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Economic aspects of the LTPD single sampling inspection plans

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Abstract: The paper deals with the LTPD (Lot Tolerance Percent Defective) single sampling plans when the remainder of rejected lots is inspected. There are considered two types of the LTPD plans – for inspection by variables and for inspection by variables and attributes (all items from the sample are inspected by variables; the remainder of rejected lots is inspected by attributes). These plans were created by the author of this paper and published in the Statistical Papers. These new plans were compared with the corresponding Dodge-Romig LTPD plans for the inspection by attributes from the economic point of view. From the results of the numerical investigations, it follows that under the same protection of consumer the LTPD plans for the inspection by variables are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans (saving of the inspection cost is 80% in any cases). The dependence of the saving of the inspection cost on the acceptance sampling characteristics is analyzed in the paper.

Key words: acceptance sampling, economic efficiency, inspection by variables

Acceptance sampling is one of the techniques used in the quality control, either in the vendor-buyer relationships or for the management of within-company processes. The aim is to meet the desired levels of the protection against risk while keeping an eye on the economic characteristics of the process. Inference is made based on the inspection of a sample of items taken from a lot. Depending on the quality of the sample, the whole lot may be either accepted or rejected, or the inspection of another sample may follow in the case of double, multiple or sequential sampling plans (Klůfa 1980). The acceptance sampling plans, specified by the sample size and the critical value (or the acceptance number), determine the rules for this decision process.

There are many ways of classifying the acceptance sampling. One such classification is according to whether an item is inspected by its attributes, i.e. just classified as either good or defective (Hald 1981) or by variables. Sampling plans for the inspection by variables in many cases allow obtaining same level of protection as the corresponding sampling plans for the inspection by attributes while using a lower sample size. The basic notions of variables sampling plans are addressed in Jennett and Welch (1939).

Under the assumption that each inspected item is classified as either good or defective (acceptance sampling by attributes), Dodge and Romig (1998) consider sampling plans which minimize the mean number of items inspected per lot of the process average quality

$$I_s = N - (N - n)\cdot L(\bar{p}; n; c) \tag{1}$$

under the condition

$$L(p_t; n; c) = \beta \tag{2}$$

where $L(p, n, c)$ is the operating characteristic (the probability of accepting a submitted lot with the proportion defective p when using plan (n, c) for the acceptance sampling), N is the number of items in the lot (the given parameter), $\bar{p}$ the process average proportion defective (the given parameter), p_t is the lot tolerance proportion defective (the given parameter, $P_t = 100\ p_t$ is the lot tolerance per cent defective, denoted LTPD), n is the number of items in the sample ($n < N$), c is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than c).

The condition (2) provides a guarantee for the consumer that the lots of an unsatisfactory quality level, with the proportion defective p_t are going to be accepted only with the specified probability β (consumer's risk). Value β = 0.1 is used for the consumer's risk in Dodge and Romig (1998).

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The LTPD plans for inspection by variables and attributes (all items from the sample are inspected by variables, the remainder of the rejected lots is inspected by attributes) have been introduced in Klufa (1994), using the approximate calculation of the plans. The exact plans, using the non-central *t* distribution in calculation of the operating characteristic Johnson and Welch (1940), have been reported in Klufa (2010) and Kaspříková (2011) – the LTPDvar is an add-on package to the R software (R Development Core Team 2008). Similar problems are solved in Klufa (1997, 2008), Chen and Chou (2001), Kaspříková and Klufa (2011), Wilrich (2012), Aslam et al. (2015).

The present paper shows the economic characteristics of the exact LTPD plans for inspection by variables (a special case of acceptance sampling by variables and attributes) and for inspection by variables and attributes and shows the impact of the input parameters values on the resulting sampling plan and its economic efficiency. A measure for the assessment of economic efficiency of these plans is proposed.

MATERIAL AND METHODS

The LTPD plans for inspection by variables and attributes have been introduced in Klufa (1994) under the following assumptions: measurements of a single quality characteristic X are independent, identically distributed normal random variables with unknown parameters μ and σ^2. For the quality characteristic X, there is given either an upper specification limit U (the item is defective if its measurement exceeds U), or a lower specification limit L (the item is defective if its measurement is smaller than L). It is further assumed that the unknown parameter σ is estimated from the sample standard deviation s.

The inspection procedure is as follows:

Draw a random sample of n items and compute the sample mean $\bar{x}$ and the sample standard deviation s. Accept the lot if

$$\frac{U-\bar{x}}{s} \geq k \quad \text{or} \quad \frac{\bar{x}-L}{s} \geq k \tag{3}$$

Suppose that c_s^* is the cost of inspection of one item by attributes and c_m^* is the cost of inspection of one item by variables and that the sample is inspected by variables. Then the inspection cost per lot with the proportion defective p, assuming that the remainder of the rejected lots is inspected by attributes (the inspection by variables and attributes), is $n \cdot c_m^*$ with probability $L(p,n,k)$ and $\left[n \cdot c_m^* + (N-n) \cdot c_s^*\right]$ with probability $\left[1 - L(\bar{p}; n,k)\right]$. The mean inspection cost per lot of the process average quality $\bar{p}$ is therefore

$$C_{ms} = n \cdot c_m^* + (N-n) \cdot c_s^* \cdot \left[1 - L(\bar{p}; n,k)\right] \tag{4}$$

Dividing (4) by c_s^* gives the objective function

$$I_{ms} = n \cdot c_m + (N-n) \cdot \left[1 - L(\bar{p}; n,k)\right] \tag{5}$$

where

$$c_m = c_m^* / c_s^* \tag{6}$$

is the ratio of the cost of the inspection of one item by variables to the cost of the inspection of this item by attributes (this new parameter has to be estimated in each real situation, usually is $c_m > 1$). Note that both the function $I_{ms} = C_{ms} / c_s^*$ and function C_{ms} have a minimum for the same acceptance plan (n, k). Therefore, we shall look for the acceptance plan (n, k) minimizing (5) instead of (4) under the condition

$$L(p_t; n; k) = \beta \tag{7}$$

Setting the value of c_m to 1 can be used in the situations, when both the sample and the remainder of the rejected lots are inspected by variables. Acceptance sampling by variables can thus be considered just as a special case of acceptance sampling by variables and attributes. Then instead of I_{ms}, we may use the notation I_m and setting $c_m = 1$ in (5) we obtain

$$I_m = N - (N-n) \cdot L(\bar{p}; n,k) \tag{8}$$

i.e. the mean number of items inspected per lot of the process average quality, assuming that both the sample and the remainder of the rejected lots are inspected by variables.

Summary: The task to be solved is to determine the plan (n, k) minimizing (5) under the condition (7) for the given values of input parameters N, c_m, p_t and $\bar{p}$.

The solution of this problem is in the paper Klufa (1994), the numerical solution is in Klufa (2010) and Kaspříková (2011).

RESULTS AND DISCUSSION

Now we shall study the economic efficiency of the LTPD plans for the inspection by variables and attributes. For the comparison of the LTPD single sam-

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pling plans for inspection by variables and attributes with the corresponding Dodge-Romig LTPD plans for inspection by attributes from economic point of view we use the parameter e defined by the relation

$$e = \frac{I_{ms}}{I_s} \cdot 100 \qquad (9)$$

According to (4), there is

$$e = \frac{I_{ms}}{I_s} \cdot 100 = \frac{I_{ms}.c_s^*}{I_s.c_s^*} \cdot 100 = \frac{C_{ms}}{C_s} \cdot 100$$

where $C_s = I_s\, c_s^*$ is the mean cost of the inspection by attributes (c_s^* is the cost of the inspection of one item by attributes). Therefore, the LTPD plan for the inspection by variables and attributes is more economically efficient than the corresponding Dodge-Romig plan when

$$e < 100$$

Expression (100 – e) then represents the percentage of savings in the inspection cost when the sampling plan for inspection by variables and attributes is used instead of the corresponding plan for the inspection by attributes.

Economic efficiency measured by the parameter e (see formula (9)) is a function of four variables, p_t, N, $\bar{p}$ and c_m, i.e.

$$e = e(p_t, N, \bar{p}, c_m) \qquad (10)$$

Some values of this function are in Table 1.

From the results of numerical investigations it follows that under the same protection of consumer the LTPD plans for inspection by variables are in many situations more economical (saving of the inspection cost is 80% in any cases) than the corresponding Dodge-Romig attribute sampling plans – see also Table 1.

For example when p_t = 0.005, N = 4000, $\bar{p}$ = 0.0005 and c_m = 2 is parameter e = 26 (see Table 1), which means that using the LTPD plan for inspection by variables and attributes it can be expected approximately (100 – e)=74% saving of the inspection cost in comparison with the corresponding Dodge-Romig plan.

Now we shall study dependence of the economic efficiency measured by parameter e on the lot size N. Let p_t, $\bar{p}$, c_m be given parameters. Function (10) for given p_t, $\bar{p}$, c_m is a function of one variable N, i.e.

$$e = e_{p_t, \bar{p}, c_m}(N) \qquad (11)$$

From the results of numerical investigations, it follows (see also Table 1) that function (11) has a decreasing trend in N, which means that *when the lot size N increases, then saving of the inspection cost* (100 – e) *increases* (using the LTPD plan for inspection by variables and attributes instead of the corresponding plan for the inspection by attributes).

In the second step, we shall study the dependence of the economic efficiency measured by the parameter e on the process average fraction defective $\bar{p}$. Let p_t, N, c_m be the given parameters. Function (10) for the given p_t, N, c_m is a function of one variable $\bar{p}$, i.e.

$$e = e_{p_t, N, c_m}(\bar{p}) \qquad (12)$$

Table 1. Values of the parameter e for p_t = 0.005

p_t = 0.005	c_m = 2			c_m = 3			c_m = 4			c_m = 5		c_m = 6	
$\bar{p}$ \N	1000	4000	50 000	1000	4000	50 000	1000	4000	50 000	4000	50 000	4000	50 000
0.000250	38	25	24	52	36	35	65	45	45	54	54	63	64
0.000500	47	26	23	63	36	32	78	45	42	54	50	63	59
0.000750	54	36	28	72	50	40	88	62	52	74	63	86	74
0.001000	60	38	26	80	52	37	96	65	48	77	58	89	68
0.001250	67	43	21	87	59	29	103	74	37	87	45	99	53
0.001500	73	47	17	94	63	24	109	78	31	92	37	104	43
0.001750	80	51	35	100	68	49	114	84	62	98	75	110	87
0.002000	86	55	33	105	74	46	117	90	59	103	71	116	82
0.002250	92	60	38	110	79	53	118	95	68	109	81	120	94
0.002500	98	65	37	112	85	52	117	101	65	114	78	124	90

Source: Own construction

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From the results of the numerical investigations, it follows (see also Table 1) that function (12) has mostly an increasing trend in $\bar{p}$, which means that *when the process average fraction defective* $\bar{p}$ *increases, then the saving of the inspection cost* (100 – *e*) *decreases* (using the LTPD plan for the inspection by variables and attributes instead of the corresponding plan for the inspection by attributes).

Finally we shall study the dependence of the economic efficiency measured by the parameter *e* on the fraction of the cost of inspection of one item by variables to the cost of the inspection of one item by attributes c_m. Let p_t, N, $\bar{p}$ be the given parameters. Function (10) for the given p_t, N, $\bar{p}$ is a function of one variable c_m, i.e.

$$e = e_{p_t, N, \bar{p}}(c_m) \tag{13}$$

From the results of numerical investigations it follows (see also Table 1 and Figure 1) that function (13) has increasing trend in c_m, which means that *when the fraction of the cost of inspection of one item by variables to the cost of inspection of one item by attributes* c_m *increases, then saving of the inspection cost* (100 – *e*) *decreases* (using the LTPD plan for inspection by variables and attributes instead of the corresponding plan for inspection by attributes).

Now we shall decide according to c_m if inspection by variables should be considered in place of inspection by attributes.

Definition. Let p_t, N, $\bar{p}$ be given parameters. Let us define

$$c_m^{BE} \tag{14}$$

as the value of c_m for which $e = 100$.

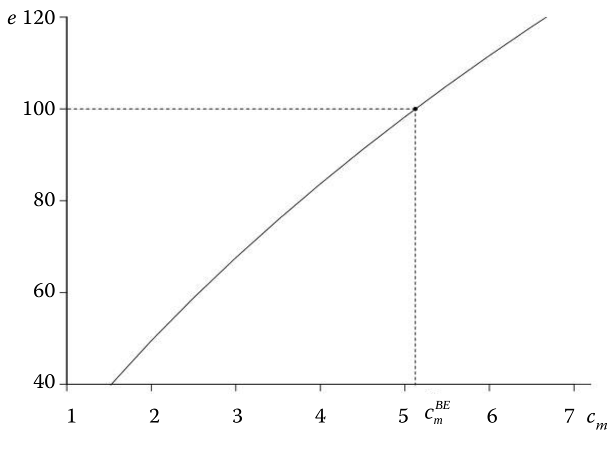

Figure 1. Graph of the function $e = e(c_m)$ for $p_t = 0.01$, $N = 1000$, $\bar{p} = 0.001$

According to (14) c_m^{BE} is such value of c_m for which mean inspection cost per lot of process average quality for inspection by variables and attributes is equal to mean inspection cost per lot of process average quality for inspection by attributes (Figure 1).

If c_m is statistically estimated and

$$c_m < c_m^{BE} \tag{15}$$

then the LTPD plans for inspection by variables and attributes are more economical than the corresponding Dodge-Romig LTPD plans. If

$$c_m > c_m^{BE} \tag{16}$$

then the Dodge-Romig LTPD plans for inspection by attributes are more economical than the corresponding LTPD plans for inspection by variables and attributes.

If value of c_m parameter is not known in some situation in practice, then c_m^{BE} (a break-even value of c_m parameter) may be calculated to provide some guidance in deciding if inspection by variables and attributes is worth considering. If c_m^{BE} is high, then using inspection by variables and attributes may be efficient (and one should try to estimate c_m to make some more precise evaluation), on the other hand if c_m^{BE} is near 1, then inspection by variables and attributes cannot be supposed to bring significant advantage over inspection by attributes. Calculation of c_m^{BE} value is implemented in LTPDvar package (Kasprikova 2012).

Parameter c_m^{BE} defined by (14) is a function of three variables p_t, N, $\bar{p}$, i.e.

$$c_m^{BE} = c_m^{BE}(p_t, N, \bar{p}) \tag{17}$$

Some values of function (17) are in Table 2 and Table 3.

In the first step we shall study dependence of c_m^{BE} (a break-even value of c_m parameter) on the lot size N. Let p_t, $\bar{p}$ be given parameters. Function (17) for given p_t, $\bar{p}$ is a function of one variable N, i.e.

$$c_m^{BE} = c_{m, p_t, \bar{p}}^{BE}(N) \tag{18}$$

From the results of numerical investigations it follows (see also Table 2 and Table 3) that function (18) has increasing trend in N.

In the second step we shall study dependence of c_m^{BE} (a break-even value of c_m parameter) on the process average fraction defective $\bar{p}$. Let p_t, N be given parameters. Function (17) for given p_t, N is a function of one variable $\bar{p}$, i.e.

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Table 2. Values of the function c_m^{BE}, for $\bar{p}$ = 0.005

$p_t \backslash N$	10 000	15 000	20 000	25 000	30 000
0.0125	3.510	3.626	3.729	3.766	3.828
0.0150	3.466	3.563	3.623	3.653	3.670
0.0200	3.253	3.295	3.321	3.354	3.358
0.0250	3.071	3.081	3.129	3.115	3.122
0.0300	2.883	2.932	2.942	2.935	2.946

Source: Own construction

Table 3. Values of the function c_m^{BE} for p_t = 0.012

$\bar{p} \backslash N$	10 000	15 000	20 000	25 000	30 000
0.002	4.525	4.580	4.675	4.666	4.684
0.003	4.173	4.267	4.315	4.371	4.394
0.004	3.820	3.938	4.010	4.072	4.096
0.005	3.514	3.666	3.748	3.809	3.857
0.006	3.219	3.391	3.500	3.566	3.618

Source: Own construction

$$c_m^{BE} = c_{m,p_t,N}^{BE}(\bar{p}) \qquad (19)$$

From the results of the numerical investigations, it follows (see also Table 3) that the function (19) has a decreasing trend in $\bar{p}$.

Finally we shall study the dependence of c_m^{BE} (a break-even value of c_m parameter) on the lot tolerance fraction defective p_t. Let $\bar{p}$, N be the given parameters. Function (17) for given $\bar{p}$, N is a function of one variable p_t, i.e.

$$c_m^{BE} = c_{m,\bar{p},N}^{BE}(p_t) \qquad (20)$$

From the results of the numerical investigations it follows (see also Table 2 and Figure 2) that the function (20) has a decreasing trend in p_t.

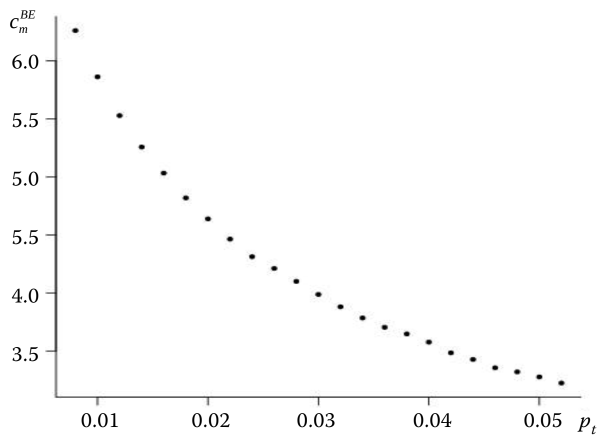

Figure 2. Graph of the function for N = 2000, $\bar{p}$ = 0.0005

It means that the economic efficiency of the LTPD plans for inspection by variables and attributes roughly speaking increases when the lot size N is increasing and decreases when the process average fraction defective $\bar{p}$ or the lot tolerance proportion defective p_t increases.

CONCLUSIONS

From the results of the numerical investigations, it follows that under the same protection of consumer the LTPD plans for the inspection by variables and attributes are in many situations more economical than the corresponding Dodge-Romig LTPD attribute sampling plans. For the chosen value of the lot tolerance fraction defective p_t, this conclusion is valid especially when:

– the number of items in the lot N is large,
– the process average fraction defective $\bar{p}$ is small,
– the cost of the inspection one item by variables is not much greater than the cost of the inspection one item by attributes, i.e. c_m is not large (see a break-even value c_m^{BE} defined in this paper).

Similar conclusions were obtained also for the comparison of the LTPD plans for the inspection by variables (special case of the LTPD plans for the inspection by variables and attributes) with the Dodge-Romig LTPD plans, but saving of the inspection cost

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is here lower than for the LTPD plans for the inspection by variables and attributes. It can be proved that under the assumption $c_m > 1$, the LTPD plans for the inspection by variables and attributes are always more economically efficient than the corresponding LTPD plans for the inspection by variables (for $c_m \leq 1$ the LTPD plans for inspection by variables are evidently most economically efficient).

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