Price and location equilibria in a circular market: a pure vs a mixed duopsony with a co-operative

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Abstract: The objective of the present paper is to analyze the location-price competition in circular markets where the power lies with the buyers. To this end, it considers two alternative market structures. Namely, the pure ones, where the buyers of a primary commodity are private firms, and mixed ones, where a private firm competes against a producer’s co-operative. According to the results, the pure-strategy location equilibrium in both cases involves a distance between the two players larger or equal to 1/4. Nevertheless, the equilibriums are qualitatively different. In the pure duopsony, a large distance is required to prevent a price war while in the mixed duopsony, the private firm tries to stay away from the co-op in order to ensure a strictly positive profit.

Key words: circular spatial competition, market power, mixed oligopsony

Following the seminal work of Hotelling (1929), the location problem of firms, either in the geographical space or in the characteristics space, has been studied extensively using a two-stage framework where in the first stage the competitors select locations and in the second stage, they select prices. The representation of space (market), however, as a straight compact line, has led to several peculiarities and analytical difficulties. Indeed, as shown by d’Aspremont et al. (1979), the Hotelling’s original model has no pure strategy location-price equilibriums in the presence of linear transportation costs. Among the approaches/modifications adopted to circumvent that problem, there has been the use of circular markets (e.g. Salop 1979; Eaton and Wooders 1985; Kopp 1993). Specifying space as the circumference of a disk ensures that each firm in the market faces competition on both sides of its location and it restores equilibrium in pure strategies (Kats 1995). Recent works on competition in circular markets are those of Ishida and Matsushima (2004), Matsumura and Shimizu (2006) and Ebina et al. (2011).

All theoretical contributions on location-price equilibriums over the circumference of a disk have considered cases where the market power lies with the sellers. The duopoly or the oligopoly is certainly a structure which adequately represents the allocation of power in many real-word finished product markets. As noted, however, by Sexton (1990) and Alvarez et al. (2000), the primary/raw commodity markets are often narrow in the geographical dimension. For example, although the markets for processed agricultural commodities may be national or international in scope, the markets for the associated primary products are usually local or regional; the bulkiness, perishability, high transportation costs, and high storage costs restrict the access of farmers to only those buyers within a limited geographical area. Similar arguments apply for other natural resource based industries, such as the forest industry (e.g. Lofgren 1986). Therefore, for analyzing markets of primary commodities, the mirror image of oligopoly (that means, the oligopsony) appears to be the most relevant theoretical model.

The presence of an oligopsony in a given market reduces the sellers’ welfare. Because of this, primary producers have the incentive to integrate around the market distortion in order to curtail the buyers’ power. That integration typically takes the form of a primary producers’ processing/marketing co-operative. According to the General Confederation of Agricultural Co-operatives, there are around 26 000 co-ops in the EU contributing more that 50 percent of the added value in the production, transformation and commercialization of farm products (COGECA 2005). In the USA, according to the National Council of Farmer Cooperatives, there are nearly 3000 local agricultural co-ops with about $50 billion in the total assets and $125 billion in the total business volume (NCFC 2008); they account for 25 to 30 percent of the total farm supply and marketing expenditures (Drivas and Giannakas 2008).

From the above, it is obvious that spatial primary commodity markets can be either pure or mixed ones. In the pure ones, the profit-maximizing investor-
owned firms (IOFs) compete among each other in the mixed ones, the IOFs compete against primary producers’ co-operatives. Although there is a number of theoretical contributions on the mixed oligopsony markets, in only few of them the spatial dimension of competition has been taken into account (e.g. Sexton 1990; Drivas and Giannakas 2008; Fousekis 2011). Even in those last works, however, the market has been represented as a compact straight line and the emphasis has been placed on the choice of prices or on pricing policies only (firm locations have been taken as given a priori).

THEORETICAL FRAMEWORK

Let a continuum of identical primary producers spread evenly on a circle with a unit circumference. Let also that two processors of the primary commodity are active in this circular market. Each processor offers a mill/Free-on-Board (FOB) price, \( p_i \), and let the primary producers pay the costs involved in transporting the commodity from their own location to the processing facility’s gate. The transportation cost are linear in distance; that is, a primary producer located at point \( x \) on the circular market bears a transportation cost equal to \( t|x – x_i| \) when shipping one unit of the commodity to processor located at \( x_i \) \((t > 0\) is the freight rate). Each primary producer has a unit supply; that means, she/he sells one unit of the primary commodity provided that the net (delivered) price she/he receives is not less than the (common to all) reservation utility level \( u \), and she/he supplies zero, otherwise. A producer will ship the primary commodity to firm \( i \) if \( m_i – t|x – x_i| ≥ m_j – t|x – x_j|, i = 1, 2 \) and \( i ≠ j \). If the delivered prices are equal, she/he will ship the commodity to the processor located closest to him/her, while if both the mill prices and the distances are equal she/he will choose a processor randomly with equal probabilities.

To convert the primary commodity into a final commodity, each firm incurs a constant average (and marginal) cost \( γ \). The final commodity is sold in a perfectly competitive market at price \( p \). In the pure duopsony case, the two processing firms aim at maximizing profits. For the primary input processing co-ops, a number of different objectives have been proposed in the relevant literature, including the maximization of processing margins, the maximization of member welfare, and the maximization of the price co-op members receive for the supply of the commodity (e.g. Le Vay 1983; Cotterill 1987). In this study, as Drivas and Giannakas (2008) and Fousekis (2011), we consider an open membership co-op pric-
The market border on the right of point 0 is determined by the equation
\[ m_i - tk = m_2 - t(x_2 - k) \Rightarrow k = \frac{m_i - m_2 + tx_2}{2t} \]  
(1)

and the market border on the left of point 0 is determined by the equation
\[ m_i - t(l - k') = m_2 - t(k' - x_2) \]
\[ \Rightarrow (1 - k') = \frac{m_1 - m_2 - tx_2 + t}{2t} \]  
(2)

From (1) and (2), there follow the market shares of the two processors as
\[ S_i(m_1, m_2, t) = \frac{\frac{2m_i - 2m_1 + t}{2t}}{m_i} \quad (i = 1, 2 \quad i \neq j) \]  
(3)

and the corresponding payoff (profit) function as
\[ \pi_i(m_1, m_2, t) = (p - \gamma - m_i) S_i(m_1, m_2, t) \]  
(4)

Subsequently, to simplify the analysis (without the loss of generality and to ensure the comparability with the earlier works on pure duopoly), we normalize the processing margin \( p - \gamma \) to 1 and the reservation utility level to 0.

In the price sub-game, the processor 1 has three strategic options to react to the competitor's mill price: (a) to set \( m_1 < m_2 - tx_2 \), something that leads to zero profit for firm 1 (b) to set \( m_2 - tx_2 \leq m_1 \leq m_2 + tx_2 \) (accommodation strategy), something that leads to positive profits for the IOF 1 given by
\[ \pi_i^a(m_1, m_2, t) = (1 - m_i) S_i(m_1, m_2, t) \]  
(5)

(c) to set \( m_1 = m_2 + tx_2 + \varepsilon \), where \( \varepsilon \) is a very small positive number (overbidding strategy), something that leads to positive profits for the IOF 1 given by
\[ \pi_i^o(m_1, m_2, t) = 1 - m_i \]  
(6)

Because the game is symmetric, the IOF 2 has the same strategic options as the IOF 1 does. The corresponding profit functions for the IOF 2 can be derived in an analogous manner.

Under the accommodation strategy, the reaction function of the processor 1 is
\[ m_i^a(m_2) = \frac{1}{2} + \frac{m_2}{2} - \frac{t}{4} \]  
(7)

If the IOF 2 uses the accommodation strategy as well, its reaction function is
\[ m^a_i(m_2) = \frac{1}{2} + \frac{m_2}{2} - \frac{t}{4} \]  
(8)

From (7) and (8) follows that the Nash equilibrium in the price sub-game under accommodation is
\[ m_i^a = m_i^o = m^a = 1 - \frac{t}{2} \]  

**Location equilibriums**

It is straightforward to show that in the first stage of the game, there can be no location equilibrium in which firm 2 selects \( x_2 = 0 \). Indeed, if \( x_2 = 0 \) the game between the two firms reduces to the textbook Bertrand symmetric duopoly, in which both firms in the price sub-game offer \( m_1 = m_2 = 1 \) resulting in zero profits. However, by selecting \( x_2 = 1/2 \) and setting its mill price \( 1 > x_2 > t/2 \) the IOF 2 can assure itself a strictly positive profit (IOF 1 cannot overbid IOF 2's price). Therefore, the equilibriums in the first sub-game as well as the sub-game perfect ones will involve \( x_2 > 0 \).

With \( x_2 > 0 \) substitution of (7) into (5) yields
\[ \pi_i^a(m_2, t) = \frac{1}{2t} \left( \frac{1}{2} - \frac{m_2}{2} + \frac{t}{4} \right) \left[ 1 - m_2 + \frac{t}{4} \right] \]  
(9)

a function which is strictly decreasing and strictly convex in \( m_2 \). Evaluating (6) at \( m_1 = m_2 + tx_2 \) yields
\[ \pi_i^o(m_2, t) = 1 - m_2 - tx_2 \]  
(10)

a function which is linear and strictly decreasing in \( m_2 \). Subtracting (9) from (10) one obtains, after some simple algebra,
\[ D_{A,O}(m_2, t) = \pi_i^o(m_2, t) - \pi_i^a(m_2, t) = -\frac{m_2^2}{4t} + \frac{m_2}{2} + \frac{t}{4} - \frac{m_2}{2} tx_2 \]  
(11)

The equation \( D_{A,O}(m_2, t) = 0 \) has two real roots,
\[ m_2^* = \frac{-3t}{2} + 1 \pm \frac{1}{2} \sqrt{2 - 4tx_2} \]  
(12)

and because \( D_{A,O}(m_2, t) \) is strictly concave in \( m_2 \), it is the case that \( D_{A,O}(m_2, t) > 0 \) for any \( m_2 \) between \( m_2^- \) and \( m_2^* \) and that \( D_{A,O}(m_2, t) < 0 \), otherwise. On the basis of the information obtained above, Figure 2 depicts the relationship between \( \pi_i^a(m_2, t) \) and \( \pi_i^o(m_2, t) \) for different values of \( m_2 \).

For \( m_2 \in (m_2^-, m_2^*) \) IOF 1 overbids IOF 2's price; for \( m_2 \geq m_2^* \) firm 1 employs the accommodation strategy. To determine the IOF 1's choice for \( m_2 \leq m_2^- \) assume momentarily that when processor 1 faces \( m_2 = m_2^- \) it responds using accommodation (relation (7)). That means, she/he sets
\[ m_i^a(m_2^-) = 1 - \frac{t}{2} + \frac{1}{2} \sqrt{2 - 4tx_2} \]  
(13)
Then, 
\[ m_1'(m_2^*) - m_2^* = \frac{t}{2}(1 + \sqrt{2 - 4x_2}) > 0 \]
\[ \Rightarrow m_1'(m_2^*) > m_2^* + \frac{t}{2}(1 + \sqrt{2 - 4x_2}) \]
\[ \Rightarrow m_1'(m_2^*) > m_2^* + \frac{t}{2} \Rightarrow m_1'(m_2^*) > m_2^* + tx_2 \]

where the last two implications follow because \( x_2 \leq 1/2 \).

From (14) follows that \( m_1'(m_2^*) \) is higher than the price required to overbid the IOF 2's price. Indeed, the IOF 1 can overbid by setting \( m_1'(m_2^*) \) and increase its profits. One may conclude, therefore, that for \( m_2 \leq m_2^* \) the IOF 1 will overbid. Because the game is symmetric an analogous argument applies for firm 2. The Nash equilibrium in pure strategies can be sustained when accommodation is the best response of each IOF to the competitor's pricing strategy. As shown above, processor 1 will accommodate when \( m_1^* \geq m_2^* \) or

\[ 1 - \frac{t}{2} \geq \frac{3t}{2} + tx_2 \Rightarrow 1 \geq \sqrt{2 - 4x_2} \Rightarrow x_2 \geq \frac{1}{4} \quad \text{(15)} \]

The sub-game perfect Nash equilibrium in pure strategies involves \( m_1 = m_2 = 1 - t/2 \) and \( x_2 \in [1/4, 1/2] \). With respect to location choices, the analysis of the pure duopoly game replicates the results obtained in the earlier works on the pure duopoly. This of course must be attributed to the choice of model parameters and to the underlying assumptions such as unit supply of the primary commodity and constant processing margin. In any case, the results of the two games suggest that when the two IOFs come sufficiently close to each other (here, at distance less than 1/4 of the unit circumference), the competition becomes so intense that all attempts to drive the other out of the market precluding an equilibrium in pure strategies.

\footnote{Note that if the assumption \( x_2 \leq 1/2 \) is relaxed the equilibrium involves \( x_2 \in [1/4, 3/4] \).}

THE MIXED DUOPSONY

Price equilibriums

Let the co-op be located at the point 0(1) and the IOF be located at point \( x_2 \leq 1/2 \) (note that the results are unaffected if we assume that the IOF is located at the point 0(1) and the co-op is located at the point \( x_2 \leq 1/2 \). Full market coverage requires \( m_c + m_1 + tx_2 - t \geq 2t \), where \( m_c \) stands for the price offered by the IOF. Working in exactly the same way as in sub-section Price equilibria we obtain the border on the right of 0 as

\[ k = \frac{m_c - m_1 + tx_2}{2t} \]

and the border on the left of 0 as

\[ k' = \frac{m_2 - m_c + tx_2 + t}{2t} \]

From (16) and (17) follow the market share of the co-op and the IOF as

\[ S_c(m_c, m_1, t) = \frac{2m_c - 2m_1 + t}{2t} \]

and

\[ S_i(m_c, m_1, t) = 1 - S_c(m_c, m_1, t) \]

respectively. Solving the IOF's profit maximization problem and normalizing the processing margin to one (that means, setting \( m_c = NAPR = p - \gamma = 1 \), one obtains the IOF's price in the second stage of the game as \( m_1 = 1 - t/4 \).

The location equilibriums

In the mixed duopsony, the co-op which prices on the basis of its NARP is the aggressive agent; any attempt of the IOF to match (or overbid), the co-op’s price will entail zero (negative) payoffs for it. In contrast with the duopsony of the IOFs where we look for locations which preclude price wars (attempts of overbidding), here, we are interesting in location choices allowing the IOF to attain positive profit in equilibrium.

For \( x_2 < 1/4 \), the market border on the right of 0 will be \( k < 1/4 \). As a result, the delivered price enjoyed by the co-op members at \( k \) will be \( 1 - tk > 1 - t/4 = m_f \). As a result, the location equilibriums involving both firms in the market with \( x_2 < 1/4 \) are precluded, since

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the NARP pricing by the co-op drives the IOF out of the circular market. For \( x_2 \geq 1/4 \), the delivered price by the co-op at \( x_2 \) will be strictly lower than the IOF’s mill price. As a result, there always exist borders \( k \) given by (16) and satisfying \( x_2 \geq k \geq 1/4 \) such that the IOF remains in the circular market. We conclude, therefore, that the location equilibriums in the mixed duopsony with a co-op are exactly the same as that in the pure duopsony.

**WELFARE EFFECTS**

The IOF’s price in the mixed duopsony, \( m_i = 1 - t/4 \), is higher than the price the profit maximizing processor offers in the pure duopsony game, \( m_i = 1 - t/2 \). The difference between the two mill prices is the result of the “competitive yardstick effect” (ability of co-ops to discipline the IOFs). In the pure duopsony game, each IOF captures one half of the circular market (from relation (3)) and obtains profit equal to \( t/4 \) (from relation (4)). The primary producer welfare is:

\[
W^{P,pure} = 2 \int_{0}^{k'} (1-t/2-ts)ds + \int_{0}^{1-k'} (1-t/2-ts)ds \tag{20}
\]

In the mixed duopsony game, the co-op captures \( 3/4 \) and the IOF \( 1/4 \) of the spatial market. The IOF’s profit is

\[
\pi^{IOF,CooP} = \int_{k}^{\frac{k'}{4}} (t/4)ds = \frac{t}{16} \tag{21}
\]

the welfare of CO-OP members is

\[
W^{P,coop} = \int_{0}^{k} ds + \int_{0}^{1-k'} ds = 0.75 \tag{22}
\]

and the welfare of the IOF’s patrons is

\[
W^{P,IOF} = \int_{0}^{x_2-k} (1-t/4-ts)ds + \int_{0}^{k'-x_2} (1-t/4-ts)ds \tag{23}
\]

In several cases, the firm profits and producer welfare depend not only on the freight rate but also on the market borders \( (k \) and \( k') \) and \( x_2 \). Therefore, in order to perform a comparison between the two market structures, we contact a simulation setting \( t = 0.1 \) and letting \( x_2 \) going from \( 1/4 \) to \( 1/2 \) (with increments of 0.05). Table 1 presents the results. As expected, the welfare in the mixed duopsony market is considerably higher (26%, in average) than the welfare in the pure IOF market. This happened because the presence of the co-op has reduced the oligopoly power of the private firm (the IOF not only offers a higher mill price to its patrons but it also loses part of its market area). The increase in the primary producer’s surplus (31.7%, in average) is more than sufficient to compensate for the reduction in the private processor’s profit.

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<th>Table 1. Welfare Comparisons</th>
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<td>(a) The pure IOF market</td>
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<td>(b) The mixed market with a co-operative</td>
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CONCLUSIONS

The specification of the geographical or of the quality space as the circumference of a disk makes it possible to overcome the analytical difficulties associated with the compact straight line spatial markets. All earlier works on the location-price equilibriums over circular markets have focused exclusively on oligopoly, a structure which very often cannot represent the allocation of power in the primary/raw commodity markets.

The present paper investigates the location-price equilibrium in circular markets where the power lies with the buyers. Moreover, it explicitly takes into account that primary commodity markets can be either pure (the competition takes place among profit maximizing firms only) or mixed ones (the competition takes place between private firms and producers’ co-operatives).

According to our results:
(a) The pure-strategy location equilibrium in the duopoly of private firms involves a distance between the two competitors larger than or equal to 1/4 on the unit circumference. For smaller distances, the pure-strategy equilibrium cannot be sustained since both firms have the incentive to wage a price war (each attempts to overbid the competitor’s price).

(b) The pure-strategy location equilibrium in the mixed duopoly also involves a distance between the co-op and the IOF larger than or equal to 1/4 on the unit circumference. The two location equilibriums, however, are qualitatively different. In the pure duopoly, a sufficient distance between the private firms is required to prevent a price war; in the mixed duopoly, the co-op is a very aggressive agent and the private firm has to stay away from it in order to survive (to attain a strictly positive profit).

(c) The market welfare in the spatial mixed duopoly is substantially higher than in the pure one, since the increase in the primary producers’ surplus outweighs by far the reduction in the private firms’ profit.

There is a number of possible future research avenues. One, for example, may relax the assumption of the unit supply and allow for a strictly positive impact of the delivered price on the supply of the primary commodity. Another is to consider alternative pricing policies such as the Uniform Delivered or the Discriminatory one instead of the FOB pricing. Finally, one also may introduce cost asymmetry either between the two IOFs or between the IOF and the co-op.

Appendix

Conditions under which there is no Full Coverage (game between IOFs)
(a) \( x_2 = 0 \) and \( m_1 = m_2 < \bar{u} - t/2 \)

Both firms have a strictly positive (and equal supply) and the market is not covered.

(b) \( x_2 > 0, \quad m_1 + m_2 + tx_2 - t \leq \bar{u} \leq m_1 + m_2 + tx_2 \)

and

\[ m_2 - tx_2 \leq m_1 \leq m_2 + tx_2 \]

Both firms have a strictly positive supply and the market is not covered to the left of \( 0 \).

(c) \( x_2 > 0, \quad \bar{u} \geq m_1 + m_2 + tx_2 \), and

\[ m_2 - tx_2 \leq m_1 \leq m_2 + tx_2 \]

Both firms have a strictly positive supply, and the market is not covered to the left of \( 0 \) nor to the left of \( x^*_2 \).

(d) \( x_2 \geq 0, \quad m_1 + m_2 - tx_2 \leq \bar{u} \leq m_1, \quad m_1 - tx_2 > m_2 \)

and \( \pi_i(m_1, m_2, \bar{u}) = (p - \gamma - m_1)S_i(m_1, m_2, \bar{u}) \)

Firm 1 is a spatial monopsonist which cannot cover the whole market.

The conditions for mixed duopoly follow from the above by replacing \( m_1 \) with \( m_c \).

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Received: 21st November 2013
Accepted: 5th February 2013

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