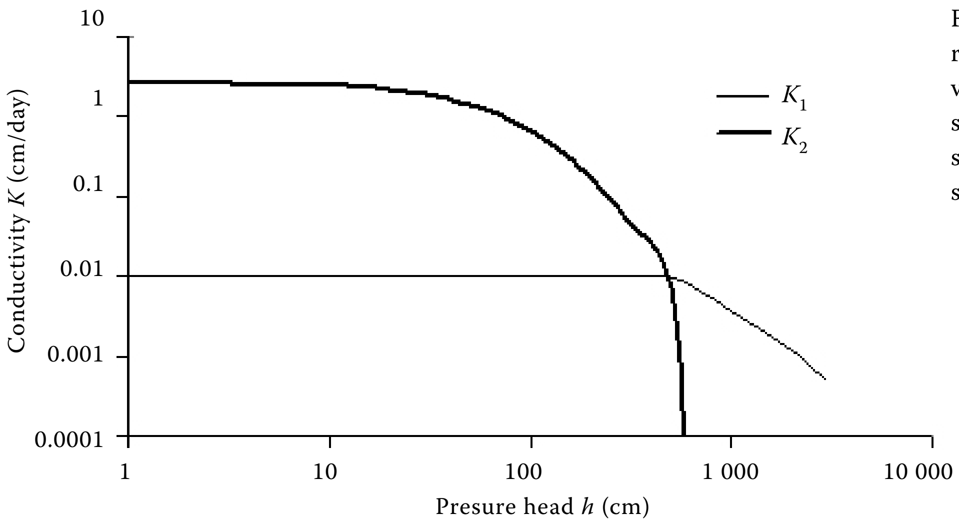

Figure 4. Fitted unsaturated hydraulic conductivities of the matrix (K_1) and structural (K_2) domains in soil UNSODA 4672 plotted separately from Figure 3

The relatively small change in the estimate of h_A and the consequent small change in parameters of the water retention function leads to substantial change of the fitted $K(h)$, see the great difference in the characteristic of model efficiency RSE for SO 60a and SO 60b in Table 9. When we compare the characteristics of model efficiency between water retention function and the unsaturated conductivity function we find that Eq. (8) for water retention is closer to experimental data than Eq. (17) valid for unsaturated conductivity. It means that the errors of estimates of $\theta(h)$ are magnified when Eq. (8) enters into Eq. (1), or in a general form, when Eq. (6) is included into Eq. (7). This is the indication that further research on formulation of pore connectivity into water retention function could bring better results in unsaturated conductivity.

The linkage between the parameters α, β, γ of the unsaturated conductivity function and the soil micromorphological characteristics is not so straight as in the Mualem model. The simplest indications of this statement are negative values of parameters in 28% of instances. But the research performed by Verwoort & Cattle (2003) on this theme for mono-modal soils in Australia looks as promising in further research where the micromorphologic observation is quantified and related to parameters.

Table 9. Characteristics of the conductivity model accuracy; RMSE (Eq. 20), RSE (Eq. 21), maximum absolute error MAE and maximum relative error MRE for optimized solution of α, β, γ in unsaturated conductivity function, Eqs (17) and (18)

Soil	RMSE	RSE	MAE	at h	MRE	at h
4040	16.66	0.034	79.9	1	0.81	307
4041	0.27	0.042	1.30	1	2.14	1846
4660	24.54	0.023	66.2	5	6.56	30
4661	31.66	0.0076	84.6	5	4.0	3000
4670	6.74	0.049	15.4	5	1.78	100
4671	0.40	0.011	0.83	50	1.68	100
4672	0.06	0.003	0.22	20	1.56	2000
SO 15	1.12	0.14	4.0	1	0.66	100
SO 60a	0.33	0.007	1.1	1	0.60	70
SO 60b[a]	0.28	0.044	0.97	6	0.60	70

[a]the same soil as 60a, but h_A was estimated by optimization

Table 10. Evaluation of maximum relative errors MRE_1 in matrix and MRE_2 in structural domains for Mualem's parameters α = 0.5, β = 1, γ = 2 in the unsaturated conductivity Equation (17); characteristics of the conductivity model accuracy RMSE (Eq. 20), RSE (Eq. 21) and maximum relative error MRE for the unsaturated conductivity in the whole range of *K*(*h*), Eqs (17) and (18)

Soil	MRE_1	MRE_2	The whole *K*(*h*)			
			RMSE	RSE	MRE	at *h*
4040	20	0.03	18.73	0.04	0.03	52
4041	0.45	9.10	0.90	0.48	0.45	319
4660	3.0E11	4.10	87.50	0.29	3.0E11	2000
4661	2.0E10	2.06	154.30	0.18	2.0E10	30
4670	22.25	37.40	31.80	1.09	37.40	50
4671	5.66	0.39	2.71	0.53	0.39	15
4672	2.58	12.25	0.84	0.55	12.25	100
SO 15	2.40	4.73	0.58	0.038	4.73	15
SO 60a	2.18	2.00	0.90	0.049	2.18	39
SO 60b[a]	1.40	5.90	1.05	0.60	5.90	6

[a]the same soil as 60a, but h_A was estimated by optimization

In the third optimization model we used Eq. (17) based upon Eq. (1) and (8) but the parameters were kept fixed according to Mualem´s model where α = 0.5, β = 1, γ = 2. The characteristics RMSE and RSE are substantially higher in Mualem´s model, in many instances by more than one order of magnitude (Table 10). The simplification introduced by α = 0.5, β = 1, γ = 2 leads to very poor efficiency of the model of *K*. In some instances the Mualem's parameters did fit better to matrix domain than to structural domain, in other instances it was vice versa, but generally the differences between

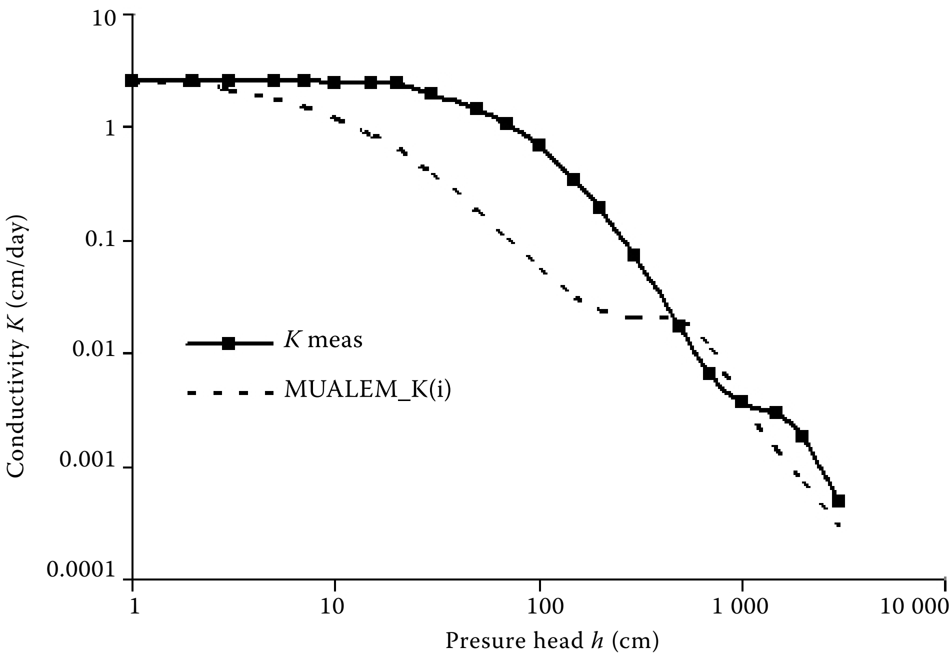

Figure 5. Unsaturated hydraulic conductivity measured (*K* meas) and computed (MUALEM_K(i)) for parameters of soil water retention curve in Table 3 with fixed values of parameters $α_i$ = 0.5, $β_i$ = 1, $γ_i$ = 2 in Eq. (17) for soil UNSODA 4672

the fitted and measured *K* data were extremely great, see also Figure 5. It means that the fixed values of parameters α, β, γ are not suitable for our model at all.

CONCLUSIONS

We modified Kosugi-Pachepsky´s water retention function and Kosugi´s unsaturated conductivity function to bi-modal soils where two porous systems exist, one in the matrix domain and the second one in the structural domain. The models of both systems have the lognormal pore size distribution. The pressure head separating the two domains is not a fixed constant value for all soils and we found its value in a very broad range. We used the optimization procedure for the construction of water retention functions in each of the two domains separately. Saturated hydraulic conductivity of each of the two domains was determined. Its value in the structural domain was by about two orders of magnitude higher than in the matrix domain. We obtained separated unsaturated hydraulic conductivity functions for matrix and structural domains. The parameters of the unsaturated hydraulic conductivity function in individual domains differ substantially and indicate that the porous systems in matrix and structural domains differ substantially, too. Assumption on $\gamma = 1$ caused a worsening of the model efficiency. The use of fixed Mualem´s parameters $\alpha = 0.5$, $\beta = 1$, $\gamma = 2$ brought great errors in the conductivity function, mainly in the structural domain.

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