

## A linkage among whole-stand model, individual-tree model and diameter-distribution model

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**ABSTRACT:** Stand growth and yield models include whole-stand models, individual-tree models and diameter-distribution models. In this study, the three models were linked by forecast combination and parameter recovery methods one after another. Individual-tree models combine with whole-stand models through forecast combination. Forecast combination method combines information from different models, disperses errors generated from different models, and then improves forecast accuracy. And then the forecast combination model was linked to diameter-distribution models via parameter recovery methods. During the moment estimation, two methods were used, arithmetic mean diameter and quadratic mean diameter method (A-Q method), and arithmetic mean diameter and diameter variance method (A-V method). Results showed that the forecast combination for predicting stand variables outperformed over the stand-level and tree-level models respectively; A-V method was superior to A-Q method on estimating Weibull parameters; these three different models could be linked very well via forecast combination and parameter recovery.

**Keywords:** forecast combination; linkage; parameter recovery; stand growth and yield model

In forest management, forest growth and yield models play a very important role in studying forest growth processes and predicting forest growth. Forest growth and yield models can be classified into three broad categories: whole-stand models, individual-tree models, and diameter-distribution models (MUNRO 1974). Whole-stand models are models that use the stand as a modelling unit (CURTIS et al. 1981; LI et al. 1988; TANG et al. 1993; WEI 2006), whereas individual-tree models take the individual tree as a studied object (ZHANG et al. 1997; CAO 2000; CAO et al. 2002; ZHANG, LEI 2009). Diameter-distribution models, in contrast, use statistical probability functions, such as the Weibull function (BAILEY, DELL 1973; MENG 1988; LIU et al. 2004; NEWTON et al. 2005), beta function (GORGOSO-VÁRELA et al. 2008) or SB function (WANG, RENNOLLS 2005). There are strengths and weaknesses of each type of model. Whole-stand

models can predict stand variables directly, but they lack detailed tree-level information. On the other hand, individual-tree models provide more detailed information, and diameter-distribution models offer the stand diameter structure, but stand-level outputs from these two types of models often suffer from an accumulation of errors and subsequently poor accuracy and precision (MENG 1996; GARCIA 2001; QIN, CAO 2006).

For further studying forest growth models, foresters proposed that these three types of models should be considered to link one model to another rather than being used completely separately. The parameter-recovery method was used to link the whole-stand model to the diameter-distribution model (HYINK, MOSER 1983; LYNCH, MOSER 1986) and the individual-tree model to the diameter-distribution model (BAILEY 1980; CAO 1997). A linkage between the whole-stand model and the

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Supported by the MOST, Projects No. 2006BAD23B02, No. 2005DIB5JI42, and No. CAFYBB2008008.

individual-tree model was established by the disaggregation method and forecast combination method to improve accuracy and compatibility (ZHANG et al. 1993; RITCHIE, HANN 1997; QIN, CAO 2006; YUE et al. 2008). However, to our knowledge, no rigorous linkage among the three types of models has been documented so far. The objective of this study was to link three different models by the forecast combination method and parameter-recovery method one after another.

## MATERIAL AND METHODS

The data, provided by the Inventory Institute of Beijing Forestry, consisted of a systematic sample of permanent plots with a 5-year re-measurement interval. The plots, 0.067 ha each, were in Chinese pine (*Pinus tabulaeformis*) plantations situated on upland sites throughout northwestern Beijing. The data consisted of 156 measurements, with a 5-year re-measurement interval, obtained in the following years: 1986, 1991, 1996 and 2001. In this study, 106 plots were used in model development, and

Table 1. Distributions of plots

Measurement time	Fit data	Validation data	Total
1986–1991	27	12	39
1991–1996	37	17	54
1996–2001	42	21	63
Total	106	50	156

Table 2. Statistics of stand variables and tree variable

Variables	Fit data				Validation data			
	Min	Max	Mean	SD	Min	Max	Mean	SD
Age (years)	11	55	30	8.12	13	60	30	8.81
Dominant height (m)	0.4	17.4	6.87	2.50	2.7	17.4	7.08	3.10
No. of trees (trees·ha <sup>-1</sup> )	238.73	2283.58	1199.63	526.81	238.81	2089.55	1178.98	469.66
Quadratic-mean diameter (cm)	5.76	17.33	10.77	2.46	5.70	17.86	10.76	2.90
Arithmetic-mean diameter (cm)	5.73	17.01	10.33	2.30	5.66	17.43	10.40	2.79
Min-diameter (cm)	5	10.1	5.50	0.87	5	9.7	5.66	1.07
Stand basal area (m <sup>2</sup> ·ha <sup>-1</sup> )	0.80	33.10	11.21	6.31	0.61	28.06	10.87	6.14
Diameter at breast (cm)	5	36.8	10.46	3.91	5	30.9	10.05	3.48

SD – standard deviation

another 50 plots for validation. Table 1 shows the distribution of plots. Summary statistics for both data sets are presented in Table 2.

CAO (2002) developed a variable rate method to predict annual diameter growth and survival for an individual tree. This method was based on the fact that rates of survival and diameter growth vary from year to year. Stand-level growth and survival were also treated in a similar manner (OCHI, CAO 2003).

Because the quadratic mean diameter ( $Dg$ ) is equal to or greater than the arithmetic mean diameter ( $Dm$ ) (CURTIS, MARSHALL 2000), the arithmetic mean diameter was modelled using the equation (DIÉGUEZ-ARANDA et al. 2006):

$$Dm = Dg - Exp(X\delta) \quad (1)$$

where:  $X$  is the vector of stand variables (e.g. dominant height, stand age and stand density) and  $\delta$  is the vector of parameters to be estimated.

The variable rate method was used in this study. Annual changes in dominant height, stand survival, quadratic mean diameter, arithmetic mean diameter, diameter standard deviation, minimum diameter, stand basal area, diameter, and survival probability were described in recursive manner (OCHI, CAO 2003; QIN et al. 2007; CAO, STRUB 2008). Table 3 lists the stand-level and tree-level growth equations.

Estimates of individual-tree diameters at age  $t+q$  were obtained by the tree diameter growth model (equation 13.h) and then  $\hat{D}g^T$ ,  $\hat{D}m^T$  and  $\hat{D}sd^T$  were calculated for each plot at age  $t+q$ . Stand survival was calculated with tree survival probability.

Table 3. List of the recursive stand-level and tree-level growth equations.

Year (t+1)	
$H_{t+1} = \text{Exp}[(A_t / A_{t+1})\text{Ln}(H_t) + (1 - A_t / A_{t+1})(\alpha_1 + \alpha_2 / A_t + \alpha_3 H_t)]$	(12.a)
$N_{t+1} = \text{Exp}\{(A_t / A_{t+1})\text{Ln}(N_t) + (1 - A_t / A_{t+1})[\beta_1 + \beta_2 / A_t + \beta_3 \text{Ln}(N_t)]\}$	(12.b)
$Dg_{t+1} = \text{Exp}[(A_t / A_{t+1})\text{Ln}(Dg_t) + (1 - A_t / A_{t+1})(\chi_1 + \chi_2 / A_t + \chi_3 H_t)]$	(12.c)
$Dm_{t+1} = Dg_{t+1} - \text{Exp}[\delta_1 + \delta_2 / A_t + \delta_3 / \text{Ln}(N_t) + \delta_4 H_t + \delta_5 Dm_t]$	(12.d)
$B_{t+1} = \text{Exp}\{(A_t / A_{t+1})\text{Ln}(B_t) + (1 - A_t / A_{t+1})[\phi_1 + \phi_2 H_t + \phi_3 / \text{Ln}(N_t)]\}$	(12.e)
$Dsd_{t+1} = \text{Exp}\{(A_t / A_{t+1})\text{Ln}(Dsd_t) + (1 - A_t / A_{t+1})[\gamma_1 + \gamma_2 \text{Ln}(H_t) + \gamma_3 \text{Ln}(N_t)]\}$	(12.f)
$D\text{min}_{t+1} = \text{Exp}\{(A_t / A_{t+1})\text{Ln}(D\text{min}_t) + (1 - A_t / A_{t+1})[\kappa_1 + \kappa_2 / A_t + \kappa_3 / \text{Ln}(N_t)]\}$	(12.g)
$D_{i,t+1} = D_{i,t} + \text{Exp}[\lambda_1 + \lambda_2 A_t / A_{t+1} + \lambda_3 B_t + \lambda_4 / Rs_t + \lambda_5 / \text{Ln}(D_{i,t})]$	(12.h)
$P_{i,t+1} = \{1 + \text{Exp}[\mu_1 + \mu_2 / A_t + \mu_3 D_t / \text{Ln}(Dg_t) + \mu_4 / \text{Ln}(N_t)]\}^{-1}$	(12.i)
Year (t + q)	
$H_{t+q} = \text{Exp}[(A_{t+q-1} / A_{t+q})\text{Ln}(H_{t+q-1}) + (1 - A_{t+q-1} / A_{t+q})(\alpha_1 + \alpha_2 / A_{t+q-1} + \alpha_3 H_{t+q-1})]$	(13.a)
$N_{t+q} = \text{Exp}\{(A_{t+q-1} / A_{t+q})\text{Ln}(N_{t+q-1}) + (1 - A_{t+q-1} / A_{t+q})[\beta_1 + \beta_2 / A_{t+q-1} + \beta_3 \text{Ln}(N_{t+q-1})]\}$	(13.b)
$Dg_{t+q} = \text{Exp}[(A_{t+q-1} / A_{t+q})\text{Ln}(Dg_{t+q-1}) + (1 - A_{t+q-1} / A_{t+q})(\chi_1 + \chi_2 / A_{t+q-1} + \chi_3 H_{t+q-1})]$	(13.c)
$Dm_{t+q} = Dg_{t+q} - \text{Exp}[\delta_1 + \delta_2 / A_{t+q-1} + \delta_3 / \text{Ln}(N_{t+q-1}) + \delta_4 H_{t+q-1} + \delta_5 Dm_{t+q-1}]$	(13.d)
$B_{t+q} = \text{Exp}\{(A_{t+q-1} / A_{t+q})\text{Ln}(B_{t+q-1}) + (1 - A_{t+q-1} / A_{t+q})[\phi_1 + \phi_2 H_{t+q-1} + \phi_3 / \text{Ln}(N_{t+q-1})]\}$	(13.e)
$Dsd_{t+q} = \text{Exp}\{(A_{t+q-1} / A_{t+q})\text{Ln}(Dsd_{t+q-1}) + (1 - A_{t+q-1} / A_{t+q})[\gamma_1 + \gamma_2 \text{Ln}(H_{t+q-1}) + \gamma_3 \text{Ln}(N_{t+q-1})]\}$	(13.f)
$D\text{min}_{t+q} = \text{Exp}\{(A_{t+q-1} / A_{t+q})\text{Ln}(D\text{min}_{t+q-1}) + (1 - A_{t+q-1} / A_{t+q})[\kappa_1 + \kappa_2 / A_{t+q-1} + \kappa_3 / \text{Ln}(N_{t+q-1})]\}$	(13.g)
$D_{i,t+q} = D_{i,t+q-1} + \text{Exp}[\lambda_1 + \lambda_2 A_{t+q-1} / A_{t+q} + \lambda_3 B_{t+q-1} + \lambda_4 / Rs_{t+q-1} + \lambda_5 / \text{Ln}(D_{i,t+q-1})]$	(13.h)
$P_{i,t+q} = \{1 + \text{Exp}[\mu_1 + \mu_2 / A_{t+q-1} + \mu_3 D_{t+q-1} / \text{Ln}(Dg_{t+q-1}) + \mu_4 / \text{Ln}(N_{t+q-1})]\}^{-1}$	(13.i)

$R_t = (10,000 / N_t)^{0.5} / H_t$  = the relative spacing at age  $A_t$ ,  $q$  = length of growth period in years (in this case,  $q = 5$ ),  $H_t$  = dominant height in m at age  $A_t$ ,  $N_t$  = number of trees per ha at age  $A_t$ ,  $D_{gt}$  = quadratic mean diameter in cm at age  $A_t$ ,  $D_{mt}$  = arithmetic mean diameter in cm at age  $A_t$ ,  $B_t$  = stand basal area in  $\text{m}^2 \cdot \text{ha}^{-1}$  at age  $A_t$ ,  $Dsd_t$  = diameter standard deviation in cm at age  $A_t$ ,  $D\text{min}_t$  = minimum diameter in cm at age  $A_t$ ,  $D_{i,t}$  = diameter of tree  $i$  at age  $A_t$ ,  $p_{i,t+1}$  = probability that tree  $i$  is survived the period for age  $A_t$  to  $A_{t+1}$ ,  $\alpha_1, \alpha_2, \dots, \mu_4$  = parameters to be estimated

Since cross-equation correlations existed among error components of the above models, to eliminate the bias and inconsistency of the regression system (equation a–h), the method of seemingly unrelated regression (SUR) was used to simultaneously estimate the regression system (equation a–h). This method was widely used in econometrics (JOHNSON 1991) and in forest biometrics (BORDERS, BAILEY 1986; BORDERS 1989; OCHI, CAO 2003). The fitting procedure involved the use of option SUR of the SAS procedure model. Parameters of the tree survival equation were separately estimated by use of NLIN procedure.

### Forecast combination

Forecast combination, introduced by BATES and GRANGER (1969), is a good method for improving forecast accuracy (NEWBOLD et al. 1987). The method combines information generated from different models and disperses errors from these models, thus improves consistency for outputs from different models. YUE et al. (2008) and ZHANG et al. (2009) applied forecast combination to combine models from stand-level and tree-level. The forecast combination model is expressed as follows:

$$Y^C = \omega Y^T + (1-\omega)Y^S \quad (2)$$

Thus, the variance of the forecast combination is as follows:

$$\sigma_C^2 = \omega^2 \sigma_T^2 + (1-\omega)^2 \sigma_S^2 + 2\omega(1-\omega)\sigma_{TS} \quad (3)$$

According to the method of calculating weights, a variance and covariance method was used broadly (ZHANG et al. 2006; YUE et al. 2008):

$$\omega = \frac{\sigma_S^2 - \sigma_{TS}}{\sigma_T^2 + \sigma_S^2 - 2\sigma_{TS}} \quad (4)$$

$$1-\omega = \frac{\sigma_T^2 - \sigma_{TS}}{\sigma_T^2 + \sigma_S^2 - 2\sigma_{TS}} \quad (5)$$

where:

- $Y^C$  – combined estimates of stand variables,
- $Y^T$  – estimates of stand variables at tree-level,
- $Y^S$  – estimates of stand variables at stand-level,
- $\omega$  – weight factor,
- $\sigma_T^2$  – variance of stand variables at tree-level,
- $\sigma_S^2$  – variance of stand variables at stand-level,
- $\sigma_{TS}$  – covariance of stand variables between the tree-level and stand-level.

### Parameter-recovery method

The Weibull function has been extensively applied in forestry because of its flexibility in describing a wide range of unimodal distributions and the relative simplicity of parameter estimation (BAILEY,

DELL 1973; KANGAS, MALTAMO 2000; MABVURI-RA et al. 2002; LEI 2008). The Weibull probability density function is expressed as follows:

$$f(x; a, b, c) = \frac{c}{b} \left(\frac{x-a}{b}\right)^{c-1} \exp\left[-\left(\frac{x-a}{b}\right)^c\right] \quad (a \leq x \leq \infty) \quad (6)$$

where:

- $x$  – diameter at breast height,
- $a$  – the location parameter,
- $b$  – the scale parameter,
- $c$  – the shape parameter.

Moment estimation is one of the methods about parameter recovery for estimating Weibull parameters and has been used broadly (LIU et al. 2004; LEI 2008). Considering that the location parameter ( $a$ ) must be smaller than the predicted minimum diameter ( $\hat{D}_{\min}$ ) in the stand, we set  $a = 0.5\hat{D}_{\min}$  since FRAZIER (1981) found that this resulted in minimum errors in terms of goodness of fit.

Two methods were used to recover  $b$  and  $c$  in the moment estimation. Method 1 is arithmetic mean diameter ( $\hat{D}m$ ) and quadratic mean diameter ( $\hat{D}g$ ) method (A-Q method) as follows (LIU et al. 2004):

$$\begin{cases} b = (\hat{D}m - a) / \Gamma_1 \\ \hat{D}g^2 + a^2 - 2a\hat{D}m - b^2\Gamma_2 = 0 \end{cases} \quad (7)$$

where:  $\Gamma_1 = \Gamma(1 + 1/c)$ ,  $\Gamma_2 = \Gamma(1 + 2/c)$ .

Method 2 is arithmetic mean diameter and diameter variance ( $\hat{D}var$ ) method (A-V method) (DIÉGUEZ-ARANDA et al. 2006; QIN et al. 2007). A possible problem of method 1 is that  $\hat{D}g$  might be too close to or too far from  $\hat{D}m$ , and can even be smaller than  $\hat{D}m$  if not properly constrained. The resulting Weibull parameters are sensitive to the difference between  $\hat{D}m$  and  $\hat{D}g$ , resulting in unstable estimators of  $b$  and  $c$ . The A-V method is expressed as follows:

$$\begin{cases} b = (\hat{D}m - a) / \Gamma_1 \\ \hat{D}var - b^2(\Gamma_2 - \Gamma_1^2) = 0 \end{cases} \quad (8)$$

Finally, the forecast combination combines stand variables from tree-level and stand-level models to predict  $\hat{D}g^c, \hat{D}m^c, \hat{D}sd^c, \hat{D}min^c$  and  $\hat{N}^c$ ; and then Weibull parameters  $b$  and  $c$  were estimated using the stand variables of the forecast combination models based on the two moment methods (equations 7 and 8). More detailed procedures of this study are shown in Fig. 1.

### Model evaluation

Model evaluation was performed for both growth models and goodness of fit for the diameter distribution model. For growth models, the following evaluation statistics were calculated:

R-square

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y}_i)^2} \quad (9)$$

Log Likelihood

$$-2\ln(L) = -2\{\sum p_i \ln(p_i) + \sum (1-p_i) \ln(1-p_i)\} \quad (10)$$

and the evaluation of goodness of fit is error index (e), expressed as follows (REYNOLDS et al. 1988; LIU et al. 2004):

$$e = \sum_j^m |P_j - O_j| \quad (11)$$

where:

- $y_i$  – observed value at age  $A_{t+q}$  of stand variables (arithmetic mean diameter, quadratic mean diameter, diameter standard deviation, minimum diameter or number of trees) or diameter of tree  $i$ ,
- $\hat{y}_i, \bar{y}_i$  – predicted value and average of  $y_i$ , respectively,
- $p_i$  – probability of tree  $i$  survival,
- $m$  – number of classes for each plot,
- $P_j, O_j$  – the predicted and observed number of trees per plot within each diameter class  $j$ , respectively.

## RESULTS AND DISCUSSION

The estimates and standard deviation errors of parameters of the different growth models are presented in Table 4. The estimates and standard deviation errors showed that all the parameters were significant ( $P$ -value < 0.0001), and  $R^2$  values were 0.9266, 0.8983, 0.8787, 0.5392, 0.8802 and 0.9148 for the quadratic mean diameter model, arithmetic mean diameter model, diameter standard deviation model, minimum diameter model, stand survival model and diameter growth model at the stand level, respectively. Log-likelihood of the tree survival model was -782.104.

Table 5 summarizes the gains in efficiency of stand variable models from tree-level, stand-level and forecast combination (e.g. YUE et al. 2008). For the data subset used for fitting the models, the efficiency for the combined quadratic mean diameter estimator was 100, as compared to 100.83, 104.38 for the tree-level and stand-level, and  $\sigma_C^2$  for the combined estimator was 0.3977 versus 0.4010, 0.4151; the efficiency for the arithmetic mean diameter was 100, as compared to 97.99, 119.03, and  $\sigma_C^2$  was 0.4219 vs. 0.4134, 0.5022; the efficiency for the diameter standard deviation was 100, as compared to 105.11, 103.03, and  $\sigma_C^2$  was 0.0958 versus 0.1007, 0.0987; the efficiency for the minimum diameter was 100,

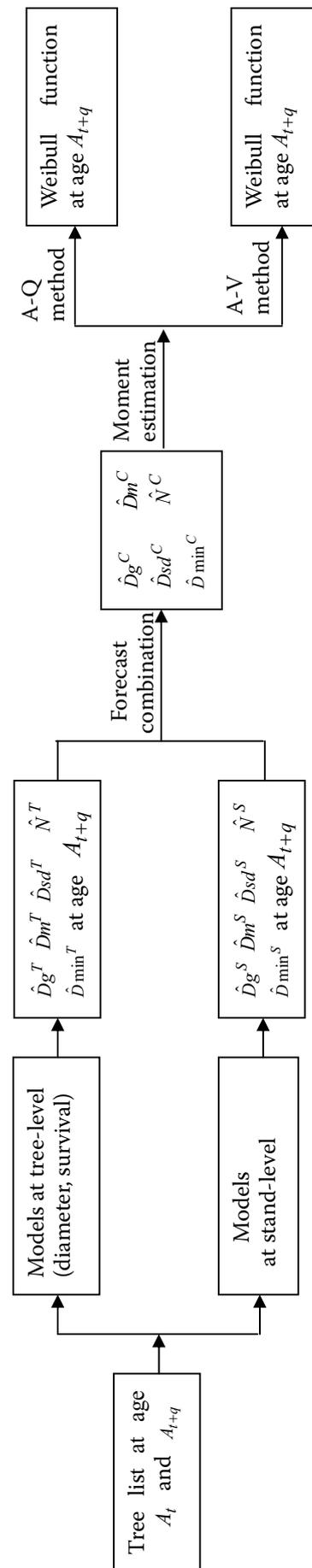


Fig. 1. Flow chart

Table 4. Parameter estimates and model evaluation

Attribute	Parameter	Estimate	SE	$R^2$
Quadratic – mean diameter (cm) (equation 13.c)	$\chi_1$	3.3940	0.0191	0.9266
	$\chi_2$	-10.5788	0.3026	
	$\chi_3$	0.0094	0.0015	
Arithmetic – mean diameter (cm) (equation 13.d)	$\delta_1$	-3.9549	0.1169	0.8983
	$\delta_2$	-27.5352	1.1346	
	$\delta_3$	21.2138	0.6141	
	$\delta_4$	0.0258	0.0024	
	$\delta_5$	0.0733	0.0038	
Diameter std. (cm) (equation 13.f)	$\gamma_1$	1.4519	0.0952	0.8787
	$\gamma_2$	0.5065	0.0187	
	$\gamma_3$	-0.0840	0.0135	
Minimum diameter (cm) (equation 13.g)	$\kappa_1$	1.9212	0.0975	0.5392
	$\kappa_2$	-8.6532	0.6425	
	$\kappa_3$	3.1075	0.6983	
Stand survival (trees-ha <sup>-1</sup> ) (equation 13.b)	$\beta_1$	2.7193	0.1625	0.8802
	$\beta_2$	17.8950	0.6520	
	$\beta_3$	0.5664	0.0215	
Diameter at breast (cm) (equation 13.h)	$\lambda_1$	16.0367	0.8744	0.9148
	$\lambda_2$	-17.2105	0.9013	
	$\lambda_3$	-0.0317	0.0029	
	$\lambda_4$	0.1382	0.0166	
	$\lambda_5$	-1.4525	0.1415	
Tree survival (equation 13.i)	$\mu_1$	7.6063	1.3892	-782.104 (-2lnL)
	$\mu_2$	-102.9	12.7234	
	$\mu_3$	-0.3895	0.0607	
	$\mu_4$	-45.0114	8.9032	

SE – standard error,  $R^2$  – multiple coefficient of determination

as compared to 121.77, 101.57, and  $\sigma_C^2$  was 0.3749 versus 0.4565, 0.3808; the efficiency for the stand survival was 100, as compared to 111.91, 100.015, and  $\sigma_C^2$  was 26,494.03, versus 29,648.46, 26,535.09. Overall, except one, the combined estimators were better than those from tree-level and stand-level models for both fit and validation data. The only exception was the arithmetic mean diameter model for the fit data. Fig. 2 illustrates the relationships between the observed quadratic mean diameter and predicted value by the three models for the validation data. It is obvious that the forecast combination achieved the beneficial effect of the highest value  $R^2$  (taking quadratic mean diameter as an example). The combined predictions were based on the optimal weights which are derived by the variance-covariance method (NEWBOLD, GRANDER1974) of the

two respective level models. Therefore, these estimators performed minimum variance and high precision (BATES, GRANDER 1969; JEONG, KIM 2009) in comparison with the single levels.

Table 6 shows the average values and standard deviations of error index ( $e$ ) calculated by two different moment estimation methods. For the data subset used for fitting the models, the average error index value for A-Q method was 509.7407, as compared to 442.1898 for A-V method. SD was 285.1731 versus 254.4337. Obviously, the average error index value and SD of A-V method are much smaller than those of A-Q method for both fit and validation data, respectively. And in the fit data, Weibull parameters of all plots (106 plots) were estimated based on A-V method. But parameters of only 96 plots were estimated by A-Q method. It means that parameters of

Table 5. Evaluation statistics from different models for fit data and validation data

	Attributes	$\sigma^2$		Efficiency (%)	
		fit	validation	fit	validation
Tree-level model	Quadratic mean diameter (cm)	0.4010	0.3340	100.83	103.50
	Arithmetic mean diameter (cm)	<b>0.4134</b>	0.3407	<b>97.99</b>	101.73
	Diameter standard deviation (cm)	0.1007	0.1252	105.11	101.95
	Minimum diameter (cm)	0.4565	0.5454	121.77	100.31
	Stand survival (trees-ha <sup>-1</sup> )	29,648.46	39,805.53	111.91	102.72
Stand-level model	Quadratic mean diameter (cm)	0.4151	0.4789	104.38	148.40
	Arithmetic mean diameter (cm)	0.5022	0.6070	119.03	181.25
	Diameter standard deviation (cm)	0.0987	0.1305	103.03	106.27
	Minimum diameter (cm)	0.3808	0.6929	101.57	127.44
	Stand survival (trees-ha <sup>-1</sup> )	26,535.09	41,340.33	100.15	106.68
Forecast combination model	Quadratic mean diameter (cm)	<b>0.3977</b>	<b>0.3227</b>	<b>100</b>	<b>100</b>
	Arithmetic mean diameter (cm)	0.4219	<b>0.3349</b>	100	<b>100</b>
	Diameter standard deviation (cm)	<b>0.0958</b>	<b>0.1228</b>	<b>100</b>	<b>100</b>
	Minimum diameter (cm)	<b>0.3749</b>	<b>0.5437</b>	<b>100</b>	<b>100</b>
	Stand survival (trees-ha <sup>-1</sup> )	<b>26,494.03</b>	<b>38,751.85</b>	<b>100</b>	<b>100</b>

Efficiency at tree-level =  $100\sigma_T^2/\sigma_C^2$ , efficiency at stand-level =  $100\sigma_S^2/\sigma_C^2$ , efficiency from forecast combination =  $100\sigma_C^2/\sigma_C^2$ , and Value in bold denotes the best statistic among models for each of the fit and validation data sets

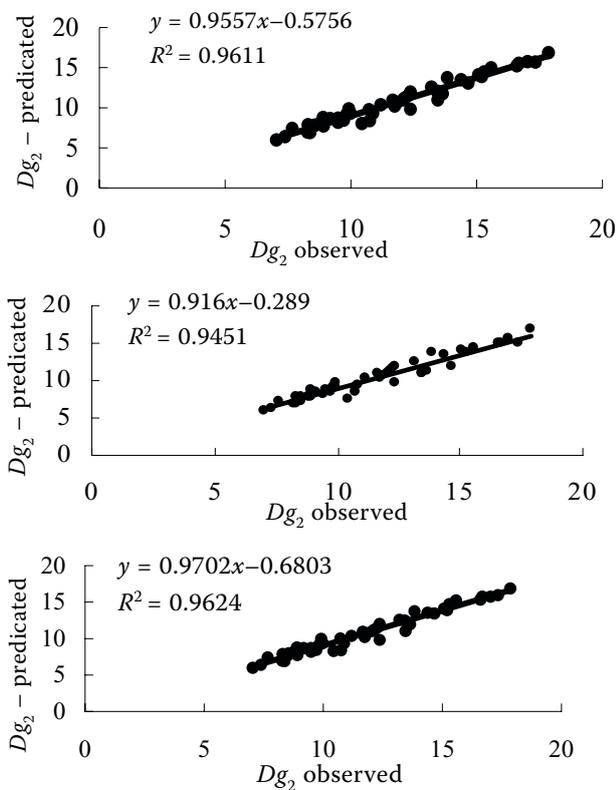


Fig. 2. Relationships between the observed quadratic mean diameter and the predicted value with three models for the validation data

the other 10 plots could not be estimated. It was because  $\hat{D}g^C$  was smaller than  $\hat{D}m^C$  of those 10 plots.

The formula for diameter variance is,  $Dvar = E(D^2) - E(D)^2$  and  $E(D) = Dm$ ,  $E(D^2) = Dg^2$ .  $E(x)$  is the expected value. And  $Dvar > 0$ , then  $Dg > Dm$ . When  $Dg$  is closer to  $Dm$ ,  $Dvar$  approaches 0, and distribution shrinks to a point at  $Dg$ . This kind of Weibull distribution does not exist. So when  $Dg$  is closer to  $Dm$  or  $Dg$  is smaller than  $Dm$ , Weibull parameters could not be estimated by A-Q method. It also verified the fact that it was not suitable to use A-Q method for estimating Weibull parameters. So A-V method outperforms A-Q method in estimating Weibull parameters.

Table 6. Error index based on A-Q method and A-V method

Attribute		A-Q	A-V
Fit data	Mean	509.7407	442.1898
	SD	285.1731	254.4337
Validation data	Mean	533.5493	479.4961
	SD	286.4376	240.311

SD – standard deviation

## CONCLUSIONS

In this study, the forecast combination was used to link tree-level models and stand-level models. It efficiently utilizes information generated from different models, reduces errors from a single model, and improves accuracy and precision. It also ensures that stand variables from tree-level and stand-level models are consistent.

Forecast combination models and diameter distribution models were linked through the parameter recovery method (moment estimation), and the two moment estimation methods were used in this study. It is much more suitable to estimate Weibull parameters on the basis of A-V method than A-Q method. And if  $\hat{D}_m$  is larger than  $\hat{D}_g$  or too close to  $\hat{D}_g$ , Weibull parameters will not be estimated by A-Q method, but they will be estimated by A-V method. So A-V method is superior to A-Q method for estimating Weibull parameters.

Whole-stand models, individual-tree models and diameter models can be linked together through the forecast combination method and the parameter-recovery method one after another. Therefore, this study provided a framework for studying the integrated system of forest models.

## Acknowledgements

The authors would like to thank the Inventory Institute of Beijing Forestry for its data and Dr. QUANG V. CAO for providing his information and SAS code.

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Received for publication October 13, 2010

Accepted after corrections April 12, 2010

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