

Generalized additive models as an alternative approach to the modelling of the tree height-diameter relationship

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ABSTRACT: Generalized additive models were tested using three types of smoothing functions as an alternative for modelling the height curve. The models were produced for 23 forest stands of Norway spruce (*Picea abies* [L.] Karst.) in the territory of the Training Forest Enterprise Masaryk Forest Křtiny. The results show that the best evaluated and recommended for practical use at the level of forest stand was the LOESS function (locally weighted scatterplot smoothing) when using a greater width of the bandwidth. Due to the frequent overfitting and the associated unrealistic behaviour of the function, smoothing by spline functions cannot be recommended for modelling the height curve at the level of forest stand. It was validated that the resulting model must be assessed not only according to the calculated quality criteria, but also depending on the graphic pattern of the model which must ensure that the height curve pattern is realistic. The quality of the resulting models (with LOESS function) was assessed to be high, mainly due to the very precise determination of model heights.

Keywords: LOESS; nonparametric method; Norway spruce; Petterson function; smoothing function; spline

A large number of variables are measured as part of the forest inventory. Of these, tree height and diameter at breast height are considered to be essential. Unlike measuring a diameter at breast height, which is easy and inexpensive, measuring height consumes considerable time and money (ADAMEC et al. 2008). Although the cost of the latter can be reduced due to the use of distance-measuring ultrasound and laser technology, height measurement is still a very labour-intensive method (VARGAS-LARRETA et al. 2009). Therefore fitted heights are often used instead of measured heights. Fitted heights are derived from the model of the height-diameter relationship which is compiled from the measured heights and diameters (HUANG et al. 1992; MARTIN, FLEWELLING 1998). A height curve is graphical expression of the height-diameter relationship. According to SCHMIDT et al. (2011), attempts to reduce costs of height measurement made the height-diameter relationship model one of the most important results of the evaluation of forest inventory data.

The height curve model is one of the regression models and can be written down using a math-

ematical relationship referred to as the height function and designed especially for this purpose. There is currently a large number of such functions; a comprehensive listing can be found e.g. in the papers of PENG (1999), HUANG et al. (2000), HUSCH et al. (2003), ŠMELKO (2007) or VAN LAAR and AKÇA (2007). MEHTÄTALO (2004) compared results of different papers regarding the comparison of height-diameter models (e. g. CURTIS 1967; ARABATZIS, BURKHART 1992; HUANG et al. 1992; FANG, BAILEY 1998). He concluded that none of these functions can generally be described as the best. A height curve can be modelled in other ways. Among the most common are adjusted models of growth functions (e.g. ZHANG 1997; PENG et al. 2001) or mixed-effects models (e.g. TRINCADO et al. 2007; CASTAÑO-SANTAMARÍA et al. 2013; LU, ZHANG 2013). The height-diameter relationship is influenced by numerous factors, particularly by site quality and silvicultural treatments. In many instances, such relationships feature a complex non-linear process which is difficult to describe using ordinary parametric models (NANOS et al. 2004). Additionally, a failure to meet the requirements

placed on the least squares method (e.g. normality and independence of residuals etc.) may often be a problem. Possible solutions to the problems outlined above include the use of nonparametric methods that are robust towards the conditions required by the least squares method. Of these, a generalized additive model (GAM) can be considered as an alternative for modelling the height-diameter relationship, a technique that employs different smooth functions to produce the model. The sum of smooth functions of covariates is included in the predictor of a GAM. Dependence of the response variable on the covariates in the GAM is specified only in terms of smooth functions rather than detailed parametric relationships (WOOD 2006). Writing down the equation with parameters is thus not possible. However, it is data that determines the resulting shape of the model. The final GAM therefore has a degree of flexibility that cannot be achieved in linear or nonlinear modelling (ROBINSON et al. 2011). The same authors referred to GAMs as being suitable for modelling in forestry at the very moment when the task is the best possible fitting rather than making the best estimate of model parameters. WANG et al. (2005) reported that GAMs are more resistant to extreme values than other linear or nonparametric methods. The same conclusions were reached by BYUN et al. (2013) in their paper. MOISEN et al. (2006) pointed to the fact that in addition to fitting quality, GAMs are easy to interpret. In forestry, this modelling technique can be encountered in a range of applications over the past few years, e.g. in the papers of FALK and MELLERT (2011) and KOUBA and ALADOS (2012), who dealt with the modelling of spatial dynamics of tree species, WANG et al. (2005), ALBERT and SCHMIDT (2010) and AERTSEN et al. (2012), who discussed the site-specific production capacity, ROBINSON et al. (2011) and KUŽELKA and MARUŠÁK (2014), who modelled the stem form or BYUN et al. (2013), who modelled the diameter growth of trees or ZHANG et al. (2008) and SCHMIDT et al. (2011), who modelled exactly height curves of various tree species, but used only different spline functions without using the LOESS function. Based on previous statements, three key advantages suggest to prefer a GAM instead of a classic height curve: (i) higher flexibility of the model, (ii) robustness against outliers, (iii) no usage limitations as in the case of OLS regression (normality or independence of residuals).

All three advantages overlap, which implies that GAM-based height curve could be utilized in forest stands where OLS-based methods do not perform satisfactorily or where preconditions for OLS ap-

plication are not met. A typical example is a forest stand with higher diameter or height variability where a few suppressed or, on the contrary, dominating trees can affect the height curve position and thus the resulting model.

The aim of this study is to assess the possibility of practical use of various types of generalized additive models as alternatives for modelling the height curve on the level of forest stand with special emphasis being placed on the resulting quality of the fitted model.

MATERIAL AND METHODS

Data were collected in the territory of Masaryk Forest Křtiny Training Forest Enterprise, the special-purpose premises of Mendel University in Brno. It involved 23 forest stands of Norway spruce (*Picea abies* [L.] Karst.), the age range being 30 to 136 years. Two sample plots were placed in each forest stand. The plot size was determined based on the requirement for a sufficient number of trees per plot and graded per age. The size of the plots was 250 m² ($r = 8.92$ m) in stands under 40 years, 700 m² in stands aged 41–80 years ($r = 14.93$ m), and 1,200 m² in stands aged 81 years and more ($r = 19.54$ m). Tree height using a Vertex Laser hypsometer to the nearest 0.1 m and diameter at breast height using a calliper to the nearest 1 cm were measured on the plots. A total of 1,590 trees were measured.

GAMs were compiled for three different smoothing functions: these involved spline, cubic spline and LOESS types. A GAM can be written down using the Equation (1):

$$\hat{h}_i = a + f_s(d_{1.3i}) + \varepsilon_i \quad (1)$$

where:

- \hat{h}_i – fitted height of a tree i ,
- a – intercept of the model,
- f_s – smoothing function,
- $d_{1.3i}$ – diameter at breast height of a tree i ,
- ε_i – residual value.

While assuming normal distribution for the dependent variable (height), the identity link function was chosen for the dependent to independent variable relationship. For smoothing functions, the task was to fit the data with a curve so that the curve most closely copies the behaviour of the data, thus minimising the degree of deviance. It is thus referred to as a data driven technique. In this type of technique, inspecting the actual curve behaviour is fundamental to avoid overfitting which causes the

resulting model becoming adjusted to the measured values too much regardless of the model's practical applicability and sense. Subsequently, such a model cannot be applied to any other data, or at least its use gives very poor results.

Different methods are used for determining the optimal degree of smoothing; this varies based on the type of smoothing function applied. In the event of the LOESS function, it is important to find the optimal width, i.e. bandwidth; indicating the degree of smoothing, it specifies the width of the interval around each of the measured values. This whole interval is fitted with a function using linear-weighted regression and a fitted value is determined for the measured value (ZUUR et al. 2009). A suitable bandwidth is searched for using the Akaike information criterion – AIC (AKAIKE 1973). In spline functions, the procedure is different. The data file is divided into segments separated by nodes. Individual segments are fitted by regression models (the spline function uses a linear model, while cubic spline uses the 3rd degree polynomials). Segments are joined together where they form nodes. For more details about joining the segments refer to the paper of WOOD (2006). The degree of smoothing is specified through the number of nodes. The optimal number of nodes is chosen using a leave-one-out cross-validation.

The models formed were compared with each other as well as with a traditional height curve computed by nonlinear regression using the Peterson function (PETTERSON 1955). This function can be written down using the Equation (2):

$$\hat{h}_i = 1.3 + 1/(a + b/d_{1.3i})^3 \quad (2)$$

where:

- \hat{h}_i – fitted height of a tree i ,
- a – intercept of the model,
- b – regression parameter,
- $d_{1.3i}$ – diameter at breast height of a tree i .

This function was chosen because it is recommended as a very good function in papers such as KRAMER and AKÇA (1995), VAN LAAR and AKÇA (2007), PRETZSCH (2009) or DRÁPELA (2011). The mutual comparison of different models took place in each of the 23 forest stands separately. To compare different models, the following criteria were chosen:

– Amount of explained deviance (DOBSON 2002) (Eq. 3):

$$\text{pseudo } R^2 = 100 \times (D_0 - D_r)/D_0 \quad (3)$$

where:

- D_0 – null deviance;
- D_r – residual deviance.

– Akaike information criterion (Eq. 4):

$$\text{AIC} = n \times \ln((\sum e_i^2)/n) + 2 \times m \quad (4)$$

where:

- e_i – residual value,
- n – sample size,
- m – number of parameters.

– Mean of residuals (Eq. 5):

$$\bar{e}_i = (\sum e_i)/n \quad (5)$$

where:

- n – sample size,
- e_i – residual value.

– Residual standard deviation (Eq. 6):

$$\sigma_{e_i} = \sqrt{(\sum (e_i - \bar{e}_i)^2)/n} \quad (6)$$

where:

- n – sample size,
- e_i – residual value,
- \bar{e}_i – mean of residuals.

– Mean of deviation of fitted values obtained from the additive model and fitted values obtained from the nonlinear regression model (Eq. 7):

$$\Delta_{ig} = \sum_{i=1}^n |\hat{y}_{i_{gam}} - \hat{y}_{i_{nlr}}|/n \quad (8)$$

where:

- $\hat{y}_{i_{gam}}$ – fitted value of a tree i ($i = 1, 2, 3, \dots, n$) from the additive model,
- $\hat{y}_{i_{nlr}}$ – fitted value of a tree i ($i = 1, 2, 3, \dots, n$) from the nonlinear regression model and n is the sample size.

– Visual similarity to the height curve model fitted by nonlinear regression,

– The realism of the model behaviour.

The realism of model behaviour implies that the model meets the general height curve requirements (ŠMELKO 2007). The most important is that the curve must be non-decreasing.

Deviance is calculated in models which use the maximum likelihood method (ZUUR et al. 2009). Zero deviance equals the total sum of squares, while residual deviance corresponds to the residual sum of squares in regression models.

All calculations were performed using the R software (R Development Core Team 2013). The results are presented with 95% confidence, i.e. at the significance level of $\alpha = 0.05$.

RESULTS

All the calculated evaluation criteria are listed in Table 1. The value in brackets for the LOESS function shows the bandwidth. According to the

Table 1. Criteria for generalized additive models

Forest stand	Smoothing function	Pseudo R^2 (%)	AIC	\bar{e}_i	σ_{ei}	Δ_{ig} (m)
1	spline	68.6	194.97	0.0000	1.0887	0.0068
	cubic spline	68.6	194.95	0.0000	1.0885	0.0068
	LOESS (0.70)	67.9	195.97	0.0000	1.1001	0.0063
2	spline	42.6	137.34	0.0000	1.2155	0.0019
	cubic spline	42.6	137.34	0.0000	1.2155	0.0019
	LOESS (0.75)	42.8	139.68	0.0000	1.2140	0.0011
3	spline	85.3	150.48	0.0000	0.9454	0.0017
	cubic spline	85.3	150.51	0.0000	0.9460	0.0017
	LOESS (0.65)	84.9	151.55	0.0000	0.9582	0.0023
4	spline	74.8	160.57	0.0000	0.9906	0.0044
	cubic spline	81.3	155.68	0.0000	0.8542	0.0121
	LOESS (0.75)	74.3	160.77	0.0000	1.0010	0.0040
5	spline	86.3	231.72	0.0000	0.9176	0.0063
	cubic spline	86.3	231.74	0.0000	0.9180	0.0063
	LOESS (0.60)	85.9	232.61	0.0000	0.9319	0.0050
6	spline	76.2	262.44	0.0000	1.3463	0.0151
	cubic spline	76.2	262.44	0.0000	1.3463	0.0151
	LOESS (0.85)	76.4	263.63	0.0000	1.3409	0.0122
7	spline	74.7	368.21	0.0000	1.2990	0.0156
	cubic spline	74.6	368.47	0.0000	1.3012	0.0148
	LOESS (0.80)	70.0	377.74	0.0000	1.4145	0.0054
8	spline	75.4	260.88	0.0000	1.3662	0.0057
	cubic spline	75.4	260.89	0.0000	1.3666	0.0057
	LOESS (0.95)	74.7	261.98	0.0000	1.3893	0.0075
9	spline	62.5	213.43	0.0000	1.4805	0.0049
	cubic spline	62.5	213.43	0.0000	1.4804	0.0049
	LOESS (0.85)	62.5	213.48	0.0000	1.4791	0.0041
10	spline	74.6	363.12	0.0000	1.8864	0.0224
	cubic spline	74.5	363.28	0.0000	1.8903	0.0218
	LOESS (0.85)	72.7	366.51	0.0000	1.9564	0.0124
11	spline	88.5	269.38	0.0000	1.2589	0.0189
	cubic spline	88.6	269.14	0.0000	1.2549	0.0194
	LOESS (0.80)	86.7	275.69	0.0000	1.3545	0.0059
12	spline	71.2	269.25	0.0000	1.4976	0.0085
	cubic spline	71.2	269.16	0.0000	1.4959	0.0087
	LOESS (0.80)	69.2	271.23	0.0000	1.5483	0.0046
13	spline	76.0	182.60	0.0000	1.2100	0.0062
	cubic spline	76.3	182.35	0.0000	1.2029	0.0069
	LOESS (0.85)	72.7	184.29	0.0000	1.2912	0.0034
14	spline	41.5	288.23	0.0000	1.9991	0.0109
	cubic spline	41.5	288.23	0.0000	1.9991	0.0109
	LOESS (0.95)	42.1	288.64	0.0000	1.9891	0.0150
15	spline	82.1	298.50	0.0000	1.5088	0.0076
	cubic spline	82.2	298.40	0.0000	1.5074	0.0078
	LOESS (0.75)	81.7	299.40	0.0000	1.5289	0.0056
16	spline	60.1	299.66	0.0000	2.4324	0.0116
	cubic spline	60.0	299.70	0.0000	2.4348	0.0114
	LOESS (0.80)	58.8	300.29	0.0000	2.4693	0.0091
17	spline	68.5	273.68	0.0000	1.4473	0.2159
	cubic spline	69.3	271.65	0.0000	1.4292	0.2187
	LOESS (0.95)	57.3	282.67	0.0000	1.6851	0.0089

Table 1. to be continued

Forest stand	Smoothing function	Pseudo R^2 (%)	AIC	\bar{e}_i	σ_{ei}	Δ_{ig} (m)
18	spline	58.3	241.33	0.0000	1.8662	0.0033
	cubic spline	58.4	241.27	0.0000	1.8633	0.0037
	LOESS (0.95)	56.5	242.39	0.0000	1.9053	0.0045
19	spline	64.5	339.13	0.0000	1.5617	0.0063
	cubic spline	68.3	338.16	0.0000	1.4746	0.0150
	LOESS (0.95)	63.9	339.61	0.0000	1.5743	0.0081
20	spline	53.6	380.35	0.0000	2.0619	0.0054
	cubic spline	59.0	379.67	0.0000	1.9385	0.0225
	LOESS (0.95)	53.0	380.50	0.0000	2.0741	0.0040
21	spline	58.1	305.99	0.0000	1.6750	0.0103
	cubic spline	58.1	305.96	0.0000	1.6749	0.0103
	LOESS (0.85)	57.6	306.00	0.0000	1.6852	0.0099
22	spline	57.8	272.45	0.0000	2.6356	0.0136
	cubic spline	57.8	272.45	0.0000	2.6356	0.0136
	LOESS (0.95)	58.4	272.79	0.0000	2.6159	0.0187
23	spline	72.0	236.27	0.0000	1.8403	0.0283
	cubic spline	73.0	234.86	0.0000	1.8089	0.0299
	LOESS (1.00)	58.5	246.38	0.0000	2.2415	0.0082

pseudo R^2 (%) – amount of explained deviance, AIC – Akaike information criterion, \bar{e}_i – mean residual, σ_{ei} – residual standard deviation, Δ_{ig} – the mean of deviations of fitted values obtained from the additive model and fitted values obtained from the nonlinear regression model (with LOESS function a bandwidth is in the brackets)

numerical results all functions used are acceptable and not a single type of smoothing function can be referred to as the best. However, in the graphics rendered by the used functions, the differences are already clearly visible. The spline functions have problems with overfitting, which caused in some cases a biologically unacceptable and unfounded shape of the function. For instance, there are several cases of the height decreasing as the diameter increases. This is due to the choice of the method for finding an optimal degree of smoothing when the cross-validation actually seeks for a model that best fits the data field regardless of the practical use of the resulting model. Of the 23 forest stands, there was a problem with the realism of the behaviour in 5 cases for spline and in 8 cases for cubic spline. It is important to note that all the numerical results (shown in Table 1) in these forest stands showed no problems; indeed, they indicated a model of very good quality.

The LOESS function very well modelled the height-diameter relationship in all stands with a very similar behaviour of the function as a model of the Petterson height curve. It is due to this fact that the resulting model was chosen as a compromise between the degree of smoothing and the realism of its behaviour. Therefore the resulting model generated through the LOESS function may not have the best values of numerical criteria com-

pared with spline functions, but will always have a biological justification. The values of the resulting criteria were never so different from those obtained through spline functions to permit speaking of a model of lesser quality. Actually, the resulting mean values of deviations of the fitted values obtained through the additive model (using the LOESS function) from the fitted values of nonlinear model (last column of Table 1) make it possible to refer to the resulting models as models with a very good capability of fitting.

When the LOESS function was used, a model with the bandwidth exceeding 0.60 was selected to be the final model in all of the forest stands. It can therefore be concluded that rather a larger bandwidth should be chosen when using the LOESS function in GAMs at a local level to prevent the overfitting of the model.

Graphical outputs of several forest stands are shown in Fig. 1. These three forest stands were selected to represent the whole set under study: the range of ages and other dendrometric variables such as heights and diameters. The ages of stands No. 3, 13 and 23 were 35, 88 and 136 years, respectively. Fig. 1a presents the behaviour of GAMs and that of the nonlinear model of the Petterson function in forest stand No. 13; it is clearly shown that the models fitted using splines and cubic splines are practically unacceptable (the tree height decreasing as the diameter increases), while the behaviour

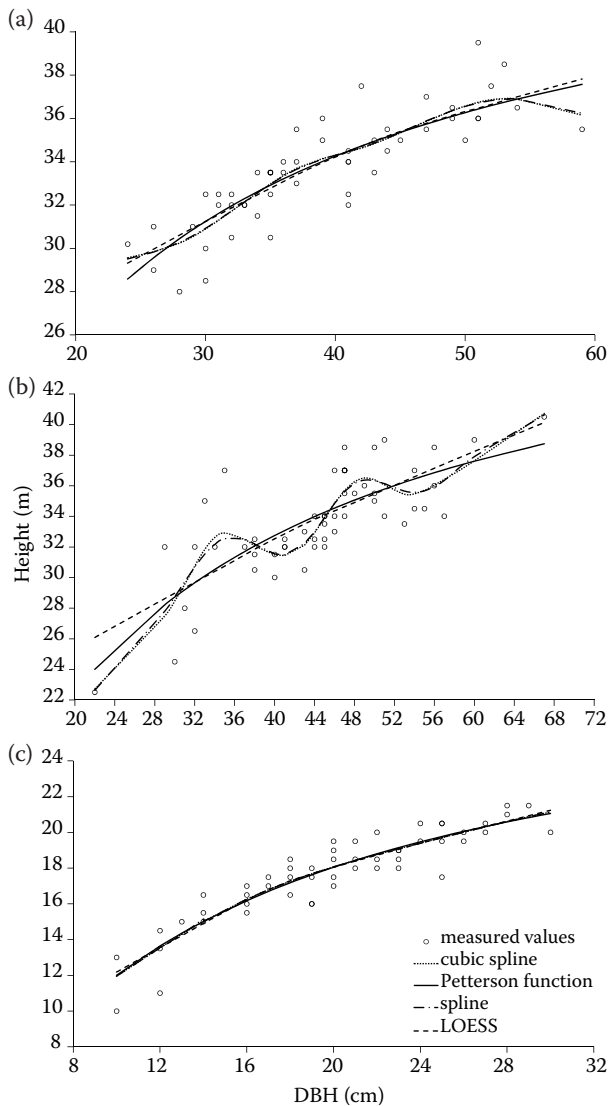


Fig. 1. Fitted GAMs and non-linear regression model in forest stands No. 13 (a), No. 23 (b), and No. 3 (c)

the LOESS function shows is very similar to that of the Petterson model and the function has a realistic behaviour.

Fig. 1b also illustrates very well the overfitting of the model fitted through the spline functions in forest stand No. 23. The lack of realism of the model seen in this case is based on the fitted values oscillating around the nonlinear model of the Petterson function. The model fitted through the LOESS function has a good and applicable behaviour again.

Fig. 1c shows the behaviour of GAMs and the non-linear model of the Petterson function in forest stand No. 3. With the behaviour of all the evaluated models being in fact consistent, the individual models overlap in the chart. All the models could be used to shape the height curve in the forest stand with no differences in the quality of fitting.

The generalized additive model using the LOESS function can be labelled a fully applicable and suit-

able alternative for modelling height curves using nonparametric methods. If this type is applied at the local level, i.e. a height curve model in the forest stand, a larger bandwidth should be used to prevent overfitting and ensure the realistic behaviour of the modelled curve.

DISCUSSION

In the case of very complicated relationships between the dependent and independent variables it is often very difficult to find a suitable mathematical function for fitting the relationship (ROBINSON et al. 2011). Finding the right estimates of the parameters of such a model is another common problem. For modelling a height curve using GAMs, three different smoothing functions were tested – spline, cubic spline and LOESS. They are recommended for use in forestry models e.g. by MOISEN and FRESCINO (2002), WANG et al. (2005), ZHANG et al. (2008), FALK and MELLERT (2011) and YUE et al. (2012).

For spline functions, cross-validation was chosen to be the method for selecting the optimum degree of smoothing. Even this method, however, failed in several cases to choose a model without overfitting. FALK and MELLERT (2011) denoted this method to be very good while adding that monitoring the ecological understandability of the resulting model was still necessary.

The fact that, in all stands, the LOESS function performed better than the spline function can be attributed to the worse performance of spline function when there was a higher data variability. This is apparent in Fig. 1. The previous statement implies that the use of GAM with LOESS function can be advantageous in stands with higher data variability.

According to the results obtained, GAMs can be considered a suitable alternative for modelling the height curve. The same conclusions were reported by ZHANG et al. (2008) and SCHMIDT et al. (2011). ZHANG et al. (2008) recommended this method as suitable for modelling the height curve after testing the GAM model using the spline function on several tree species in Ontario, Canada. He also confirmed the hypothesis that the mean residuals of GAMs are lower than those of models calculated by the least squares method. SCHMIDT et al. (2011) examined the compilation of a height curve in Scots pine (*Pinus sylvestris* L.) using the GAM model for the territory of Estonia with a penalised cubic spline used as the smoothing function. This function does not use different numbers of nodes;

instead, a large number of nodes are chosen and the degree of smoothing is determined by means of the parameter λ , the optimum value which is searched for applying cross-validation. Furthermore, this GAM model included also a stand variable; more specifically mean diameter at breast height of the stand, so that the author was able to produce a system of uniform height curves.

From a practical aspect, a height curve produced using GAM where the emphasis is placed on the most accurate determination of heights can be used. This means that it can find its application not only when identifying the standing volume using volume tables, but particularly also in modelling heights on research plots where the heights can be used for calculating the increment in height and subsequently in volume as part of repeated forest inventory operations. Modelling missing heights in stands that serve as a basis for growth simulation is another alternative. Using GAMs instead of ordinary parametric models thus eliminates the problem of initial estimates of function parameters.

One of the results of our research was the statement that GAMs provided a very accurate fitting. ZHANG and GOVE (2005) argued that in GAMs, the good quality of fitting is due to their robustness and flexibility. Although they modelled the crown area – diameter at breast height relationship, their results can be regarded as a general statement for this method. Therefore it is possible to compare our results with those of ZHANG and GOVE (2005).

It is stated in various papers that GAMs are capable of describing strongly nonlinear relationships occurring in an ecosystem (BYUN et al. 2013) and also the behaviour of biological processes (FRESCINO et al. 2001; AUSTIN 2002; LEHMANN et al. 2003). This was only partially confirmed in our study because only the LOESS-based model was ecologically plausible and capable of modelling the nonlinear relationship of local scale height curve.

CONCLUSIONS

Based on the results obtained, GAMs can be considered as a suitable alternative for modelling a height curve. Differences were shown between smoothing functions when using them at the stand level. The LOESS type with a greater bandwidth can be described as the best as they prevented the overfitting of the resulting model. Overfitting was a problem for both the spline functions. The insufficient performance of spline functions could be caused by higher data variability

in forest stands under study. When selecting the resulting model, the decision cannot be driven solely by the calculated values of benchmarks; the graphical pattern of the model needs to be monitored with respect to the behaviour realism. The resulting model should be a trade-off between the interleaving smoothness and the realism of the model behaviour. Since the deviation of GAMs from nonlinear models is very small, very good results were obtained for fitted heights. GAMs can therefore be used not only for modelling the height curve in determining the standing volume using volume tables, but also for modelling heights in growth simulators or modelling missing heights on research plots.

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