Road network optimization using heuristic and linear programming

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ABSTRACT: To minimize the cost of logging, it is necessary to optimize the road density. The aim of this study was to determine optimal road spacing (ORS) in Northern Austria. The stepwise regression method was used in modeling. The production rate of tower yarder was 10.4 m³/PSHo (Productive system hours) and cost of 19.71 €.m⁻³. ORS was studied by calculating road construction cost, installation cost and yarding cost per m³ for different road spacing. The minimum total cost occurred at 39.15 €.m⁻³ and ORS would be 474 m assuming uphill and downhill yarding. The optimal road density and yarding distance are 21.1 m.ha⁻¹ and 90 m, respectively. A sample logging area was used to plan different roads and, using network analysis, the best solution was found based on a modified shortest path algorithm. The network analysis results were very different from the optimal road spacing results that assumed roads and logging corridors could be located anywhere in the planning area at a constant cost. Mixed integer programming was also used to get a real optimal solution.

Keywords: cable yarding; mixed integer programming; network analysis; optimum road density; production

Austria is a mountainous country in Central Europe. About 60% of the forest has slope greater than 30% and 22% of the forestlands are located on the steep ground with slope greater than 60%. Hilly terrain and mountainous forest regions have led to the use of cable yarding systems in this country. About 19% of harvesting operations are done by cable yarding systems. The common types of cable logging systems are sled winch, tower yarder and self-propelled carriage.

Road planning is an important step in planning the forest operation. Optimization of the road network can help minimize the total cost of harvesting. The average road density in Austrian forests is 45 m.ha⁻¹ (www.bfw.ac.at). It is necessary to determine the optimal road density to minimize the combined yarding and road costs. Matthews (1942) first studied optimal road spacing. He developed a model to define optimum road spacing based on minimizing the total cost of skidding and road construction from the viewpoint of a landowner. For downhill forwarding operation an ORS of 503 m was reported in Southern Austria (Ghaffariyan et al. 2007). In two and three-stage cable yarding systems in British Colombia, three-stage yarding provided cost savings and a substantial increase in road spacing once critical road costs were exceeded (Howard, Tanz 1990).

In the last years, mixed integer mathematical programming and heuristic algorithms such as TIMBRI (Sullivan 1974), TRANSHIP (Kirby et al. 1981), MINCOST (Wong 1981), NETCOST (Weintraub 1986), NETWORK (Sessions 1978) and NETWORK 2000 (Sessions, Chung 2003) have been used to find the lowest cost solution for certain fixed and variable cost problems. NETWORK (Sessions 1978) and NETWORK 2000 (Sessions, Chung 2003) have been used to search for the low-
est cost solutions for certain fixed and variable cost problems. Sessions (1992) introduced the method of using network analysis for road and harvesting planning which is applied in this study. Tan (1999) developed a spatial and heuristic procedure to locate forest roads. Stueckelberger et al. (2006) considered road construction cost, ecological effects and suitability for cable yarding landings in their automatic road-network planning using multi-objective optimization in Switzerland.

Cable yarding systems are more expensive than ground based skidding systems in areas where ground based skidding systems can operate. A yarding time prediction model is developed using multiple regression based on the data collected by Limbeck-Lilienau (2002) for cable yarding in Northern Austria. Optimal road spacing formulas (Matthews 1942) can provide a guide to optimal road density and a cost target but does not suggest where the roads should be actually placed. Thus, the road spacing formulas provide a lower bound on the logging costs since roads cannot be placed everywhere in the landscape and either more roads or fewer roads will be necessary. Mixed integer programming and network analysis have been used to solve different logging problems, but they have not been applied for optimization of road spacing. Therefore, the aim of this study is the application of mixed integer programming and network analysis to optimize the road network in a mountainous area. The results are compared with minimization of total cost method using road spacing formulas which assume roads can be placed almost anywhere, the unit cost of road construction is constant everywhere in the landscape, and road gradient is not limiting. The effects of harvesting volume per ha and road construction costs on ORS are discussed.

**MATERIAL AND METHODS**

**Site of study**

The study site is located in Steyr and Gmünden in Northern Austria, and was harvested using the Syncrofalke tower yarder (Table 1). The Syncrofalke tower yarder is usually used to yard the logs at a medium yarding distance less than 800 m. This yarder was combined with a Wolf 50 B processor for whole tree yarding. The crew consisted of two persons, a yarder operator and the chainsaw operator who felled and topped and set chokers. Yarding was performed both uphill and downhill (Limbeck-Lilienau 2002).

<table>
<thead>
<tr>
<th>Table 1. Site study description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–300</td>
</tr>
<tr>
<td>28–32</td>
</tr>
<tr>
<td>0.67–1.06</td>
</tr>
<tr>
<td>32–60</td>
</tr>
<tr>
<td>Fir-larch and beech stand composition</td>
</tr>
<tr>
<td>551–745</td>
</tr>
</tbody>
</table>

Assumed that the road construction cost increased as a function of ground slope (Table 2)

To study the optimal road network using the network analysis and mixed integer programming methods, a digital map of a mountainous forest area of 196 ha in the northeast of Austria was used. Most of this sample area was steeper than 35%. It was assumed the whole area would be harvested in the same year.

**Yarding time model**

It was assumed that the yarding cycle time is a function of the variables such as yarding distance, lateral yarding distance, load volume, tree volume, harvest intensity, stand density, yarding direction, harvesting time (summer or winter) and slope of the cableway. The multiple regression and stepwise method was applied to develop a model to predict the yarding time per cycle. In this method, if the desired variables have a significant effect on residual mean squares of the model, they enter the model.

**Road spacing**

Application of graphing method of minimization of road construction and yarding cost

The road construction costs in Steyr and Gmünden vary from 14 to 100 €.m⁻¹. The logging volume ranges from 100 to 230 m³.ha⁻¹. The hourly cost of the tower yarder is about 205 €.h⁻¹. The road construction, installation and yarding cost per m³ were computed for different yarding distances in the range of the observations of this study.

Models developed by Stampfer et al. (2006) were used to estimate installation cost. Stamper’s models estimate the time to set up and take down cable yarding settings in Austria for different systems.

Installation time (h) = Set-up (h) + Take-down (h)
Set-up time (h) = EXP (1.42 + 0.00229 × corridor length (m) + 0.03 × int. support height (m) +
0.256 × corridor type – 0.65 × extraction direction + 0.11 × yarder size + 0.491 × extraction direction × yarder size).

Take-down (h) = EXP (0.96 + 0.00233 × corridor length (m) – 0.31 × extraction direction – 0.31 × int. support + 0.33 × yarder size).

The yarder size is a dummy variable. For a larger yarder with the mainline pull higher than 35 kN it has a value of 1 and for a smaller yarder it is 0.

Road density was evaluated from this formula:

\[
\text{Road density (m.ha}^{-1}\text{)} = K / \text{Yarding distance (km)}
\]

where:

\[ K \] – the road efficiency factor which normally varies between 5 and 9 for skidding and forwarding operation (FAO 1974).

Based on the site study condition in cable yarding operation \( K \) value of 3.8 was used in the calculation. The existing road density and average yarding distance are 33.27 m.ha\(^{-1}\) and 114.1 m, respectively, in cable yarding operation sites of this study. Substituting these values in the above formula, \( K = 3.8 \). Road spacing and road density are related by the formula:

\[
\text{Equivalent Road spacing (m)} = 10,000 / \text{Road density (m.ha}^{-1}\text{)}
\]

Road density and road spacing were computed for different yarding distances in the range of observation. The yarding cost per m\(^3\) was obtained through the model considering the average load volume. Road construction costs per m\(^3\) were computed using road density, harvesting volume of 167 m\(^3\).ha\(^{-1}\), and average road construction cost of 35 €.m\(^{-1}\).

Installation costs were calculated using the time prediction model developed by Stampfer et al. (2006), hourly cost and average harvested volume per corridor. The harvesting volume per corridor ranged from 82 to 170 m\(^3\), with an average of 121.2 m\(^3\) for the sites studied by Limbeck-Lilienau (2002).

**Network Analysis application**

For the Network Analysis method, feasible road locations were planned in the sample logging area. The length of potential roads was 16,305 m or a road density of 83.2 m.ha\(^{-1}\) (Fig. 1). A maximum longitudinal gradient of 12% was used to plan the roads. The slope was classified to four categories of 0% to 35%, 35% to 60%, 60% to 90% and 90% to 132%. This classification helps estimate the road construction cost depending on the terrain slope. Skyline corridors

![Fig. 1. Slope classification and planned roads in the sample area](image1)

![Fig. 2. Grouped corridors (segments) in the sample area](image2)
were planned for the whole area based on the topography and road locations. The spacing of corridors was assumed as 30 m. Since uphill yarding is more productive than downhill yarding, uphill yarding was used where possible.

The area covered by each corridor was measured and, using the average harvest volume of 167 m$^3$.ha$^{-1}$, the harvest volume per corridor was calculated.

The average harvest volume and corridor length were 45.3 m$^3$ and 98.7 m, respectively. Yarding and installation costs per m$^3$ were determined using the time prediction models and hourly cost of the system for each corridor. The corridors which had similar characteristics such as yarding direction, landing or length of the cable way were grouped (Fig. 2). The sum of yarding and installation cost per m$^3$ was calculated for each group. An entry node into the road network was designed for each group of corridors.

Where different road options were available for a given area, alternative yarding costs were estimated and entered into the logging/transportation link file to model the alternatives.

Between each node, the length of road was measured on the map classified by the slope. It was assumed that the road construction cost increased as a function of ground slope (Table 2).

The road construction cost was computed based on the ground slope.

The planned network consisted of 121 nodes. Four mills were considered at the end of roads to the boundary of the area. Since the logs could go to any of the four mills, a “super mill” or final destination was the node created as a terminal destination for the four mills. The nodes were linked together using yarding cost and road construction cost in the possible cases. The neighbour nodes were connected with the road segments. Some nodes were connected using a longer yarding distance if the terrain allowed this connection.

NETWORK 2000 was used to identify which road segments should be built and which transport routes should be taken. NETWORK 2000 was developed to optimize large fixed and variable cost transportation problems and it provides three different heuristic algorithms; one based on the shortest path algorithm, simulated annealing and great deluge. The first algorithm solves the network problem using a heuristic method that prorates the fixed costs in an iterative mode. The algorithm can solve a large fixed and variable cost transportation problem quickly. The best solution found by this algorithm cannot be said to be optimal, however it is usually a high quality solution. It is possible to search for a better solution with additional iterations with this program option or to use other program options to search for higher quality solutions.

To solve the shortest path subproblem within the shortest path heuristic, a variant of the Dijkstra (1959) algorithm is used. The basic premise of this algorithm is to find the length of the shortest path between the starting vertex and first vertex; then the length of the shortest path between the starting vertex and second vertex; continuing until the length of the shortest path between the starting vertex and ending vertex is found.

Simulated annealing (SA) is a search technique which exploits an analogy with the way in which a metal cools to a minimum energy crystalline structure (the annealing process). It forms the basis of an optimization technique for combinatorial and other problems. The algorithm employs a neighbourhood random search which not only accepts changes that decrease the objective function (assuming a minimization problem), but also some changes that increase it as a way for avoiding being trapped in local minima.

The great deluge algorithm (GDA) (Dueck 1993) is a variant of the random neighbourhood search method, with slightly different acceptance rules (Bettinger et al. 2002). It uses one parameter rather than two, as in the simulated annealing algorithm, which reportedly desensitizes the algorithm leading to good results even when the parameter estimation and formulation is poor.

The objective of the network problem is to minimize yarding and road costs. The network model for the 196 ha problem is expressed mathematically as:

\[
\text{Minimize} \quad z = \sum_{i=0}^{i=120} y_i \cdot \sum_{i=0}^{i=120} x_i + \sum_{i=0}^{i=120} f_i \cdot y_i
\]

Subject to:

Conservation of flow at each node

\[
\sum_{i} x_{in} - \sum_{i} x_{out} = 0
\]

Road triggers:

\[
M \cdot y_i \geq x_i
\]

Table 2. Assumed road construction cost as a function of ground slope

<table>
<thead>
<tr>
<th>Ground slope (%)</th>
<th>Road construction cost (€.m$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–35</td>
<td>14</td>
</tr>
<tr>
<td>35–60</td>
<td>28</td>
</tr>
<tr>
<td>60–90</td>
<td>50</td>
</tr>
<tr>
<td>90–132</td>
<td>100</td>
</tr>
</tbody>
</table>
Allowable values of decision variables:
\[ x_i \geq 0 \]
\[ y_i = \{0, 1\} \]
where:
\[ v_i \] – variable cost (€.m$^{-3}$) of link $i$,
\[ f_i \] – fixed cost (€) of each link $i$,
\[ x_i \] – traffic on each link (m$^3$),
\[ y_i \] – road construction variable (binary),
\[ M \] – value greater than the total volume on the network.

**Mixed integer programming**

The same data for installation, yarding and road construction for the Network Analysis model for the sample cable yarding area were used to minimize the total cost using mixed integer programming. A database including the variable, fixed cost and logging volume per each node was designed to be read by optimization software. The program was written to solve the Network Analysis model under the constraints to solve for the road segments that should be built in the case study area.

**RESULTS**

The production rate based on productive machine hours was 10.4 m$^3$/PSHo. The cost of yarding is 19.71 €.m$^{-3}$.

**Yarding time predicting model**

Time (min/cycle) = 0.005 × Yarding distance (m) + 0.054 × Lateral yarding distance (m) + 1.019 × Load volume (m$^3$) + 0.023 × Harvest intensity (%) + 0.002 × Stand density (N.ha$^{-1}$) + 0.028 × Slope (%) + 0.376 × Extraction direction

\[ R^2 = 0.894, \text{ adjusted } R^2 = 0.893 \text{ and number of observations } = 752. \]

From the analysis, 89.3% of the variation can be explained by the model. The table of analysis of variance (Table 3) shows that the model makes sense at a probability level of 5%. The values for uphill and downhill yarding are 0 and 1, respectively, as a dummy variable.

Table 4 presents the summary statistics of the parameters in this case study.

**Optimal road spacing using minimization of road construction and yarding cost method**

For different forwarding distances in the range of observation, optimal road density and road spacing were computed assuming that any desired road spacing is feasible. The yarding cost per cubic meter was obtained by means of the model and using the average load volume. Road construction costs per cubic meter were computed using road density, harvesting volume of 167 m$^3$.ha$^{-1}$, and average road construction cost of 35 €.m$^{-1}$.

Installation costs were computed using the time prediction model developed by Stampfer et al. (2006), hourly cost and average harvested volume per corridor.

The total cost was graphed for different road spacing (Fig. 3). The minimum total cost was 39.15 €.m$^{-3}$ and optimal road spacing would be 474 m. The optimal road density and yarding distance are 21.1 m.ha$^{-1}$ and 90 m, respectively.

If one-way yarding is applied, the ORS, optimum road density and optimum yarding distance would be 329 m, 30.4 m.ha$^{-1}$ and 125 m, respectively. The minimum total cost is 42.9 €.m$^{-3}$.

**Table 3. ANOVA of the model**

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F value</th>
<th>Significance level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>6</td>
<td>22,411.57</td>
<td>3,201.65</td>
<td>895.35</td>
<td>0.000</td>
</tr>
<tr>
<td>Residual</td>
<td>746</td>
<td>2,664.01</td>
<td>3.57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>752</td>
<td>25,075.58</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4. Summary statistics of the parameters in time study**

<table>
<thead>
<tr>
<th>Variable</th>
<th>PSHo (min)</th>
<th>Yarding distance (m)</th>
<th>LYD (m)</th>
<th>Load volume (m$^3$)</th>
<th>Tree volume (m$^3$)</th>
<th>Harvest intensity (%)</th>
<th>Stand density (N.ha$^{-1}$)</th>
<th>Slope (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>13.89</td>
<td>300.0</td>
<td>22.00</td>
<td>3.457</td>
<td>3.457</td>
<td>95.26</td>
<td>1,045.11</td>
<td>67.00</td>
</tr>
<tr>
<td>Mean</td>
<td>5.44</td>
<td>114.1</td>
<td>6.35</td>
<td>0.877</td>
<td>0.679</td>
<td>34.88</td>
<td>969.47</td>
<td>45.72</td>
</tr>
<tr>
<td>Minimum</td>
<td>1.55</td>
<td>0</td>
<td>0</td>
<td>0.076</td>
<td>0.076</td>
<td>1</td>
<td>369.68</td>
<td>6.00</td>
</tr>
</tbody>
</table>

PSHo – Productive system hours; LYD – lateral yarding distance
Effect of road construction cost and logging volume on ORS

Based on the information from Styr and Gmünden, the road construction cost ranges from 14 to 100 €.m\(^{-1}\) depending on the ground conditions. In Fig. 4, the road spacing for different average road construction costs is presented with the logging volume held constant at the average of 167 m\(^3\).ha\(^{-1}\). The increasing road construction cost will increase total cost.

This equation can be used to predict the ORS for two-way yarding for different average road construction costs.

\[
\text{ORS (m)} = 95.525 \times [\text{Road construction cost (€.m}^{-1})]^{0.4514}
\]

If the average road construction cost is held at 35 €.m\(^{-1}\) and logging volume is changed from 50 to 250 m\(^3\).ha\(^{-1}\), the ORS for two-way yarding will decrease and can be predicted by the following model.

\[
\text{ORS (m)} = 4194.7 \times [\text{Logging volume (m}^3\text{.ha}^{-1})]^{-0.4812}
\]

The increasing logging volume will decrease total unit cost and decrease ORS (Fig. 5).

Optimal road density using Network Analysis

The shortest path algorithm was run. The best solution was found at the total of 78.87 €.m\(^{-3}\) with a road cost of 22.05 €.m\(^{-3}\) and logging cost of 56.81 €.m\(^{-3}\). The best solution road locations found
by the network analysis approach are shown in Fig. 6. The road segments between nodes 13, 14, 15, 19, 54 and 76 are eliminated by the network approach (Fig. 6). Therefore the proposed road density would be about 75.5 m.ha⁻¹. It was assumed that the logging for all segments is done at the same time. Simulated annealing and great deluge algorithms were run based on the data base but could not find a better solution than the shortest path heuristic.

Solution of mixed integer programming

The program written in Xpress was run for the database of the sample logging area. The function (total cost) was minimized at 2,595,530 €. The total volume extracted is 32,910.2 m³. This yielded a combined unit cost of harvesting and road construction of 78.87 €.m⁻³. The roads to be built in the solution are shown in Fig. 7. The road segments of nodes 19, 47, 49, and 76 are proposed.
to be eliminated by the mixed integer programming solution.

The mixed integer programming solution was compared to NETWORK 2000 solution. Comparing the solutions, the harvest (flow) volume was multiplied by variable cost for each link and the fixed cost was added. The flow volumes were similar for both methods. But on links where the flow volumes were different, the variable cost was zero and for the links which had the variable cost more than zero, the flow volume was the same. Therefore the total cost was similar for the mixed integer programming and Network Analysis methods.

**DISCUSSION**

The results of minimization of total costs showed that ORS for the study area would be 474 m with road density of 21.1 m.ha$^{-1}$ for two-way yarding. If one-way yarding were applied, the total cost would be higher than in two-way yarding and ORS would be only 329 m.

The increasing road construction cost in minimization of total cost resulted in higher ORS and if the harvesting volume per ha increases, the ORS will decrease as shown in Figs. 4 and 5.

Using actual feasible road locations and the Network Analysis approach in a sample logging area, the best solution was found at 73.9 m.ha$^{-1}$. This compared to an optimum road density through minimization of total cost of 30.4 m.ha$^{-1}$ for one-way yarding. The large difference can be caused by different logging volume per corridor and average road construction cost per meter. The sub-optimal solution was found in the Network Analysis method results because many road locations that would result in optimal cost are not feasible road construction sites because of the terrain. The average logging volume per corridor for cost minimization and network procedure were 121.2 m$^3$ and 45.3 m$^3$, respectively.

These results in an increase in installation cost in the network and mixed integer programming solutions compared to the optimization solution. Also, the average road construction cost per meter for the cost minimization method assumed to be 35 €.m$^{-1}$ but in the Network Analysis solution the average cost of road construction was higher.

If Network Analysis or mixed integer programming is used to optimize the road density, the planner can input different road construction costs based on ground conditions (Table 2). Planners can also identify different feasible road construction locations and choose the best using this approach while in minimization of road construction and yarding cost this is not possible. For each segment if different harvesting system or machines are used, it is possible to input different harvesting costs to find ORS but in the method of minimization of total cost used in this paper only one harvesting (yarding, skidding or forwarding) unit cost was used.

In this study, the same cost of yarding and installation, road segment costs and harvest volume per node were used for network and mixed integer programming procedures. It should be noted that the solution found by mixed integer programming is a real optimum while the solution of running NETWORK 2000 is a local optimum. The Network Analysis method with fixed costs does not guarantee the optimal solution whereas the mixed integer linear programming method does guarantee an optimal solution. Nevertheless, for the sample problem the network procedure solution yielded the same total cost as the mixed integer programming procedure solution, though the two procedures proposed to eliminate different road segments. For this case study the same harvesting time for all nodes was assumed. If the planners have different harvesting periods in their planning area, then the Network Analysis would be an appropriate method.

**CONCLUSIONS**

Optimization of road spacing for cable yarding operations can help planners minimize the cost of logging. The ORS prediction models can be used as a guide but in hilly and mountainous terrain harvested by cable yarding systems, feasible road construction locations are limited and the ORS models generally underestimate the total road construction and yarding costs due to the model assumptions of road placement opportunities in the landscape, equal road cost anywhere in the landscape, and lack of consideration of maximum road gradient. We conclude that the cost optimization approach, while providing some guidance for road spacing, would not be appropriate for road planning in the mountainous terrains in our planning area. Because of the importance of ground slope in road construction costs, additional research is needed to refine this relationship in steep terrain. The next studies can also use the soil stability maps to plan the possible road variants more carefully.

**References**


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