

# Modelling farmer participation to a revenue insurance scheme by the means of the Positive Mathematical Programming

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**Abstract:** European farmers face an increasing income uncertainty. The debate is growing on the role of the insurance schemes and on the support provided by the CAP in this field. Therefore, there is a need for the empirical analysis and the tools aimed at providing empirical evidences on this subject. This paper develops an innovative Positive Mathematical Programming model that takes into explicit consideration the risk aversion behaviour. This is used to investigate the implications of participating in the crop revenue insurance scheme. In particular, a quadratic mix integer program has been developed on a small group of crop farms in Italy and it has been used to simulate the impact of changes in the level of the insurance premium. The objective of this paper is to assess the soundness of the proposed approach and to identify its limitations. The obtained results suggest that this could be a useful tool to investigate the impact of participating in insurance schemes on production patterns, farm profitability and the role of public support.

**Key words:** insurance schemes, PMP, farmers' participation, risk aversion, non-linear mix-integer programming.

Farmers are perceived to face an increasing income uncertainty. Commodity prices have been characterised by an increasing volatility in the recent years. This has been experienced also in the domestic EU market given that the Common Agricultural Policy (CAP) has reduced its role in price stabilisation. Production risk is also expected to increase in the future because the current climate changes may bring about higher yield variability due to the increasing occurrence of extreme events and the weather variability. For these reasons, the debate is growing on the potential role of the private and of publicly funded instruments to manage the farm risk including those measures financed by the CAP.

Because of all these elements, it seems relevant to develop evaluation approaches able to provide insights on the management strategies to cope with risk, including the insurance schemes. In order to do so, models used in the empirical analysis should explicitly take into consideration the farmers' risk aversion behaviour (Moschini and Hennessy 2001).

While other approaches have been developed to modelling the revenue crop insurance at the farm level (see, for example, Hansen and de Frahan 2010), this paper focuses on the Positive Mathematical Programming (PMP). In particular, it applies a PMP approach proposed to taking into explicit consideration the risk aversion behaviour (Cortignani and Severini 2010) in order to test whether it can be used to evaluate the potential impact of insurance schemes.

This is done by introducing a revenue insurance scheme into a model developed on a small group of field crop farms located in the Central Italy. Unlike a recent paper (Severini and Cortignani 2011), this model has been developed to explicitly depict the choice of farmers to participating in the proposed scheme by means of a non-linear mix-integer approach. This allows to a better investigation of the impact of the insurance scheme.

The objective of the paper is to develop a preliminary attempt to assess the soundness and applicability of the proposed approach, to consider its strengths and weaknesses and to identify the future developments needed to improve it. Indeed, the paper is presented with the aim of exchanging opinions with other researchers interested in the topic and to receive critiques and suggestions with the aim of improving the approach.

Despite the limited scope of the empirical application, some very preliminary and tentative considerations on the usefulness and drawbacks of the analysis to explore the policy relevant questions are also derived.

## MATERIAL AND METHODS

### Insurance schemes and the role of governmental policies

Revenue insurance is the kind of insurance scheme considered in the empirical application of the model. It

combines the yield and price risk coverage in a single insurance product and it can be product-specific or including whole farm (EC 2006). This insurance could be cheaper than insuring independently the price and yield, as the risk of a bad outcome is smaller: indeed, low yields may be compensated by high prices and vice-versa. Nevertheless, this kind of insurance is not very common in the EU, but available in the USA (EC 2006; Edwards 2009).

Governments have traditionally developed public policies aimed at increasing the risk management ability of farmers including subsidies to premium (Cafiero et al. 2005). This is a very common instrument that is often justified on the grounds that the premium must be affordable, that a sufficient volume of insurance contracts must be underwritten and that insurance companies have to find the insurance product attractive enough to remain in the business.

The emphasis on this instrument has increased also within the CAP. The reform of the CMO wine (Reg. (EC) No. 479/2008) has introduced the possibility of providing public funds for the harvest insurance in order to contribute to safeguarding the producers' incomes where these are affected by natural disasters, adverse climatic events, diseases or pest infestations. A broader instrument has been introduced after the 2009 Health Check of the CAP. Art. 68 of Reg. (EC) No. 73/2009 allows the Member States to use up to 10% of their first pillar funds to grant a specific support to farmers, among others, in the form of the contributions for insurance premiums.

The role of the CAP in supporting the insurance scheme is expected to increase in the near future. The Commission has proposed that "a risk management toolkit should be included to deal more effectively with income uncertainties and market volatility that hamper the agricultural sector's possibility to invest in staying competitive. The toolkit would be made available to the Member States to address both production and income risks, ranging from a new WTO green box compatible income stabilization tool, to the strengthened support to insurance instruments and mutual funds" (EC 2010: p. 11).

## Methodology

The Positive Mathematical Programming (PMP) models have been extensively used to evaluate the farmers' adjustment to changes in the market and policy conditions. However, these models generally consider the risk aversion behaviour only implicitly by the means of the estimated cost function included in their objective functions. Few authors have gone

forward proposing ways to explicitly consider the risk aversion behaviour (Paris and Arfini 2000; Heckeley 2002).

Recently, a way to explicitly incorporate such behaviour into the PMP models has been proposed and empirically tested (Cortignani and Severini 2010; Severini and Cortignani 2011). This approach, formally described in the appendix, is based on a simple expected utility framework under the uncertainty of the activity gross margins and assuming the constant absolute risk aversion coefficients (McCarl and Spreen 1997).

The model has the following general structure:

$$\begin{aligned} \max_{x_n} Z = & E(\bar{g}m_n)' x_n - d_n' x_n - \frac{1}{2} x_n' Q_n x_n - \\ & - \frac{1}{2} \phi_n x_n' \Sigma_{gm} x_n \end{aligned} \quad (1)$$

s. to

$$A' x_n \leq b_n \quad [\lambda_n] \quad (2)$$

where  $E(\bar{g}m_n)$  are the expected unitary gross margin values;  $x_n$  are the model variables that refer to the land allocated to each activity in the  $n$ -th farms;  $d_n$  and  $Q_n$  are the parameters of the quadratic cost function;  $\phi_n$  are the farm specific coefficients of the absolute risk aversion and  $\Sigma_{gm}$  the covariance matrix of the unitary gross margins.

The parameters  $d_n, Q_n$  the  $\lambda_n$  dual values and  $\phi_n$  are estimated by imposing the first-order conditions of the considered farm model taking into account the exogenous information (i.e. supply elasticities) and all the observations over the considered period in which the data is available (Heckeley 2002). The  $\Sigma_{gm}$  has been calculated by taking into consideration the variability of the gross activity margins observed in the same period in the farm sample. The estimation model is described in the Appendix.

This paper develops this kind of model in order to assess its potential use to evaluate the potential role of the revenue insurance schemes. The model considers the possibility to participate in a revenue insurance scheme for a single crop (i.e. durum wheat in the empirical application). When participating in the program, the farmer pays an insurance premium and, if the unitary revenue of that crop falls below the expected level, he/she receives an indemnity calculated on the basis of the difference between the expected and the actual revenue level. In this case, the expected gross margin vector and the covariance matrix of gross margins are recalculated and differ from the case without the insurance scheme.

In a preliminary application of this model, it has been assumed that all farmers participate in the in-

insurance scheme whenever they grow durum wheat in a sort of “compulsory participation” (Severini and Cortignani 2011). This paper goes further by removing such a very restrictive hypothesis by explicitly modelling the participation choice: the model has been developed to allow for the discrete choice of participating or not in the proposed scheme by the means of a quadratic mix integer formulation.

In order to do so, the simulation models have the following general structure:

$$\begin{aligned} \max_{x_{in}, x_{un}} Z = & E(\tilde{g}m_{n,un})' x_{in,un} - \frac{1}{2} \phi_n x_{in,un}' \Sigma_{un} x_{in,un} \\ & + E(\tilde{g}m_{n,in})' x_{in,in} - \frac{1}{2} \phi_n x_{in,in}' \Sigma_{in} x_{in,in} \\ & - d_n' x_n - \frac{1}{2} x_n' Q_n x_n \end{aligned} \quad (3)$$

s. to

$$A'x_n \leq b_n \quad (4)$$

$$x_n = x_{in,un} + x_{in,in} \quad (5)$$

$$x_{in,in} \leq b_n \times \delta_n \quad (6)$$

$$x_{in,un} \leq b_n \times (1 - \delta_n) \quad (7)$$

$E(\tilde{g}m_{n,un})$  and  $E(\tilde{g}m_{n,in})$  are the expected unitary gross margin values for the crop  $j$  without (index  $un$ ) and with (index  $in$ ) the insurance. This latter vector takes into account both the insurance premium and the indemnities.

Variables  $x_n$  are split into two further variables:  $x_{in,un}$  and  $x_{in,in}$ . These refer to the amount of land of each crop grown with and without insurance, respectively.

$\Sigma_{un}$  and  $\Sigma_{in}$  are the variance-covariance matrixes of the activity gross margins without and with insurance;  $\delta_n$  is a farm specific dichotomous variable that can take the values 1 or 0.

The portion of the objective function that accounts for the participation case is given by the second and the third lines of (3). The second line accounts for the expected values and the covariance matrix of the gross margins taking into account the role of the insurance scheme.

Constraint (5) requires that the sum of the variable  $x_i$  for each crop (with and without insurance) is equal to the variable  $x$ .

Constraints (6) and (7) allow making the participation choice discrete. Indeed, when the variable  $\delta_n$  for a specific farm is equal to 1, this forces the farmer to participate in the program with all available land and vice-versa.

Therefore, for a farm participating in the scheme (variable  $\delta = 1$ ), the first line of the objective function (3) cancels out and the objective function only refers to the case with insurance. The opposite occurs in the non-participation case (variable  $\delta = 0$ ).

## Empirical analysis

A sample of 27 FADN farms (constant in the period 2005–2007) specialized in cereals, oilseed and protein crops – located in the province of Ancona (Marche, Italy) – has been taken into consideration<sup>1</sup>.

Most of the area is cultivated to durum wheat which, in average, uses around 60% of the cropped area (Table 1). Other important crops are sunflower and maize.

Before turning to the simulation results, it seems useful to briefly discuss the calibration results and, in particular, the levels of the recovered absolute risk aversion (ARA) coefficients. Two over the 27 farms show a null ARA coefficient suggesting non-risk aversion behaviour. The remaining 25 farms show low levels of the ARA coefficients: in 12 cases these coefficients are non-zero but lower than 0.0002, in 11 farms these range between 0.0002 and 0.0004, while only in two farms these coefficients are higher than 0.0004. The level of the ARA coefficients seems to be negatively but weakly correlated with the farm size (correlation index = –0.553). No correlation is found between the level of these coefficients and the degree of the production specialization of the considered farms.

While the calibrated model relies on the assumption that the analysed insurance scheme is not available to the farmers (BASELINE), all simulations refer to the case in which farmers can decide whether to participate in the insurance scheme or not. The baseline insurance simulation case (BLINS) is described first.

Table 1. Share of each crop in terms of the total cropped area per year and three year average (%)

	2005	2006	2007	Average
Durum wheat	63.3	49.9	64.1	59.1
Maize	3.8	3.8	6.4	4.7
Other cereals	4.8	3.7	5.1	4.5
Sunflower	14.4	18.9	13.8	15.7
Other crops	13.6	23.8	10.7	16

Source: Own calculation on the FADN data

<sup>1</sup>We thank the Italian Institute of Agricultural Economics (INEA) of Rome that has supplied the FADN farm data.

Table 2. Synthesis of the simulation scenarios

Simulation code	Short description of the simulations
BLINS	Baseline insurance simulation case. It refers to durum wheat only and considers the full coverage (100% indemnity) and a premium set at 197.7 €/ha
PREM	It considers a different level of the premium paid by farmers: increases and decreases of: 25%, 50%, 75%, 100% from the BLINS case

Then, another set of simulations considers changes in the level of the unitary premium (PREM) (Table 2).

All simulations assume that an indemnity (*ind*) is paid to farmers whenever the level of the unitary revenues from durum wheat is below its expected revenue level ( $E(rev)$ ). This latter level is calculated on the basis of the weighted average of the unitary revenues from the observations in the following way:

$$E(rev_n) = \frac{\sum_t rev_{n,t} \times x_{n,t}^0}{\sum_t x_{n,t}^0} \quad (8)$$

where  $x(n,t)$  are the amount of land devoted to durum wheat in each farm and period.

The unitary premium paid (*pre*) is identified on the basis of the arbitrary hypothesis that the expected total amount of indemnities ( $E(TIND)$ ) should be equal to 80% of the expected total amount of premiums ( $E(TPRE)$ )<sup>2</sup>. These are calculated *ex-ante* on the basis of the available three-year data set in the following way:

$$E(TIND) = \sum_{n,t} (E(rev) - rev_{n,t}) \times x_{n,t}^0 \quad (9)$$

$$E(TPRE) = \sum_{n,t} pre \times x_{n,t}^0 \quad (10)$$

Note that a uniform unitary premium per 1 hectare of durum wheat (*pre*) is assumed to be applied to all farmers that decide to participate in the insurance scheme.

The unitary revenues for durum wheat in all observations (i.e. for all  $n$  and  $t$ ) are then recalculated introducing the insurance scheme previously described. This generates a new set of the unitary gross

margins that differs from the original one only in the gross margins of durum wheat. This set is then used to recalculate the variance-covariance matrix for the unitary gross margins.

## RESULTS

The empirical analysis has been developed mainly for testing the model and to assess how it responds to: (a) the introduction of the insurance scheme; (b) changes in the levels of the premium paid by farmers. Table 3 reports some basic parameters for durum wheat under the baseline and the BLINS scenario.

The introduction of the revenue insurance generates that 7 over 27 farm models participate in the insurance scheme and that around 28% of the grown durum wheat is insured (Table 4). This also causes a small increase of the total area devoted to durum wheat.

The possibility to participate in the scheme increases slightly with the overall farm expected gross margins (Table 5). This is due to the fact that, in average, the expected indemnities are greater than the premium paid by farmers and that, despite the previous analysis, the model depicts the participation choice on an individual farm basis.

The loss ratio is even higher than 100%, clearly indicating that the total expected indemnities exceed the total amount of premiums paid by farmers (Table 5)<sup>3</sup>. This implies that the insurance companies cannot find this market attractive without the government support. In order to reach the 80% loss ratio,

Table 3. Durum wheat. Basic economic parameters

		BASELINE	BLINS
Expected gross margin	(€/ha)	646	605
Total variance of durum wheat gross margins	€ <sup>2</sup>	130 160	99 629
Premium	(€/ha)	0	197.7
Expected indemnity	(€/ha)	0	263.83

Data are calculated as the weighted average of the whole farm sample

<sup>2</sup>This *ex-ante* evaluation may not be satisfied *ex-post* because farmers can decide whether or not to subscribe the insurance contract.

Table 4. Cropping patterns under the baseline (no insurance) and different scenarios. Whole sample

	Baseline without insurance (BASELINE)	Baseline with insurance (BLINS)	Changes of the premium rate from the BLINS (PREM scenario)							
			relative increases of the premium				relative decreases of the premium			
			100%	75%	50%	25%	-25%	-50%	-75%	-100%
Insured farms (n°)	0	7	0	0	2	4	11	18	23	27
	ha	ha	Percentage change from the BLINS values (%)							
Durum wheat	460	468	-1.7	-1.7	-1.5	-1.0	1.7	4.0	8.9	11.6
<i>insured</i>	0	133	-100.0	-100.0	-66.0	-55.4	58.4	199.3	246.9	293.4
<i>uninsured</i>	460	335	37.2	37.2	24.1	20.5	-20.8	-73.4	-85.4	-100.0
Maize	37	38	-1.1	-1.1	-1.1	-1.1	-5.2	-16.2	-35.1	-42.9
Other cereals	35	32	9.6	9.6	6.9	3.6	-5.1	-5.3	-1.0	-0.6
Sunflower	122	120	1.7	1.7	1.5	1.2	-1.0	-3.2	-8.6	-12.4
Other crops	91	88	3.9	3.9	3.9	2.9	-3.5	-7.9	-20.4	-26.5

the government support must be relatively large: it represents around 2/3 of the total revenues of the insurance companies (Table 5).

Under the conditions set by the scenario BLINS, the increase of the expected gross margin due to the introduction of the insurance scheme is only around 18% of the total amount of the government support (Table 5). This suggests that only a small share of this support translates into an increase of the farmer's income.

The model has also been tested considering the increases and decreases in the level of the premium paid by the farmers (Scenario PREM).

As expected, when the premium increases, the number of the farm models participating in the scheme declines (Table 4): when the premium is increased by at least 50%, no farm models participate in the scheme. Increasing the premium has a very negative impact on the amount of durum wheat enrolled in the insurance scheme and, to a very lower extent, also on the total amount of durum wheat (Table 4).

Increasing the premium rate clearly has a negative impact on the expected farm gross margins that, for the increases of 50% and higher, go back to the baseline level (Table 5). The reduction of the amount of land enrolled in the scheme generates a decrease of the total expected indemnities and of the total premiums paid. This latter result suggests that the increase of the premium does not compensate for the reduction of the land enrolled in the scheme. Because the decrease of the total indemnities is greater than that of the total premium paid, the loss ratio declines from the BLINS case (Table 5). However, it remains

always higher than 100% generating a need for the government support.

The relative importance of this support declines from the BLINS case but only slightly: with the premium by 50% higher than in the BLINS case, the government support still accounts for more than one half of the overall revenues of the insurance companies (Table 5). Given that the farmers are asked to pay a higher premium than in the BLINS case, lower shares of the government support are translated into increases of the farm expected gross margins. With a premium by 50% higher than in the BLINS case, the increase of the expected gross margins is only around 11% of the overall government support (Table 5). This ratio is lower than that observed under the BLINS conditions also because only the farm models with relatively high expected indemnities remain enrolled in the insurance scheme.

Opposite results are obtained when the premium decreases. In this case, more farm models participate in the scheme: all of them are enrolled when the premium is fully paid by the means of the government funds (Table 4). Decreasing the premium has a very positive impact on the amount of durum wheat enrolled in the insurance scheme. However, this also generates a not negligible increase of the total area devoted to durum wheat. For example, when the premium is decreased by 50% from the BLINS, the total durum wheat area increases by around 4% (Table 4).

Decreasing the premium rate clearly has a positive impact on the expected farm gross margins: for example, when it is set at half the BLINS level, these increase by almost 4% (Table 5). The increase

<sup>3</sup>The ex-post loss ratio is higher than the one used ex-ante to identify the BLINS premium (80%). This seems consistent with the way the participation choice has been modelled.

of the amount of land enrolled in the scheme generates an increase of the total expected indemnities and, at least for the decreases up to 50%, of the total amount of premiums paid by the farmers. Clearly this increases the expected loss ratio and, in order to ensure the 80% loss ratio, the amount of support must be strongly increased in order to compensate for the reduction of the unitary farm payments and the increase of participation (Table 5).

Because of the positive effect of the decrease of the premium on the expected gross margins, higher shares of the government support translate into increases of such margins. With a premium by 50% lower than in the BLINS case, the increase of the expected gross margins is only around 22% of the overall government support (Table 5). This ratio is higher than under the BLINS conditions also because now the farm models with relatively low expected indemnities participate in the insurance scheme.

## CONCLUSION

This paper has used a PMP modelling approach that includes the exogenous information on the gross margin variability. This permits to recover the farm specific risk aversion coefficients and to develop a model that

has been found to respond to the simulation scenarios in a different way than other PMP models (Cortignani and Severini 2010). Furthermore, this kind of model can be used to evaluate the likely impact of changes in the variability of gross margins and of introducing an insurance scheme (Severini and Cortignani 2011).

In this paper, the model has been used to evaluate the impact of introducing a revenue insurance scheme for a single activity and of changing the level of premium paid by the farmers. The analysis presented here has overcome an important limitation that affected the previous work on this subject. By using a non-linear mixed integer programming approach, it has represented the choice of the farmers to participate or not to participate in the insurance scheme. This approach has been applied to the data from a small group of field crop farms located in the Central Italy in order to develop a first preliminary empirical test.

The analysis has a couple of limitations which are important to mention before summarising its main results. First, the modelling approach relies on a simplified and restrictive expected utility framework that assumes constant absolute risk aversion coefficients. Second, the empirical test considers only one specific type of the insurance scheme and a very limited and specific sample of farms.

Table 5. Main economic results under the baseline (no insurance) and different scenarios. Whole sample

	Baseline without insurance (BASELINE)	Baseline with insurance (BLINS)	Changes of the premium rate from the BLINS (PREM scenario)							
			relative increases of the premium				relative decreases of the premium			
			100%	75%	50%	25%	-25%	-50%	-75%	-100%
	€ 1 000	€ 1 000	Percentage change from the BLINS values (%)							
Total Gross Margins	662	671	-1.4	-1.4	-1.1	-0.7	1.3	3.7	6.9	10.4
Total amount of premiums	0	26	-100.0	-100.0	-49.1	-44.2	18.8	49.7	-13.3	-100.0
Total amount of indemnities	0	35	-100.0	-100.0	-57.5	-47.0	38.8	110.7	127.4	134.0
Loss ratio w/out gov. support* (%)	–	133	–	–	111	127	156.0	188	350	–
Government support** (1000 €)	0	53	0	0	16	26	89	159	231	308
Loss ratio with gov. support (%)	–	80	–	–	80	80	80	80	80	80
Premium paid by farmers (€/ha)	–	198	–	–	297	247	148	99	49	0
Unitary gov. support (€/ha)	–	132	–	–	116	144	141	133	167	196

\*The total expected amount of premiums paid by farmers divided by the total expected value of indemnities

\*\*Government support to insurance companies to ensure ex-post a 80% loss ratio

Source: Own elaboration on the FADN data

Despite these limitations, the analysis has produced some interesting results. The model has been able to investigate the impact of introducing an insurance scheme and of changing the level of the premium paid by the farmers. It has permitted to assess how this affects the participation, the production choices and the relative profitability of both farmers and insurance companies. Furthermore, it allows assessing if and under which conditions such scheme could remain in place.

The results of the empirical test suggest that the proposed model responds in a coherent way to the considered simulations. Introducing the insurance scheme provides an incentive for some farmers to participate and to increase the land used to grow the

insured crop. Decreasing (increasing) the level of the premium paid by farmers increases (decreases) the participation in the insurance scheme and the acreage of the insured but also of the total durum wheat. Thus, under the considered case, providing the government subsidies increases, the production of the insured crop is showing its small production-distorting nature. Finally, decreasing (increasing) the level of premium positively (negatively) affects the farm economic results. However, in the considered empirical conditions, the proposed insurance scheme seems not to be profitable for the insurance companies. Thus, such market could be developed only if the government provides subsidies in order to cover a large share of the premium.

## APPENDIX. DESCRIPTION OF THE ESTIMATION MODEL

In our analysis, we use the method proposed by Heckeleei (2002) extending it to explicitly considering the risk aversion<sup>4</sup>. This uses the Generalized Maximum Entropy (GME) approach covered by the restrictions needed to determine the appropriate curvature of the cost function and incorporates the exogenous supply elasticities (Heckeleei 2002).

Considering that the data refer to several years ( $t = 1, \dots, T$ ), the GME problem is specified as follows:

$$\begin{aligned} \max_{\mathbf{w}_t, \mathbf{d}, \phi, \mathbf{Q}, L, \lambda_t} H(\mathbf{w}_t) &= - \sum_{t=1}^T \mathbf{w}'_t \ln \mathbf{w}_t - \mathbf{w}^\varepsilon' \ln \mathbf{w}^\varepsilon \\ \text{s. to} \\ E(\tilde{\mathbf{g}}\mathbf{m}_t) - \lambda_t \mathbf{A} - \mathbf{d}_t - \mathbf{Q}(\mathbf{x}_t^0 - \mathbf{V}\mathbf{w}_t) - \phi \Sigma_{gm}(\mathbf{x}_t^0 - \mathbf{V}\mathbf{w}_t) &= 0 \\ E(\tilde{\mathbf{g}}\mathbf{m}_t) - \lambda_t \mathbf{A} - \mathbf{d}_t - \mathbf{Q}(\mathbf{x}_t^0 - \mathbf{V}\mathbf{w}_t) - \phi \Sigma_{gm}(\mathbf{x}_t^0 - \mathbf{V}\mathbf{w}_t) &< 0 \\ \mathbf{A}'(\mathbf{x}_t^0 - \mathbf{V}\mathbf{w}_t) &= b_t \\ \mathbf{Q} = \mathbf{L}\mathbf{L}' \quad \text{con } L_{j,i'} &= 0 \quad \forall i' > j \end{aligned}$$

$$\mathbf{V}^\varepsilon \mathbf{w}^\varepsilon = \left[ (\mathbf{Q}^{-1} - \mathbf{Q}^{-1} \mathbf{A} (\mathbf{A}' \mathbf{Q}^{-1} \mathbf{A})^{-1} \mathbf{A}' \mathbf{Q}^{-1}) \odot \left[ \frac{\mathbf{g}\mathbf{m}^0}{\mathbf{x}^0} \right]' \right]$$

$$\sum_{s=1}^S w_{j,t,s} = 1$$

$$\sum_{s=1}^S w_{j,t,s}^\varepsilon = 1$$

where  $H(\mathbf{w}_t)$  is the level of entropy, the errors vector ( $\mathbf{V}\mathbf{w}_t$ ) is re-parameterized as the expected value of a discrete probability distribution by defining the  $\mathbf{V}$  support matrix and the  $\mathbf{w}_t$  probabilities vector; elasticities ( $\mathbf{V}^\varepsilon \mathbf{w}^\varepsilon$ ) are re-parameterised in the same way as the error terms by defining the  $\mathbf{V}^\varepsilon$  support matrix and the  $\mathbf{w}^\varepsilon$  probabilities vector<sup>5</sup>;  $\mathbf{g}\mathbf{m}_t$  are the gross margins of each activity;  $\lambda_t$  is the shadow price of land over several

<sup>4</sup>For details see Cortignani and Severini (2010).

<sup>5</sup>The intuition behind the objective function is that the entropy criterion pulls towards the centre of the elasticity support range, in opposition to the error terms of the data constraints. The smaller the elasticity support range, the higher the penalty for deviating from the support centre. Consequently, the width of the support range reflects the precision of the *a priori* information (Heckeleei and Wolff 2003).

years;  $A$  is the technical coefficients matrix;  $d_t$  and  $Q$  are respectively the parameters associated with the linear term and the quadratic term of the cost function;  $x_t^0$  are the observed levels of activity in different years;  $\phi$  are the coefficients of the absolute risk aversion for each farm  $n$  and  $\Sigma_{gm}$  the covariance matrix of the gross margins<sup>6</sup>;  $L$  is the lower triangular matrix of the Cholesky decomposition. The first two constraints impose the first order conditions for the observed and for the not observed activities. The following two equations ensure that the land allocated to different crops in each year is equal to the total available land, and the proper curvature of the cost function. The fifth constraint is the combination between the elasticity re-parameterization ( $V^e w^e$ ) with the Jacobian matrix that contains the partial derivatives of the land demand functions  $\left(\frac{\partial x_i}{\partial gm_i}\right)$ ; the matrix  $\left[\frac{gm^0}{x^0}\right]$  is defined as the sample mean of the activity gross margin ( $gm^0$ ) divided by the sample mean of the observed land allocation ( $x^0$ ). The last two constraints relate to the probability law (where  $s$  is the number of support values).

<sup>6</sup>Upper and lower bounds on the level of the coefficient of the absolute risk aversion have been imposed. The E-V risk aversion coefficient equal the E-standard error risk aversion coefficient divided by twice the standard error. Because the E-standard error risk aversion coefficient usually ranges from 0–3 (McCarl and Spreen 1997), these values have been chosen as lower and upper bounds. The  $\Sigma_{gm}$  has been calculated taking into consideration the variability of gross activity margins observed during the three-year period.

## REFERENCES

- Cafiero C., Capitanio F., Cioffi A., Coppola A. (2005): Risk and crisis management in agriculture. Final Report prepared for European Parliament. October 2005.
- Cortignani R., Severini S. (2010): Taking into explicit consideration risk aversion behaviour in Positive Mathematical Programming models: an empirical application. Paper presented at III Workshop on Valuation Methods in Agro-food and Environmental Economics. CREDA-UPS-IRTA, Castelldefels (Barcelona), Spain, 1–2 July 2010.
- Edwards W. (2009): Crop Revenue Insurance. Ag Decision Maker, Iowa State University, University Extension. Available at <http://www.extension.iastate.edu/agdm/>
- European Commission (EC) (2006): Agricultural Insurance Schemes. European Commission – Joint Research Centre, Ispra (Italy).
- European Commission (EC) (2010): The CAP towards 2020: Meeting the food, natural resources and territorial challenges of the future. COM(2010) 672 final. Brussels, 18 November 2010.
- Hansen K., de Frahan B.H. (2010): Risk in Agriculture: Modeling Revenue Insurance for Crop Farms in Belgium. Paper presented at the III Workshop on Valuation Methods in Agro-food and Environmental Economics. CREDA-UPS-IRTA, Castelldefels (Barcelona), Spain, 1–2 July 2010.
- Heckeley T. (2002): Calibration and estimation of programming models for agricultural supply analysis. University of Bonn.
- McCarl B.A., Spreen T.H. (1997): Applied Mathematical Programming Using Algebraic Systems. Available at <http://agecon2.tamu.edu/people/faculty/mccarl-bruce/books.htm>
- Moschini G., Hennessy D.A. (2001): Uncertainty, risk aversion and risk management for agricultural producers. *Handbook of Agricultural Economics*, 1: 87–153.
- Paris Q., Arfini F. (2000): Funzioni di costo di frontiera, auto-selezione, rischio di prezzo, PMP e Agenda 2000. *Rivista di Economia Agraria*, 15: 211–242.
- Severini S., Cortignani R. (2011): An attempt to modelling revenue insurance schemes at the farm level by means of Positive Mathematical Programming. Paper accepted for presentation at the 122<sup>nd</sup> EAAE Seminar. Ancona, February 17–18, 2011.

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