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Influences of determined and estimated dendrometric variables on the precision of volumetric modelling

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Abstract: The use of independent variables in volumetric modelling is an important step in fitting models to represent tree or stand characteristics. The *DBH* measured at 1.3 m from the ground level and total tree height (*Ht*) are the most commonly used independent variables when modelling individual tree volumes. This work aimed to analyze the importance of independent variables in fitting and selecting volumetric equations. A total of 750 trees from an experiment with three Eucalyptus spp. clones planted in five spacings in the semi-arid region of Pernambuco were used. Four statistical procedures were applied to compare the equations: Adjusted Fit Index (*AFI*), Akaike information criterion (*AIC*), mean absolute percentage error (*MAPE*), and a completely random design having the real tree volume as control and the fit equations as treatments. The error measuring heights in the field (*EH*) was also analyzed. Four heights were evaluated: *Ht*, height estimated in the field (*He*) and heights adjusted (*Ha*) from hypsometric relationships using the *DBH* [*Ha* (a)] and $D_{1.7}$ [*Ha* (b)], which was the diameter most correlated with the volume. The result indicates that all 18 fitted models provided high precision volumetric equations which do not differ at the 5% significance level.

Keywords: estimated height; adjusted height; hypsometric relationships; tree volume

Many dendrometric variables can be used to represent quantitative characteristics of trees or forest stands which are usually represented in volumetric terms. Considering that volume is one of the most important variables to accurately represent the productive capacity of a forest stand, its determination or estimate must be carried out very precisely (Vanclay 1994; Burkhart, Tomé 2012).

Measurements for estimations are generally performed in the field using instruments such as tapes and callipers for measuring diameters or circumferences and hypsometers to estimate heights. Another procedure which has been very frequently used in volumetric estimates of forest stands is a variety of remote sensing techniques (LANDSAT, LiDAR, CBERS and others) (Rex et al. 2019; De Oliveira et al. 2021).

However, the most commonly used technique to estimate tree and forest stand volumes, including hypsometric relationships, is the use of statistical modelling by means of regression analysis (Schumacher, Chapman 1948; Pretzsch 2009; Arney 2016).

The number of equations which predict tree and stand volumes is immense in the forestry literature, but the Schumacher-Hall model (Schumacher, Hall 1933) and the Spurr or combined variable model ($DBH^2 \times H$) (Spurr 1952) are often used.

Artificial intelligence procedures such as artificial neural networks (ANN), machine learning, support vector machine (SVM) and random forests have recently been very frequently used in the forest sector for volumetric modelling (Özçelik et al. 2017; Guera et al. 2018; Silva 2020; Lima et al. 2021). However, they present similar results to traditional methods for large samples ($N > 200$) (De Souza et al. 2018).

In fitting the volumetric models which use the total height (*Ht*) as an independent variable, its mea-

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surement is error-free, because *Ht* is measured directly on the tree during rigorous cubing. However, after the models are fitted, the resulting equations use *He* which generally presents different values when compared with the *Ht* and it is a source of error. Its minimizing will depend a lot on the operator's ability to use hypsometers, precision of the hypsometer, population density and the distance from the tree where the height is estimated.

One way to try to minimize errors in measuring tree heights in the field is to employ regression techniques using hypsometric relationships. Thus, the systematic error of the *Ht* estimate with the use of hypsometers does not exist, but the model fitting error is incorporated into the volumetric estimates.

Da Silva et al. (1992) developed a methodology for fitting volumetric models in which the tree volume (dependent variable) is modelled according to a combination of volumes at the tree trunk base whose height ranges from 0.3 m to 1.7 m. In addition to eliminating the *Ht* or *He* variable from the model, results have shown similar precision to the traditional models used in forest measurement (Da Silva, Borders 1993; Lynch 1995; Ferreira et al. 2011).

Thus, the objective of this work was to analyze the behaviour of different dendrometric variables used as independent variables, being measured directly and/or indirectly in fitting volumetric models.

MATERIAL AND METHODS

Data from the rigorous cubing of 750 *Eucalyptus* clone trees were used. The trees were randomly selected from an experiment at the Experimental Station of the Instituto Agronômico de Pernambuco (IPA), Araripina, Pernambuco, with geographic coordinates 07° 27'37''S and 40°24'36''W, and an altitude of 831 m. The soils are type LA 19 defined as yellow Latosol + red-yellow Latosol (Santos et al. 2006), with a tropical semi-arid climate characterized as BshW' type according to the Köppen classification. An average annual rainfall of 760 mm results in water deficiencies accumulated during the months when there is minimal precipitation. Three *Eucalyptus* clones were used: (a) *Eucalyptus urophylla* hybrid (natural crossing; C39); (b) *Eucalyptus brassiana* hybrid (natural crossing; C11); (c) *Eucalyptus urophylla* hybrid (natural crossing; C41).

The clones were planted at five spacings 2 × 1 m; 2 × 2 m; 3 × 2 m; 3 × 3 m and 4 × 2 m, totalling 15 clone combinations with spacing. Next, 50 trees were randomly selected from each combination and grouped by diametric classes with an amplitude of 3.0 cm. The *DBH* ranged from 3.0 to 21.2 cm and the height *Ht* ranged from 4.5 to 20.5 m.

All tree heights were estimated (*He*) in the field using the Haglöf hypsometer to the nearest 10 cm. Afterwards, the trees were strictly cubed by the Smalian method, with the total tree height (*Ht*) and the diameters at 0.30, 0.50, 0.70, 0.90, 1.10, 1.30, 1.50, and 1.70 m being measured with a calliper to the nearest 1.0 cm; then diameters from 2.30 m were measured at each meter up to the height of the commercial volume (minimum diameter of 2.0 cm). Thus, 28 section volumes between 0.30 m and 1.70 m were calculated with the different diameters in the first log using the Smalian method (Table 1).

Table 1. Different volumes with respective sectional areas and section lengths

Volume	D_i	D_s	L (m)
v_1	$D_{0.3}$	$D_{0.5}$	0.2
v_2	$D_{0.5}$	$D_{0.7}$	0.2
⋮	⋮	⋮	⋮
v_7	$D_{1.5}$	$D_{1.7}$	0.2
v_8	$D_{0.3}$	$D_{0.7}$	0.4
v_9	$D_{0.5}$	$D_{0.9}$	0.4
⋮	⋮	⋮	⋮
v_{13}	*DBH*	$D_{1.7}$	0.4
v_{14}	$D_{0.3}$	$D_{0.9}$	0.6
v_{15}	$D_{0.5}$	$D_{1.1}$	0.6
⋮	⋮	⋮	⋮
v_{18}	$D_{1.1}$	$D_{1.7}$	0.6
v_{19}	$D_{0.3}$	$D_{1.1}$	0.8
v_{20}	$D_{0.5}$	*DBH*	0.8
⋮	⋮	⋮	⋮
v_{22}	$D_{0.9}$	$D_{1.7}$	0.8
v_{23}	$D_{0.3}$	*DBH*	1.0
v_{24}	$D_{0.5}$	$D_{1.5}$	1.0
v_{25}	$D_{0.7}$	$D_{1.7}$	1.0
v_{26}	$D_{0.3}$	$D_{1.5}$	1.2
v_{27}	$D_{0.5}$	$D_{1.7}$	1.2
v_{28}	$D_{0.3}$	$D_{1.7}$	1.4

D_i – diameter at the section base; *Ds* – diameter at the end of the section; *DBH* – diameter at breast height; *L* – section length

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The different volumes in the first log (v_i) for model (d) were calculated as follows (Equations 1–28):

$$v_1 = \frac{(g_{0.3} + g_{0.5})}{2} \times (0.2) = \left(\frac{\pi}{4}\right) \times \left(\frac{D_{0.3}^2 + D_{0.5}^2}{2}\right) \times (0.2) \quad (1)$$

$$\vdots$$

$$v_{28} = \frac{(g_{0.3} + g_{1.7})}{2} \times (1.4) = \left(\frac{\pi}{4}\right) \times \left(\frac{D_{0.3}^2 + D_{1.7}^2}{2}\right) \times (1.4) \quad (28)$$

The simple linear model was adjusted in two situations for the tree height (*Ha*) as a function of the *DBH*, and as a function of the diameter of the first (*Dk*) log most correlated with the volume.

Next, descriptive statistics and the correlation matrix among all measured variables were calculated with the volume data per tree (*V*), total height (*Ht*), estimated height (*He*), adjusted height (*Ha*) and diameters: $D_{0.3}$; $D_{0.5}$; $D_{0.7}$; $D_{0.9}$; $D_{1.1}$; *DBH*; $D_{1.5}$ and $D_{1.7}$. Table 2 relates the hypsometric and fitted volumetric models.

Models (a) and (b) were fit to be introduced in the Schumacher-Hall (g), (h), (k) and (l) and Spurr (o), (p), (s) and (t) models. Although the total determined height *Ht* is not used in the volumetric equations since *He* estimates are used, it was considered for the purpose of comparison with equations which use *He* or *Ha* as a function of *DBH* and *Dk*.

The stepwise technique was applied in the fit multiple models (c) and (d) at the significance level of 1% to select the equation with only significant independent variables.

Measurement errors from height estimates in the field with respect to the determined height (*EH* = *Ht* – *He*) were also calculated to be incorporated into hypsometric models which use the independent variable He.

The adjusted Fit Index (FI_{adj}) (Schlaegel 1981) was considered to assess the fit of the models. The Akaike information criterion (*AIC*) (Akaike 1974) and the mean absolute percentage error (*MAPE*) were also used as follows (Equations 29 and 30):

$$FI = \frac{\sum_{i=1}^{n} (\hat{V}_i - \bar{V})^2}{\sum_{i=1}^{n} (V_i - \bar{V})^2} = \frac{SS_{\text{Reg}}}{TSS} = 1 - \frac{SS_{\text{Res}}}{TSS} \quad (29)$$

where:
FI – Fit Index;
$\hat{V}_i$ – estimated volume;
V_i – actual volume;
SS_{Reg} – sum of squares of the regression;
SS_{Res} – sum of squares of the residuals;
TSS – total sum of squares.

$$FI_{adj} = 1 - \left(\frac{n-1}{n-p}\right) \times (1 - FI) \quad (30)$$

where:
FI_{adj} – adjusted Fit Index;
n – number of observations;
p – number of independent variables of the model.

Similar to the adjusted coefficient of determination (R^2_{adj}), the closer the FI_{adj} is to 1.0, the better the fit, meaning the more accurate the resulting equation is.

Akaike Information Criterion (AIC). To compare equations resulting from fitting various models, the most accurate is the one with the lowest AIC value that enables comparing models of different nature involving different numbers of parameters (Burnham, Anderson 2004) (Equation 31):

$$AIC = n \times \ln(\hat{\sigma}^2) + 2k \quad (31)$$

where:
$\hat{\sigma}^2$ – maximum likelihood estimator of the error (MS Residual);
k – number of parameters in the model including the intersection.

Mean absolute percentage error (MAPE) (Equation 32):

$$MAPE = \frac{100}{n} \times \sum_{i=1}^{n} \left| \frac{V_i - \hat{V}_i}{V_i} \right| \quad (32)$$

where:
V_i – actual volume;
$\hat{V}_i$ – estimated volume.

A completely random design was also applied to the data considering the actual tree volume determined by the Smalian method as control and each equation generated as a treatment, with the following mathematical model (Da Silva, Silva 1995) (Equation 33):

$$V_i = \mu + \tau_i + \varepsilon_{ij} \quad (33)$$

where:
V_i – response variable (actual volume) in treatment *i* in repetition *j*;
μ – overall mean;
τ_i – effect of treatment *i* (estimated volumes for each equation);
ε_{ij} – random error.

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Table 2. Linear and non-linear hypsometric and volumetric models considered in the study

Name	Model
(a) Hypsometric with *DBH*	$Ha_i = \beta_0 + \beta_1 \times DBH_i + \varepsilon_i$
(b) Hypsometric with *Dk*	$Ha_i = \beta_0 + \beta_1 \times Dk_i + \varepsilon_i$
(c) Multiple linear: D_i in the first log	$V_i = \beta_0 + \beta_1 \times D_{0.3i} + \ldots + \beta_8 \times D_{1.7i} + \varepsilon_i$
(d) Silva-Borders-Brister	$V_i = \beta_0 + \beta_1 \nu_{1_i} + \beta_1 \nu_{2_i} + \ldots + \beta_{28} \nu_{28_i} + \varepsilon_i$
(e) Schumacher-Hall with *DBH* and *Ht*	$V_i = \beta_0 \times DBH_i^{\beta_1} \times Ht_i^{\beta_2} \times \varepsilon_i$
(f) Schumacher-Hall with *DBH* and *He*	$V_i = \beta_0 \times DBH_i^{\beta_1} \times He_i^{\beta_2} \times \varepsilon_i$
(g) Schumacher-Hall with *DBH* and *Ha**	$V_i = \beta_0 \times DBH_i^{\beta_1} \times Ha_i^{\beta_2} \times \varepsilon_i$
(h) Schumacher-Hall with *DBH* and *Ha***	$V_i = \beta_0 \times DBH_i^{\beta_1} \times Ha_i^{\beta_2} \times \varepsilon_i$
(i) Schumacher-Hall with *Dk* and *Ht*	$V_i = \beta_0 \times Dk_i^{\beta_1} \times Ht_i^{\beta_2} \times \varepsilon_i$
(j) Schumacher-Hall with *Dk* and *He*	$V_i = \beta_0 \times Dk_i^{\beta_1} \times He_i^{\beta_2} \times \varepsilon_i$
(k) Schumacher-Hall with *Dk* and *Ha**	$V_i = \beta_0 \times Dk_i^{\beta_1} \times Ha_i^{\beta_2} \times \varepsilon_i$
(l) Schumacher-Hall with *Dk* and *Ha***	$V_i = \beta_0 \times Dk_i^{\beta_1} \times Ha_i^{\beta_2} \times \varepsilon_i$
(m) Spurr with *DBH* and *Ht*	$V_i = \beta_0 \times \left(DBH_i^2 \times Ht_i\right)^{\beta_1} \times \varepsilon_i$
(n) Spurr with *DBH* and *He*	$V_i = \beta_0 \times \left(DBH_i^2 \times He_i\right)^{\beta_1} \times \varepsilon_i$
(o) Spurr with *DBH* and *Ha**	$V_i = \beta_0 \times \left(DBH_i^2 \times Ha_i\right)^{\beta_1} \times \varepsilon_i$
(p) Spurr with *DBH* and *Ha***	$V_i = \beta_0 \times \left(DBH_i^2 \times Ha_i\right)^{\beta_1} \times \varepsilon_i$
(q) Spurr with *Dk* and *Ht*	$V_i = \beta_0 \times \left(Dk_i^2 \times Ht_i\right)^{\beta_1} \times \varepsilon_i$
(r) Spurr with *Dk* and *He*	$V_i = \beta_0 \times \left(Dk_i^2 \times He_i\right)^{\beta_1} \times \varepsilon_i$
(s) Spurr with *Dk* and *Ha**	$V_i = \beta_0 \times \left(Dk_i^2 \times Ha_i\right)^{\beta_1} \times \varepsilon_i$
(t) Spurr with *Dk* and *Ha***	$V_i = \beta_0 \times \left(Dk_i^2 \times Ha_i\right)^{\beta_1} \times \varepsilon_i$

*using Equation (a); **using Equation (b); *Ht* – total height; *He* – height estimated in the field; *Ha* – adjusted height using the hypsometric relationship as a function of *DBH* and diameter in the first log most correlated with the volume *Dk*; *Dk* – diameter of the first log most correlated with the volume; D_i – diameters in the first log; *DBH* – diameter at breast height; ν_i – volume *i* in the first log ranging from 0.30 m to 1.70 mm

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The null hypothesis (H_0) considered was that there are no significant differences between the actual tree volume and different volumes estimated by the equations. The same statistical analysis procedure was used to analyze the four heights (*Ht*, *He*, *Ha* with *DBH* and with $D_{1.7}$). All statistical analyses were performed using the MS Excel (Ver. 2019, 2019) and SYSTAT (Version 13.2, 2021) software programs.

RESULTS AND DISCUSSION

The percent correlation coefficients among measurement errors at the heights *EH* and the variables *V*, *Ht*, *He*, *Ha* with *DBH* and *Hai* with $D_{1.7}$ presented the values of 18.04%, 21.10%, –38.33%, 17.36% and 17.82%, respectively. Thus, due to the variable *EH* presenting very low correlation coefficients with these variables, models including EH_i were not fitted.

It is observed that all the independent variables D_i and v_i presented high correlation coefficients with the dependent variable V_i (Table 3). This means that any of these independent variables can be used for volumetric modelling. For diameters, $D_{1.7}$ (0.9638) was the most correlated with volume and is also present in variable v_7, which is the log volume from 1.50 m to 1.70 m (the most correlated with volume).

The percent correlations among independent variables of the same nature, meaning among diameters and volumes in the first log, varied from 97.34% (v_1 with v_7) and 99.89% (v_3 and v_{10}), because the volumes in the first log are not independent; a volume with a length of 0.20 m is included in a volume of 0.40 m and so on. The percent correlations for diameters ranged from 97.58% ($D_{0.3}$ with $D_{1.7}$) to 99.75% (*DBH* with $D_{1.5}$).

Table 4 shows that the FI_{adj}, *AIC* and *MAPE* statistic values for the equations resulting from the model fittings varied little, because in terms of FI_{adj} the variation was 5.59% between equations (c) and (i). Equation (c) uses three diameters as independent variables: $D_{0.3}$, $D_{0.7}$ and $D_{1.7}$. Such a result was expected to be inferior to the other equations which also use heights or volumes of the first log as independent variables, in addition to the *DBH* and $D_{1.7}$. Except for equation (c), this difference becomes 3.21% between equations (i) and (r).

The best results in terms of the statistics used were for equations (i) and (q), in this case the equation resulting from the Schumacher-Hall model with $D_{1.7}$ and the total height of the tree (*Ht*) and the Spurr model as these two independent variables. It should be noted that *DBH* is not present in equations (i) and (q), even though it is the most frequently used dendrometric variable in volumetric modelling; it was replaced by $D_{1.7}$, which was the most correlated with the volume.

In reality, these two equations (i) and (q) are not often used because the independent variable height (*Ht*) corresponds to the total height of the tree, and in field work the estimated height (*He*) or the adjusted height (*Ha*) by a hypsometric relationship is used. Thus, the results for equations (i) and (q) are theoretical.

Considering the estimated height (*He*) and adjusted height (*Ha*) by two simple linear hypsometric relationships involving *DBH* and $D_{1.7}$, the results were

Table 3. Correlation matrix between the actual volume (V_i) and the independent variables

$D_{0.3}$	$D_{0.5}$	$D_{0.7}$	$D_{0.9}$	$D_{1.1}$	*DBH*	$D_{1.5}$	$D_{1.7}$
0.9420	0.9494	0.9546	0.9571	0.9578	0.9618	0.9631	0.9638
Ht	*He*	*Ha*(*DBH*)	*Ha*($D_{1.7}$)	v_1	v_2	v_3	v_4
0.8301	0.6792	0. 9618	0.9638	0.9586	0.9658	0.9705	0.9730
v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}
0.9786	0.9826	0.9845	0.9630	09685	0.9727	0.9780	0.9808
v_{13}	v_{14}	v_{15}	v_{16}	v_{17}	v_{18}	v_{19}	v_{20}
0.9841	0.9662	0.9705	0.9772	0.9799	0.9822	0.9678	0.9748
v_{21}	v_{22}	v_{23}	v_{24}	v_{25}	v_{26}	v_{27}	v_{28}
0.9790	0.9813	0.9715	0.9764	0.9804	0.9729	0.9776	0.9739

D_i – diameter at the section base; *DBH* – diameter at breast height; v_i – volume of the section *i*; *Ht* – total height; *He* – height estimated in the field; *Ha* – adjusted height using the hypsometric relationship as a function of *DBH* and diameter in the first log most correlated with the volume *Dk*

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Table 4. Equations resulting from adjustments of hypsometric and volumetric models, best results indicated in bold

Equations		FI_{adj}	*AIC*	MAPE
$\hat{H}a_i = 6.170679 + 0.685167 DBH_i$	(a)	0.7674	555.60	9.9450
$\hat{H}a_i = 6.177450 + 0.714868.D_{1.7i}$	(b)	0.7763	526.61	9.7567
$\widehat{V}_i = -0.060015 - 0.001525 D_{0.3_i} + 0.003450.D_{0.7_i} + 0.010619 D_{1.7_i}$	(c)	0.9284	−6 506.20	57.4279
$\hat{V}_i = -0.005002 + 32.642911 v_{7_i} + 2.065824 v_{18_i}$	(d)	0.9694	−7 148.13	11.9646
$\hat{V}_i = 0.000042 \times DBH_i^{1.703763} \times Ht_i^{1.217876}$	(e)	0.9820	−7 534.73	7.2591
$\hat{V}_i = 0.000216 \times DBH_i^{2.030522} \times He_i^{0.313144}$	(f)	0.9656	−7 050.59	11.7952
$\hat{V}_i = 0.015544 \times DBH_i^{3.876687} \times Ha_i^{-3.003455}$	(g)*	0.9625	−6 989.16	10.8740
$\hat{V}_i = 0.015840 \times DBH_i^{3.915067} \times Ha_i^{-3.018076}$	(h)**	0.9624	−6 989.16	15.1157
$\hat{V}_i = 0.000058 \times D_{1.7i}^{1.753060} \times Ht_i^{1.081607}$	(i)	**0.9843**	**−7 647.44**	**6.5733**
$\hat{V}_i = 0.000257 \times D_{1.7i}^{2.051602} \times He_i^{0.259893}$	(j)	0.9715	−7 191.00	10.5010
$\hat{V}_i = 0.000308 \times D_{1.7i}^{1.976254} \times Ha_i^{0.261879}$	(k)*	0.9685	−7 117.52	13.4296
$\hat{V}_i = 0.008540 \times D_{1.7i}^{3.537648} \times Ha_i^{-2.408032}$	(l)**	0.9694	−7 137.79	9.8357
$\hat{V}_i = 0.000073 \times \left(DBH_i^2 \times Ht_i\right)^{0.909033}$	(m)	0.9806	−7 486.16	7.1158
$\hat{V}_i = 0.000082 \times \left(DBH_i^2 \times He_i\right)^{0.896316}$	(n)	0.9488	−6 760.79	12.4764
$\hat{V}_i = 0.000149 \times \left(DBH_i^2 \times Ha_i\right)^{0.818530}$	(o)*	0.9598	−6 942.23	17.3995
$\hat{V}_i = 0.000150 \times \left(DBH_i^2 \times Ha_i\right)^{0.815421}$	(p)**	0.9599	−6 942.22	16.4369
$\hat{V}_i = 0.000079 \times \left(D_{1.7i}^2 \times Ht_i\right)^{0.909040}$	(q)	**0.9839**	**−7 629.44**	**6.4551**
$\hat{V}_i = 0.000091 \times \left(D_{1.7i}^2 \times He_i\right)^{0.892741}$	(r)	0.9522	−6 812.09	11.8106
$\hat{V}_i = 0.000152 \times \left(D_{1.7i}^2 \times Ha_i\right)^{0.824838}$	(s)*	0.9681	−7 195.52	14.9988
$\hat{V}_i = 0.000153 \times \left(D_{1.7i}^2 \times Ha_i\right)^{0.823366}$	(t)**	0.9675	−7 099.78	15.1455

*using Equation (a); **using Equation (b); V_i – volume of tree *i*; D_i – diameter at the section base; *Ha* – adjusted height using the hypsometric relationship as a function of *DBH* and diameter in the first log most correlated with the volume *Dk*; *DBH* – diameter at breast height; FI_{adj} – adjusted Fit Index; *AIC* – Akaike information criterion; *MAPE* – mean absolute percentage error

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lower than those using the total height (*Ht*), but very close considering the FI_{adj}, *AIC* and *MAPE* statistics.

It is observed that the equations which use $D_{1.7}$ presented better statistics (*AIC*, *MAPE*, FI_{adj}) than those that used *DBH*, but with very close values. Thus, based on the statistics considered, it is not possible to identify significant differences between the equations resulting from the adjustments.

When considering a completely randomized design to compare the tree total height (*Ht*) used as a control with the height estimated in the field (*He*) and with the adjusted heights (*Ha*) as a function of *DBH* and $D_{1.7}$ (which was the diameter most correlated with the volume), a non-significant *F* is observed in the analysis of variance (Table 5); therefore, the null hypothesis (H_0) is accepted, which indicates the non-existence of significant differences between the heights. Thus, the heights are similar and there is no need to apply a means comparison test.

This result was also a function of height estimates in the field with little significant measurement errors, meaning that the trees had their height estimates very close to the total heights measured directly on the trees. The variable *EH* (height measurement error) presented random values, which means it did not follow a pattern associated with the total tree height. Thus, the measurement errors which occur more easily when the stand is higher, but underestimate or overestimate the total tree height, were random in this work.

It is worth mentioning that the work on estimating total tree height was facilitated by the fact that the average total height of the sample of 750 trees was 13.6 m, with few trees reaching 20 m.

Moreover, the *F* value was not significant in the analysis of variance to compare the actual volume *V* (control) with those estimated by the 18 equations resulting from the fittings (Table 6), so there is no significant difference between the volumes. Therefore, the actual volume and the volumes estimated by the equations are similar and there is no need to apply a means comparison test.

The selected equation has two volumes in the first log as independent variables, in this case v_7 and v_{18} (Table 7). After replacing these volumes in equation (d), which means placing the diameters $D_{1.1}$, $D_{1.5}$ and $D_{1.7}$ with the lengths of 0.2 m for v_7 and 0.6 m for v_{18}, the equation becomes Equation (34):

$$\widehat{V}_i = -0.005002 + 0.486749 D_{1.1_i}^2 + 2.5662774 D_{1.5_i}^2 + 3.050523 D_{1.7_i}^2 \quad (34)$$

In this case, the tree height estimate (*He*) is excluded from the equation. The use of volumetric equations using only different tree diameters can streamline the forest inventory process in the more complex structure of forest stands, where mea-

Table 5. Analysis of variance for the variables *Ht*, *He*, *Ha* with *DBH* and *Ha* with with $D_{1.7}$

SV	*DF*	*SS*	*MS*	*F*	*Pr > F*
Equations	3	9.45822	3.15274	0.38459	0.7641
Residual	1 996	24 560.27280	8.19769	–	–
Total	1 999	24 569.73102	–	–	–

Ht – total height; *He* – height estimated in the field; *Ha* – height adjusted; *SV* – sources of variation; *DF* – degrees of freedom; *SS* – sums of squares; *MS* – mean squares; *F* – *F*-test; *Pr > F* – value of the probability of accepting the null hypothesis H_0

Table 6. Analysis of variance for the variables *Ht*, *He*, *Ha* with *DBH* and *Ha* with with $D_{1.7}$

SV	*DF*	*SS*	*MS*	*F*	*Pr > F*
Equations	18	0.026121	0.001451	0.6345	0.8755
Residual	14 231	32.543678	0.002287	–	–
Total	14 249	32.569799	–	–	–

Ht – total height; *He* – height estimated in the field; *Ha* – height adjusted; *SV* – sources of variation; *DF* – degrees of freedom; *SS* – sums of squares; *MS* – mean squares; *F* – *F*-test; *Pr > F* – value of the probability of accepting the null hypothesis H_0

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Table 7. Means and coefficients of variation for the actual volume and adjusted equations (see Table 4)

Equation	Mean (m^3)	Coefficient of variation (%)
Actual volume	0.071704	67.85
(c)	0.071622	65.36
(d)	0.071703	66.80
(e)	0.071387	67.48
(f)	0.071993	65.90
(g)	0.071642	66.74
(h)	0.077591	68.10
(i)	0.071709	67.50
(j)	0.071802	66.17
(k)	0.071799	67.11
(l)	0.071609	65.72
(m)	0.072417	64.91
(n)	0.070674	64.66
(o)	0.071963	67.33
(p)	0.071647	65.80
(q)	0.072153	65.68
(r)	0.071665	66.92
(s)	0.071497	65.29
(t)	0.072024	65.38

surement of heights is less accurate or more time-consuming than directly measuring three different diameters. The results are expected to be similar to Schumacher-Hall and Spurr models.

Another factor which must be considered is the sample size. A total of 750 trees were used in this work, constituting a relatively large sample which provided fits for the models used with high precision, even with high coefficients of variation for each combination between clones and spacing. Thus, even if models with many variables and recent computing techniques are used in this database, the results would certainly be most similar to those found in this work.

CONCLUSION

In volumetric modelling procedures, the use of a large sample associated with the use of models which have traditionally been used with success in forest measurement (such as those of Schumacher-Hall and Spurr or combined variables), the use of independent variables of DBH or diameters in the first log and estimated tree height or adjusted by a hypsometric relationship will produce high precision fits.

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