Combining the crisp outputs of multiple fuzzy expert systems using the MPDI along with the AHP

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Abstract: Business, economic, and agricultural YES-or-NO decision making problems often require multiple, different and specific expertises. This is due to the nature of such problems in which decisions may be influenced by multiple different, relevant aspects, and accordingly multiple corresponding expertises are required. Fuzzy expert systems (FESs) are widely used to model expertises due to their capability to model the real world values, which are not always exact, but frequently vague, or uncertain. In this paper, different expertises relevant to the decision solution are modelled using several corresponding FESs. These systems are then integrated to comprehensibly judge the YES-or-NO binary decision making problem, which requires all such expertises. This integration involves several independent and autonomous FESs arranged synergistically to suit a varying problem context. Then, the main focus of this paper is to realize such integration through combining the crisp numerical outputs produced by multiple FESs. The newly developed methods MPDI and WMPDI are utilized to combine the crisp outputs of multiple parallel FESs, whilst weights are determined through the analytical hierarchy process (AHP). The presented approach of utilizing the proved efficient MPDI combining criteria along with AHP will encourage practitioners to take advantage of integration and cooperation among multiple numerically outputting knowledge sources in general.

Key words: fuzzy expert system (FES), combining criteria, group decision making (GDM), binary decision making, output combination/aggregation, AHP, knowledge integration

Integrating multiple intelligent or decision support systems contributes to obtaining, a high quality, more comprehensible and reliable decision solution. This paper attempts to treat one of complex problems that have not gained much attention before, in spite of the wide prevalence of situation in which a group of expertises concurrently evaluate YES/NO decision problems. The problem of integrating multiple FESs involves combining or aggregating the crisp outputs produced by the individual systems to obtain a final, consolidated, YES/NO output decision. The need for multiple expert systems (ESs) can occur frequently when a complex problem in hand has multiple related aspects for which the existence of multiple independent and separated expertises is necessary, and there is no available expertise that covers whole aspects of the problem (see e.g. Arumugam et al. 2010; Beranová and Martinovičová 2010; Kubon and Krasnodębski 2010; Tomšík and Svoboda 2010). There are still some other practical reasons behind independence amongst ESs which have been described in Aly and Vrana (2010).

Next we shall review the basic component of a FES, to give more insights about the combined units and the nature of their combined output values. The fuzzy expert system consists of four components: a fuzzification subsystem, a knowledge-base, an inference mechanism, and a defuzzification subsystem which converts the implied fuzzy sets into crisp values expressing the YES/NO decisive degree (Figure 1).

The special concern around fuzzy expert systems is attributed to their wide applicability and use due to their capability to treat vagueness, and subjectivity. Especially important is the possibility to convert subjective non-sharp factors into the corresponding easily manageable and comprehensible numerical scale. More description of the fuzzy set theory and FES can be found in Zadeh (1965) and Kilagiz et al. (2004).

Various possible configurations for the cooperation among several expert systems have been described in Beeri and Spiegler (1996) and Aly and Vrana (2010). However, our proposed configuration is that FESs are arranged in such a way that the same problem is pre-

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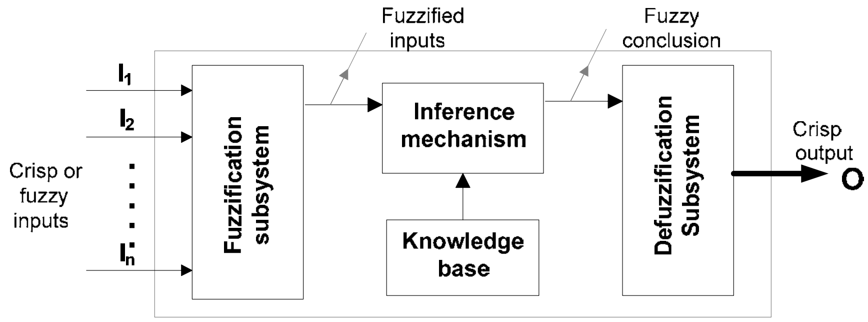

Figure 1. Comparison of yields in the Agro Žlunice with the CR (t/ha)

sented to every expert system concurrently in order to reach one consolidated output decision. Every system uses its tools, skills and expertises to reach its individual crisp output about whether the decision answer should be "Yes" or "No". Here the problem is how to combine the crisp outputs of multiple parallel expert systems to obtain such a representative consolidated output. The problem configuration is shown in Figure 2. Solving this problem specifically for fuzzy expert systems will be our concern in this research.

Only few researches have been recorded up to date that involve the combination of multiple fuzzy expert systems. This research idea is novel. Also, there was no past research attempts that involved the integration of multiple expert systems through combining their final numerical outputs. However, considerable researches have been investigated that involve the integration or synergetic cooperation of multiple knowledge sources (Venkatasubramanian and Chen 1986; Beeri and Spiegler 1996; Gams et al. 1997). In addition, some researches in the field of pattern recognition have considered the integration between multiple pattern classifiers (Ho et al. 1992; Xu et al. 1992; Kittler et al. 1998; Constantindis et al. 2001). Other research attempts have been conducted to combine the experts' judgments (Ashton 1986; Lipscomb et al. 1998).

Combining multiple fuzzy expert systems belongs to the GDM type problems. It involves obtaining a finally consolidated output decision for a group of multiple expert systems or decision makers. Every system produces crisp numerical value within a certain range, e.g. [0, 10], which expresses the degree of bias toward Yes or No decisions. Group decision techniques, from the simple majority voting rule to more elaborate techniques, are available for knowledge aggregation (e.g., Jessup and Valacich 1993).In the literature, various combining rules or criteria are divided into three types depending on the output which the information experts or systems provide (Xu et al. 1992). Since combination methods span a wide variety of research areas, the term module is used to refer to the individual units to be combined. A module can be an expert, an expert system, a forecaster, an estimator, or a classifier. Depending on what level of information received from the module, there are three types of combination (Al-Ghoneim and Kumar 1998):

(a) Combination at abstract level

At this level, combination criteria or algorithms use only the abstract level information, the identity of the top class, provided by the modules. These methods are based on voting procedures that are adopted from the group decision-making theory such as unanimity, majority, plurality, ... etc. The majority and plurality voting rules are the most widely used ones. They are further divided based on whether or not they take into account the relative importances of modules. They are as follows:

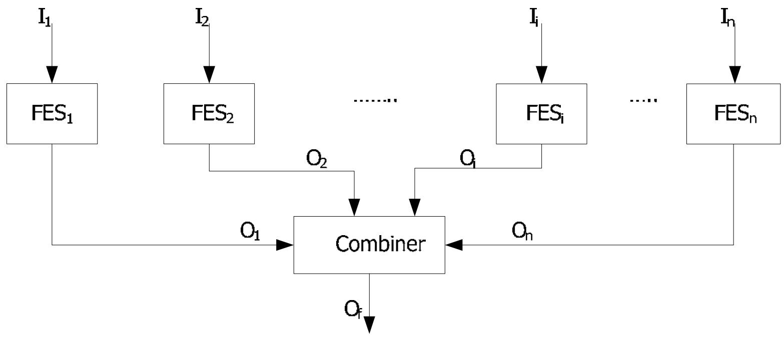

Figure 2. Graphic presentation of the profit/losses of the Agro Žlunice and the CR 2003–2009 (CZK/ha)

(b) Combination at rank level

At this level, the classifier modules provide rank information; that is the preference ordering of classes from the top to the bottom rank. Every module provides a sorted list of classes arranged in order of preference. A well-known combination method at the rank level is the Borda count.

(c) Combination at a measurement level

At the measurement level, the combination criteria or algorithms have access to a set of numerical scores provided by the classifier modules. Cordella et al. in1999 stated that combining schemes that exploit information from the classifiers at the measurement level allow us to define combing rules that are more sophisticated and potentially more effective. The combination at the measurement level is our main focus in this research.

Combining criteria like the Arithmetic Mean (AM), Geometric Mean (GM), and Harmonic Mean (HM), belongs to combination methods at the measurement level. In Vrana et al. (2010) the widely used arithmetic mean combining criterion has proven to be a compromising, not decisive criterion, and the results in most situations led to the information loss due to its smoothing effect. It works always by pointing to the center of the numerical data based on their values, not on the direction of answer, and this is considered not adequate in our situation in which we need a considerable degree of decisiveness to reach either "Yes" or "No" final decisions. However, the AM is still is a useful formula used to refer to the central tendency of a group of numerical values, as long as there is an effective way to interpret the resulting combined value.

In Vrana et al. (2010), a new measurement-level combining criterion, called the MPDI (an acronym for Multiplicative Proportional Deviative Influence) was proposed. The MPDI was compared to the well-known classical combining criteria: AM, GM, and HM, based on some uniformly created random numerical data. The results of such experiment have distinguishingly reported the superiority of the MPDI over other considered classical combining criteria.

In this paper, we propose an approach based on our newly developed combining criteria, MPDI along with the AHP (Saaty 1980) to combine the final crisp outputs of the multiple FESs, evaluating the binary-type GDM problems. First we shall outline the basic idea of the MPDI criterion and its weighted version WMPDI and we shall also discuss the offered advantage of the MPDI. Then we shall provide an example to demonstrate how the proposed two heuristics could be used.

THE MPDI AND WMPDI COMBINING CRITERION

The MPDI was introduced in Vrana et al. (2010) is a Black-board (Chi et al. 2001) inspired new combining criteria. It imitates some processing aspect of the blackboard concepts in integrating the multiple knowledge sources. In the black-board, knowledge sources, which can be experts or any intelligent systems, interact via a shared global data structure – the blackboard that organizes and stores the intermediate problem solving data. Knowledge sources produce changes to the blackboard that lead incrementally to a solution of the problem. Communication between the knowledge sources is conducted solely through changing the blackboard. Similarly, the MPDI combination is based on this idea in that there are multiple knowledge sources, each of which changes a numerical value initially existing on the black-board. All knowledge sources bear numerical value within the range [0, 10] expressing the degree of bias to either Yes or No answers. Initially existing in the black-board is the middle value 5 of the psychometric scale used, which expresses that initially there is no bias. Then, all knowledge sources fairly participate in changing this initial value based on their *deviation* from that middle value. That is, the *influence* of every numerical value is *proportional* to its deviation from the middle. Then, all deviation are accumulated on the middle by *multiplicatively* augmentig it if the deviation is positive, and decrementing if the deviation is negative. This is why the criterion is called the Multiplicative Proportional Deviative Influence, or MPDI. It is mathematically defined and expressed as follows:

O_m : initially non-biased middle value, 5

O_i : i^{th} FES's crisp output, $i = 1, 2, ..., n$

ΔO_i^+: absolute deviation of the i^{th} FES's crisp output, O_i, below from the middle

$$\Delta O_i^+ = \begin{cases} O_i - O_m & \text{if } O_i \geq O_m \\ 0 & \text{otherwise} \end{cases}$$

ΔO_i^- : absolute deviation of the i^{th} FES's crisp output, O_i, above from the middle

$$\Delta O_i^- = \begin{cases} O_m - O_i & \text{if } O_i \leq O_m \\ 0 & \text{otherwise} \end{cases}$$

Then

$$\text{MPDI} = \left[\frac{\prod_{i=1}^{n} \left[1 + (\Delta O_i^+ / O_m)\right]}{\prod_{i=1}^{n} \left[1 + (\Delta O_i^- / O_m)\right]} \right] \times O_m \qquad (1)$$

The limiting range of values the MPDI criterion can take is within $[5/2^n, 5 \times 2^n]$, which is a function of the number of experts or decision makers involved in judgment.

The WMPDI – the weighted version of MPDI – is based on a simple notion that as the relative importances of the experts' or expert systems' outputs differ, then in this case their computed deviative influences are weighted to reflect the varying importances in imposing influences. It is formally stated as follows:

Let

O_m : initially non-biased middle value, 5

O_i : i^{th} FES's crisp output, $i = 1, 2, ..., n$

W_i : i^{th} FES's weight

ΔO_i^+: absolute deviation of the i^{th} expert judgment O_i, above from the middle

$$\Delta O_i^+ = \begin{cases} O_i - O_m & \text{if } O_i \geq O_m \\ 0 & \text{otherwise} \end{cases}$$

ΔO_i^- : the absolute deviation of the i^{th} expert judgment O_i, below from the middle

$$\Delta O_i^- = \begin{cases} O_m - O_i & \text{if } O_i \leq O_m \\ 0 & \text{otherwise} \end{cases}$$

Then:

$$\text{WMPDI} = \left[\frac{\prod_{i=1}^{n} \left[1 + W_i (\Delta O_i^+ / O_m) \right]}{\prod_{i=1}^{n} \left[1 + W_i (\Delta O_i^- / O_m) \right]} \right] \times O_m \qquad (2)$$

THE ADVANTAGEOUS COMBINING CAPABILITY OF THE MPDI

The distinct features and inherent combining characteristics of the MPDI have been described in the introductory paper (Vrana et al. 2010). We review it here only briefly. First, the MPDI notion is logically understood as a group of numerical values imposing influences on an existing non-biased value in a fair way, based on their deviation from that middle value. It can be considered a more logical formula than the widely used arithmetic average which only measures the central tendency of a group of numerical values. Arithmetic mean (AM) has important limitations in that it exhibits a smoothing effect on a group of numerical values which results in some information loss. In addition, it never gives the extreme value of a group of numbers, but in our case of two binary classes and the utilized numerical scale, decisive extremes value are more required than the comprising values. In contrast, our proposed formula is considered more decisive than the arithmetic mean in the sense that the multiplication process always gives a more magnified value than a sum and magnifies the agreement more than the AM. Sometimes the combined value rises over 10, the extreme limit of "Yes" decision answer. The overflow of values means only a saturation, in which high degree of consensus have been attained. This can be logically interpreted as an increase of the number of experts who agree on a particular option, this should be reflected into the reinforcement of their concordant answers.

The experimental work conducted in (Vrana et al. 2010) has showed clearly that the criterion MPDI can be considered superior to the well-known arithmetic average and the others as well, and that the MPDI outperformed all other combining criteria under the most diverse values of the consensus level. The MPDI is particularly useful in that it is more decisive in attributing the combined outputs to the correct binary class.

THE ANALYTICAL HIERARCHY PROCESS (AHP)

The AHP is a basic approach to decision making (Dweiri and Meier 1996 ; Saaty 1980). In this process, the decision maker carries out simple pair-wise comparative judgments, which are then used to develop overall priorities for ranking alternatives, factors or criteria. The AHP allows for inconsistency in the judgments and provides a mean to improve consistency. The decision makers assign an importance intensity value from the fundamental scale shown in Table 1, which represents the true preference of each reason with respect to another reason. The importance intensity of factor (also criterion, or alternative) i over j is denoted by a_{ij}, and the reciprocal importance

Table 1. The fundamental scale of the AHP importance intensity value

Importance intensity a_{ji}	Definition
1	Equal importance of i and j
2	Between equal and weak importance of i over j
3	Weak importance of i over j
4	Between weak and strong importance of i over j
5	Strong importance of i over j
6	Between strong and demonstrated importance of i over j
7	Demonstrated importance of i over j
8	Between demonstrated and absolute importance of i over j
9	Absolute importance of i over j

intensity of factor j over i is denoted by $a_{ji} = 1/a_{ij}$, $a_{ji} = 1$, *iff* $i = j$. It is clear that a_{ij} is greater than 1 if factor i is more important than factor j, and is less than 1 if factor i is less important than factor j.

The AHP procedure by Saaty (1980), suggested by Dweiri and Meier (1996,) is as follows:

(1) Developing the importance intensity matrix **A**:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

Where:

n = the number of factors or alternatives to be compared.

(2) Finding the vector (**P**) of the priorities of factors P_i:
  (a) Multiplying the n elements in each row in the intensity importance matrix by each other. The result is the vector ($\mathbf{X_i}$).
  (b) Taking the n-th root of the vector ($\mathbf{X_i}$) for each row. The result is the vector ($\mathbf{Y_i}$).
  (c) Normalizing by dividing each number in the vector ($\mathbf{Y_i}$) by the sum of all the numbers $\sum Y_i$. The result is a vector (**P**).

(3) Determining the consistency of judgments:
  (a) Finding the vector **F** by multiplying matrix **A** by vector **P**.
  (b) Dividing every F_i by P_i to determine Z_i.
  (c) Summing Z_i, and dividing by n to obtain the maximum eigen value = λ_{max}, which is the average.
  (d) Computing the "consistency index" ${}^iC = (\lambda_{max} - n)/(n - 1)$.
  (e) Finding the "random index" ${}^iR$ from the Table 2, for the corresponding number of compared factors n.
  (f) Computing the consistency ratio ${}^rC = {}^iC/ {}^iR$. Any value of ${}^rC \leq 0.1$ is considered an acceptable ratio of consistency.

See Saaty (1980) for more details.

We shall utilize the AHP in our proposed approach to weigh the importance of every FES selected to judge the decision problem in the matched set. In the next section, we will demonstrate how the AHP could be used along with the MPDI combining criterion to obtain a reliable consolidated output for the crisp outputs of multiple FESs. An example will illustrate how the proposed approaches can be simply used to integrate the multiple parallel FESs.

AN ILLUSTRATIVE EXAMPLE

Decision making processes are applied in enterprises across all sectors, including agriculture. Besides applying general approaches, agricultural enterprises should further consider many specific aspects as the biological character of production, the influence of climatic conditions, etc. This often leads to the multi-dimensional, multi-aspect decision-making problem, which might become very complex and should be simplified for the practical utilization in agricultural enterprises, e.g by reducing the number of dimensions where only a limited number of aspects and criteria are considered. As an example, we introduce a typical situation, when an agricultural enterprise wants to incorporate a new main crop (or to replace the existing one) into its production program. Given suitable natural and other conditions for the given crop, mainly the following criteria are considered:

– Economic profitability (which is given by an economic effectiveness factor) – it incorporates aspects of the commodity direct costs, overhead costs, investment costs and the expected financial gain to the enterprise.
– Marketing potential (gaining a share at the local/regional/broader market) – mainly the current competition in the given market segment is considered and also an inherent competitiveness.
– Social impact – it is possible to consider achieving support from the structural framework programs for the development of agriculture and rural areas, which is bound at the new production program.
– Environmental impacts – analogically as above, for adopting a final decision, it is important to consider a chance to achieve support related to environmental aspects from the structural framework development programs.

With respect to their importance, the individual aspects and criteria can have different weights in the decision making processes. The common objective is usually gaining financial resources – i.e. economic effect. It can be either a direct effect or an indirect one, which brings finances from the framework of the supporting structural policy.

In our illustrative example, a large agricultural farm ABCD is considering the decision of whether or not to migrate from the present portfolio of poor croppers to better croppers, where a higher overall gain and sustainability is expected in the near future.

Table 2. The random index ${}^iR$ versus the number of factors assessed

N	1	2	3	4	5	6	7	8	9	10	11	12
${}^iR$	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.58

Such migration of the production portfolio can either require changes in the technology or adopting a new technology, it can increase environmental risks, it can lead to the new marketing strategy with changes in the supplier/customer relationships. It also would have a significant impact on the qualification structure of employees with consequences in the social aspects. The farm management is seeking the help of four considered relevant FESs holding expertise and knowledge in marketing, environmental, technological, and social domains. The four FESs' crisp outputs are as follows: O_1 = 9, O_2 = 3, O_3 = 5, O_4 = 8.

The company managers are relying on such FESs to reach the correct decision, and consequently there must be a mean to combine the outputs of such systems. These systems may have different relative importances with respect to the decision problem.

The multiple FESs are to be combined first using the MPDI assuming equal relative importances, and then with the difference in the relative importances as follows:

(a) Combining multiple FESs with equal relative importances

First the deviative influence of each FESs is computed as follows:

– The middle value of the scale [0, 10] is O_m = 5.

$$\Delta O_1^+ = 4, \Delta O_1^- = 0$$
$$\Delta O_2^+ = 0, \Delta O_2^- = 2$$
$$\Delta O_3^+ = 0, \Delta O_3^- = 0$$
$$\Delta O_4^+ = 3, \Delta O_4^- = 0$$

Then, according to eq. (1); in case of equal weights:

$$MPDI = \left[\frac{1.8\times 1\times 1\times 1.6}{1\times 1.4\times 1\times 1}\right]\times 5 = 10.3\ (10)\ \text{“Yes”}$$

AM = 6.25 (> 5 ± 0.5) “Yes”.

The two combining criteria agree, but the MPDI is more decisive into pointing to the correct direction.

(b) Combining multiple FESs with different relative importance's

Assume that the company's product manager, who has a sufficient knowledge and experience to assess the relative importance of every expert system, will assign relative importance intensity values using the AHP fundamental scale in pair-wise comparisons. He/she considered three criteria to be used in comparisons:

Knowledge: the amount of the important knowledge and information each FES bears.

Experience: the age and historical depth of the expertise contained in each FES.

Relevance: the degree of how much each FES has knowledge pertaining and relating to the decision problem.

Stage 1: computing FESs' absolute weights using the AHP:

The AHP hierarchy is shown in Figure 3.

1. Computing absolute weights of the criteria

the importance intensity matrix:

$$A_{3x3} = \begin{bmatrix} 1 & 1/2 & 1/4 \\ 2 & 1 & 1/2 \\ 4 & 2 & 1 \end{bmatrix}$$

$$X_i = \begin{bmatrix} 0.125 \\ 1 \\ 8 \end{bmatrix},\quad Y_i = \begin{bmatrix} 0.5 \\ 1 \\ 2 \end{bmatrix},\quad P_i = \begin{bmatrix} 0.143 \\ 0.286 \\ 0.571 \end{bmatrix}$$

$$Fi = \begin{bmatrix} 1 & 1/2 & 1/4 \\ 2 & 1 & 1/2 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0.143 \\ 0.286 \\ 0.571 \end{bmatrix} = \begin{bmatrix} 0.429 \\ 0.858 \\ 1.715 \end{bmatrix},\quad Z_i = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix},$$

$$\lambda_{\max} = \frac{\sum Zi}{n} = 3$$

$CI = \dfrac{\lambda_{\max} - n}{n-1} = \dfrac{3-3}{3-1}$, and from table ^iR = 0.58 for n = 3, then CR = 0 ≤ 0.1 (acceptable).

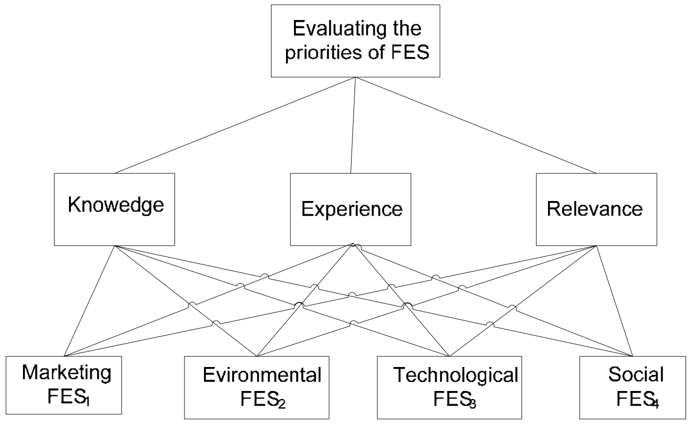

Figure 3. Expert systems prioriies evaluation hierarchy

2. Computing the absolute weights of FESs under each criterion

a. The knowledge criterion

The importance intensity matrix:

$$A_{4x4}=\begin{bmatrix}1&2&\frac{1}{4}&5\\ \frac{1}{2}&1&\frac{1}{3}&4\\ 4&3&1&6\\ \frac{1}{5}&\frac{1}{4}&\frac{1}{6}&1\end{bmatrix}$$

$$X_i=\begin{bmatrix}2.5\\0.667\\72\\8.333^{-3}\end{bmatrix},\quad Y_i=\begin{bmatrix}1.257\\0.904\\2.913\\0.302\end{bmatrix},\quad P_i=\begin{bmatrix}1.257\\0.904\\2.913\\0.302\end{bmatrix}$$

$$F_i=\begin{bmatrix}1&2&\frac{1}{4}&5\\ \frac{1}{2}&1&\frac{1}{3}&4\\ 4&3&1&6\\ \frac{1}{5}&\frac{1}{4}&\frac{1}{6}&1\end{bmatrix}\begin{bmatrix}1.257\\0.904\\2.913\\0.302\end{bmatrix}=\begin{bmatrix}0.9855\\0.9747\\2.318\\0.2351\end{bmatrix},\quad Z_i=\begin{bmatrix}4.212\\5.802\\4.277\\4.198\end{bmatrix},$$

$$\lambda_{\max}=\frac{\sum Zi}{n}=4.622$$

$CI=\dfrac{\lambda_{\max}-n}{n-1}=0.2074$, and from table ^iR = 0.9 for n = 4, then CR = 0.23 > 0.1 (not acceptable), so the manager must repeat judgments until his judgment becomes consistent. He should investigate the relative importance of FESs over each other.

The revised importance intensity matrix:

$$A_{3x3}=\begin{bmatrix}1&2&\frac{1}{4}&5\\ \frac{1}{2}&1&\frac{1}{6}&2\\ 4&6&1&6\\ \frac{1}{5}&\frac{1}{2}&\frac{1}{7}&1\end{bmatrix}$$

$$X_i=\begin{bmatrix}2.5\\0.1667\\168\\0.014\end{bmatrix},\quad Y_i=\begin{bmatrix}1.257\\0.639\\3.6\\0.344\end{bmatrix},\quad P_i=\begin{bmatrix}0.215\\0.109\\0.616\\0.059\end{bmatrix}$$

$$F_i=\begin{bmatrix}1&2&\frac{1}{4}&5\\ \frac{1}{2}&1&\frac{1}{6}&4\\ 4&6&1&6\\ \frac{1}{5}&\frac{1}{2}&\frac{1}{7}&1\end{bmatrix}\begin{bmatrix}0.215\\0.109\\0.616\\0.059\end{bmatrix}=\begin{bmatrix}0.882\\0.437\\2.543\\0.2445\end{bmatrix},\quad Z_i=\begin{bmatrix}4.102\\4.009\\4.128\\4.144\end{bmatrix},$$

$$\lambda_{\max}=\frac{\sum Zi}{n}=4.096$$

$CI=\dfrac{\lambda_{\max}-n}{n-1}=0.032$, and from table ^iR = 0.9 for n = 4, then CR = 0.036 ≤ 0.1 (acceptable).

b. The experience criterion

The importance intensity matrix:

$$A_{3x3}=\begin{bmatrix}1&\frac{1}{5}&2&\frac{1}{6}\\ 5&1&7&2\\ \frac{1}{2}&\frac{1}{7}&1&\frac{1}{8}\\ 6&\frac{1}{2}&8&1\end{bmatrix}$$

$$X_i=\begin{bmatrix}0.067\\70\\8.929^{-3}\\24\end{bmatrix},\quad Y_i=\begin{bmatrix}0.509\\2.893\\0.307\\2.214\end{bmatrix},\quad P_i=\begin{bmatrix}0.086\\0.488\\0.052\\0.374\end{bmatrix}$$

$$F_i=\begin{bmatrix}1&\frac{1}{5}&2&\frac{1}{6}\\ 5&1&7&2\\ \frac{1}{2}&\frac{1}{7}&1&\frac{1}{8}\\ 6&\frac{1}{2}&8&1\end{bmatrix}\begin{bmatrix}0.086\\0.488\\0.052\\0.374\end{bmatrix}=\begin{bmatrix}0.35\\2.03\\0.212\\1.55\end{bmatrix},\quad Z_i=\begin{bmatrix}4.07\\4.16\\4.077\\4.144\end{bmatrix},$$

$$\lambda_{\max}=\frac{\sum Zi}{n}=4.113$$

$CI=\dfrac{\lambda_{\max}-n}{n-1}=0.038$, and from table ^iR = 0.9 for n = 4, then CR = 0.042 ≤ 0.1 (acceptable).

c. The relevance criterion

The importance intensity matrix:

$$A_{3x3}=\begin{bmatrix}1&4&2&\frac{1}{6}\\ \frac{1}{4}&1&\frac{1}{3}&\frac{1}{4}\\ \frac{1}{2}&3&1&\frac{1}{2}\\ 1&4&2&1\end{bmatrix}$$

$$X_i=\begin{bmatrix}8\\0.021\\0.75\\8\end{bmatrix},\quad Y_i=\begin{bmatrix}1.682\\0.381\\0.931\\1.682\end{bmatrix},\quad P_i=\begin{bmatrix}0.36\\0.082\\0.199\\0.36\end{bmatrix}$$

$$F_i=\begin{bmatrix}1&4&2&\frac{1}{6}\\ \frac{1}{4}&1&\frac{1}{3}&\frac{1}{4}\\ \frac{1}{2}&3&1&\frac{1}{2}\\ 1&4&2&1\end{bmatrix}\begin{bmatrix}0.36\\0.082\\0.199\\0.36\end{bmatrix}=\begin{bmatrix}1.446\\0.328\\0.805\\1.446\end{bmatrix},\quad Z_i=\begin{bmatrix}4.017\\4\\4.045\\4.017\end{bmatrix},$$

$$\lambda_{\max}=\frac{\sum Zi}{n}=4.02$$

$CI=\dfrac{\lambda_{\max}-n}{n-1}=(6.583)\times10^{-}$, and from table ^iR = 0.9 for n = 4, then CR = $(7.315)\times10^{-3}$ ≤ 0.1 (acceptable).

3. Synthesis of priorities

Given the priorities of each FES under each criterion, now we follow the recommended additive synthesis method of the AHP for the distributive mode as follows (Table 3):

Then, first the weighted deviative influences of each FES is computed as follows:

O_m = 5

$$\Delta O_1^+=4,\ \Delta O_1^-=0,\quad W_1\times(\frac{\Delta O_1^+}{O_m})=0.26\times0.8=0.208$$

$$\Delta O_2^+=0,\ \Delta O_2^-=2,\quad W_2\times(\frac{\Delta O_2^-}{O_m})=0.202\times0.4=0.0808$$

$$\Delta O_3^+=0,\ \Delta O_3^-=0,\quad \text{(no influence)}$$

$$\Delta O_4^+=3,\ \Delta O_4^-=0,\quad W_4\times(\frac{\Delta O_4^+}{O_m})=0.321\times0.6=0.1926$$

Table 3. The random index ^iR versus the number of factors assessed

FES_i	Criterion			Final multi-criterion weights
	knowledge	experience	relevance	
	0.143	0.286	0.571	
FES_1	0.215	0.086	0.36	0.26
FES_2	0.109	0.488	0.082	0.202
FES_3	0.616	0.052	0.199	0.217
FES_4	0.059	0.374	0.36	0.321

Then, according to eq. (2); in case of equal weights:

$$WMPDI = \left[\frac{1.208 \times 1 \times 1 \times 1.1926}{1 \times 1.0808 \times 1 \times 1}\right] \times 5 = 6.67 \ (> 5) \text{ “Yes”}$$

The weighted arithmetic means gives:
WAM = 6.6 (> 5) “Yes”

The two combining criteria agree, and the WMPDI is still slightly more decisive in pointing to the correct direction.

It should be noted that some direction threshold can be utilized to more reliably attribute the resulting combined value of both criteria to the specific decision class. For instance, a threshold ± 0.5 can be utilized. Then, if the combined value was greater than 5.5, it is classified as “Yes”; if the combined value was lower than 4.5, it is classified as “No”; otherwise the resulted value should be classified as non-biased. This threshold may contribute to the increasing reliability of results, and usually could be set based on the vision of the decision analysts and characteristics of the participating FESs or the decision problem.

CONCLUSION

In this paper, we have proposed a combining method for obtaining a final consolidated decision of numerical judgments made by the multiple cooperative FESs. This method is based on a newly promising criterion, the MPDI, and the AHP. The role of the AHP was to compute the weights of the individual FESs. These weights are to be used then in the presented weighted version of the MPDI. The illustrative example has shown that the WMPDI is still slightly more decisive than the weighted arithmetic mean (WAM) criterion as was its original MPDI. Both the MPDI and WMPDI are superior to the AM and WAM, respectively. The whole approach of integrating multiple FESs, effectively realized by a reliable combining criterion, actually improves the decision making process in general, and the group decision making in particular. Since the individual FESs are able to model the expertise in both quantitative and qualitative contexts, to handle vague input variables and each of them incorporates a relevant source of knowledge and expertise, it is expected that the finally obtained consolidated output is a highly realistic and reliable decision answer. The proposed integration scheme of multiple FESs is novel and could be utilized in any field, such as economics, business, medicine, engineering areas, military decision making, ..., etc. It is especially relevant, when we have multiple aspects and ill-structured, decision making problems.

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