

# Economic aspects of the LTPD single sampling inspection plans

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**Abstract:** The paper deals with the LTPD (Lot Tolerance Percent Defective) single sampling plans when the remainder of rejected lots is inspected. There are considered two types of the LTPD plans – for inspection by variables and for inspection by variables and attributes (all items from the sample are inspected by variables; the remainder of rejected lots is inspected by attributes). These plans were created by the author of this paper and published in the Statistical Papers. These new plans were compared with the corresponding Dodge-Romig LTPD plans for the inspection by attributes from the economic point of view. From the results of the numerical investigations, it follows that under the same protection of consumer the LTPD plans for the inspection by variables are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans (saving of the inspection cost is 80% in any cases). The dependence of the saving of the inspection cost on the acceptance sampling characteristics is analyzed in the paper.

**Key words:** acceptance sampling, economic efficiency, inspection by variables

Acceptance sampling is one of the techniques used in the quality control, either in the vendor-buyer relationships or for the management of within-company processes. The aim is to meet the desired levels of the protection against risk while keeping an eye on the economic characteristics of the process. Inference is made based on the inspection of a sample of items taken from a lot. Depending on the quality of the sample, the whole lot may be either accepted or rejected, or the inspection of another sample may follow in the case of double, multiple or sequential sampling plans (Klůfa 1980). The acceptance sampling plans, specified by the sample size and the critical value (or the acceptance number), determine the rules for this decision process.

There are many ways of classifying the acceptance sampling. One such classification is according to whether an item is inspected by its attributes, i.e. just classified as either good or defective (Hald 1981) or by variables. Sampling plans for the inspection by variables in many cases allow obtaining same level of protection as the corresponding sampling plans for the inspection by attributes while using a lower sample size. The basic notions of variables sampling plans are addressed in Jennett and Welch (1939).

Under the assumption that each inspected item is classified as either good or defective (acceptance

sampling by attributes), Dodge and Romig (1998) consider sampling plans which minimize the mean number of items inspected per lot of the process average quality

$$I_s = N - (N - n) \cdot L(p; n; c) \quad (1)$$

under the condition

$$L(p; n; c) = \beta \quad (2)$$

where  $L(p, n, c)$  is the operating characteristic (the probability of accepting a submitted lot with the proportion defective  $p$  when using plan  $(n, c)$  for the acceptance sampling),  $N$  is the number of items in the lot (the given parameter),  $\bar{p}$  the process average proportion defective (the given parameter),  $p_t$  is the lot tolerance proportion defective (the given parameter),  $P_t = 100 p_t$  is the lot tolerance per cent defective, denoted LTPD),  $n$  is the number of items in the sample ( $n < N$ ),  $c$  is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than  $c$ ).

The condition (2) provides a guarantee for the consumer that the lots of an unsatisfactory quality level, with the proportion defective  $p_t$  are going to be accepted only with the specified probability  $\beta$  (consumer's risk). Value  $\beta = 0.1$  is used for the consumer's risk in Dodge and Romig (1998).

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The LTPD plans for inspection by variables and attributes (all items from the sample are inspected by variables, the remainder of the rejected lots is inspected by attributes) have been introduced in Klufa (1994), using the approximate calculation of the plans. The exact plans, using the non-central  $t$  distribution in calculation of the operating characteristic Johnson and Welch (1940), have been reported in Klufa (2010) and Kaspříková (2011) – the LTPDvar is an add-on package to the R software (R Development Core Team 2008). Similar problems are solved in Klufa (1997, 2008), Chen and Chou (2001), Kaspříková and Klufa (2011), Wilrich (2012), Aslam et al. (2015).

The present paper shows the economic characteristics of the exact LTPD plans for inspection by variables (a special case of acceptance sampling by variables and attributes) and for inspection by variables and attributes and shows the impact of the input parameters values on the resulting sampling plan and its economic efficiency. A measure for the assessment of economic efficiency of these plans is proposed.

## MATERIAL AND METHODS

The LTPD plans for inspection by variables and attributes have been introduced in Klufa (1994) under the following assumptions: measurements of a single quality characteristic  $X$  are independent, identically distributed normal random variables with unknown parameters  $\mu$  and  $\sigma^2$ . For the quality characteristic  $X$ , there is given either an upper specification limit  $U$  (the item is defective if its measurement exceeds  $U$ ), or a lower specification limit  $L$  (the item is defective if its measurement is smaller than  $L$ ). It is further assumed that the unknown parameter  $\sigma$  is estimated from the sample standard deviation  $s$ .

The inspection procedure is as follows:

Draw a random sample of  $n$  items and compute the sample mean  $\bar{x}$  and the sample standard deviation  $s$ . Accept the lot if

$$\frac{U - \bar{x}}{s} \geq k \quad \text{or} \quad \frac{\bar{x} - L}{s} \geq k. \quad (3)$$

Suppose that  $c_s^*$  is the cost of inspection of one item by attributes and  $c_m^*$  is the cost of inspection of one item by variables and that the sample is inspected by variables. Then the inspection cost per lot with the proportion defective  $p$ , assuming that the remainder of the rejected lots is inspected by attributes

(the inspection by variables and attributes), is  $n \cdot c_m^*$  with probability  $L(p, n, k)$  and  $[n \cdot c_m^* + (N - n) \cdot c_s^*]$  with probability  $[1 - L(\bar{p}; n, k)]$ . The mean inspection cost per lot of the process average quality  $\bar{p}$  is therefore

$$C_{ms} = n \cdot c_m^* + (N - n) \cdot c_s^* \cdot [1 - L(\bar{p}; n, k)] \quad (4)$$

Dividing (4) by  $c_s^*$  gives the objective function

$$I_{ms} = n \cdot c_m + (N - n) \cdot [1 - L(\bar{p}; n, k)] \quad (5)$$

where

$$c_m = c_m^* / c_s^* \quad (6)$$

is the ratio of the cost of the inspection of one item by variables to the cost of the inspection of this item by attributes (this new parameter has to be estimated in each real situation, usually is  $c_m > 1$ ). Note that both the function  $I_{ms} = C_{ms} / c_s^*$  and function  $C_{ms}$  have a minimum for the same acceptance plan  $(n, k)$ . Therefore, we shall look for the acceptance plan  $(n, k)$  minimizing (5) instead of (4) under the condition

$$L(p; n; k) = \beta \quad (7)$$

Setting the value of  $c_m$  to 1 can be used in the situations, when both the sample and the remainder of the rejected lots are inspected by variables. Acceptance sampling by variables can thus be considered just as a special case of acceptance sampling by variables and attributes. Then instead of  $I_{ms}$ , we may use the notation  $I_m$  and setting  $c_m = 1$  in (5) we obtain

$$I_m = N - (N - n) \cdot L(\bar{p}; n, k) \quad (8)$$

i.e. the mean number of items inspected per lot of the process average quality, assuming that both the sample and the remainder of the rejected lots are inspected by variables.

*Summary:* The task to be solved is to determine the plan  $(n, k)$  minimizing (5) under the condition (7) for the given values of input parameters  $N, c_m, p_t$  and  $\bar{p}$ .

The solution of this problem is in the paper Klufa (1994), the numerical solution is in Klufa (2010) and Kaspříková (2011).

## RESULTS AND DISCUSSION

Now we shall study the economic efficiency of the LTPD plans for the inspection by variables and attributes. For the comparison of the LTPD single sam-

pling plans for inspection by variables and attributes with the corresponding Dodge-Romig LTPD plans for inspection by attributes from economic point of view we use the parameter  $e$  defined by the relation

$$e = \frac{I_{ms}}{I_s} \cdot 100 \tag{9}$$

According to (4), there is

$$e = \frac{I_{ms}}{I_s} \cdot 100 = \frac{I_{ms} \cdot c_s^*}{I_s \cdot c_s^*} \cdot 100 = \frac{C_{ms}}{C_s} \cdot 100$$

where  $C_s = I_s \cdot c_s^*$  is the mean cost of the inspection by attributes ( $c_s^*$  is the cost of the inspection of one item by attributes). Therefore, the LTPD plan for the inspection by variables and attributes is more economically efficient than the corresponding Dodge-Romig plan when

$$e < 100$$

Expression  $(100 - e)$  then represents the percentage of savings in the inspection cost when the sampling plan for inspection by variables and attributes is used instead of the corresponding plan for the inspection by attributes.

Economic efficiency measured by the parameter  $e$  (see formula (9)) is a function of four variables,  $p_t$ ,  $N$ ,  $\bar{p}$  and  $c_m$ , i.e.

$$e = e(p_t, N, \bar{p}, c_m) \tag{10}$$

Some values of this function are in Table 1.

From the results of numerical investigations it follows that under the same protection of consumer the

LTPD plans for inspection by variables are in many situations more economical (saving of the inspection cost is 80% in any cases) than the corresponding Dodge-Romig attribute sampling plans – see also Table 1.

For example when  $p_t = 0.005$ ,  $N = 4000$ ,  $\bar{p} = 0.0005$  and  $c_m = 2$  is parameter  $e = 26$  (see Table 1), which means that using the LTPD plan for inspection by variables and attributes it can be expected approximately  $(100 - e) = 74\%$  saving of the inspection cost in comparison with the corresponding Dodge-Romig plan.

Now we shall study dependence of the economic efficiency measured by parameter  $e$  on the lot size  $N$ . Let  $p_t$ ,  $\bar{p}$ ,  $c_m$  be given parameters. Function (10) for given  $p_t$ ,  $\bar{p}$ ,  $c_m$  is a function of one variable  $N$ , i.e.

$$e = e_{p_t, \bar{p}, c_m}(N) \tag{11}$$

From the results of numerical investigations, it follows (see also Table 1) that function (11) has a decreasing trend in  $N$ , which means that *when the lot size  $N$  increases, then saving of the inspection cost  $(100 - e)$  increases* (using the LTPD plan for inspection by variables and attributes instead of the corresponding plan for the inspection by attributes).

In the second step, we shall study the dependence of the economic efficiency measured by the parameter  $e$  on the process average fraction defective  $\bar{p}$ . Let  $p_t$ ,  $N$ ,  $c_m$  be the given parameters. Function (10) for the given  $p_t$ ,  $N$ ,  $c_m$  is a function of one variable  $\bar{p}$ , i.e.

$$e = e_{p_t, N, c_m}(\bar{p}) \tag{12}$$

Table 1. Values of the parameter  $e$  for  $p_t = 0.005$

$p_t = 0.005$	$c_m = 2$			$c_m = 3$			$c_m = 4$			$c_m = 5$		$c_m = 6$	
	$\bar{p} \setminus N$	1000	4000	50 000	1000	4000	50 000	1000	4000	50 000	4000	50 000	4000
0.000250	38	25	24	52	36	35	65	45	45	54	54	63	64
0.000500	47	26	23	63	36	32	78	45	42	54	50	63	59
0.000750	54	36	28	72	50	40	88	62	52	74	63	86	74
0.001000	60	38	26	80	52	37	96	65	48	77	58	89	68
0.001250	67	43	21	87	59	29	103	74	37	87	45	99	53
0.001500	73	47	17	94	63	24	109	78	31	92	37	104	43
0.001750	80	51	35	100	68	49	114	84	62	98	75	110	87
0.002000	86	55	33	105	74	46	117	90	59	103	71	116	82
0.002250	92	60	38	110	79	53	118	95	68	109	81	120	94
0.002500	98	65	37	112	85	52	117	101	65	114	78	124	90

Source: Own construction

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From the results of the numerical investigations, it follows (see also Table 1) that function (12) has mostly an increasing trend in  $\bar{p}$ , which means that *when the process average fraction defective  $\bar{p}$  increases, then the saving of the inspection cost  $(100 - e)$  decreases* (using the LTPD plan for the inspection by variables and attributes instead of the corresponding plan for the inspection by attributes).

Finally we shall study the dependence of the economic efficiency measured by the parameter  $e$  on the fraction of the cost of inspection of one item by variables to the cost of the inspection of one item by attributes  $c_m$ . Let  $p_t, N, \bar{p}$  be the given parameters. Function (10) for the given  $p_t, N, \bar{p}$  is a function of one variable  $c_m$  i.e.

$$e = e_{p_t, N, \bar{p}}(c_m) \tag{13}$$

From the results of numerical investigations it follows (see also Table 1 and Figure 1) that function (13) has increasing trend in  $c_m$ , which means that *when the fraction of the cost of inspection of one item by variables to the cost of inspection of one item by attributes  $c_m$  increases, then saving of the inspection cost  $(100 - e)$  decreases* (using the LTPD plan for inspection by variables and attributes instead of the corresponding plan for inspection by attributes).

Now we shall decide according to  $c_m$  if inspection by variables should be considered in place of inspection by attributes.

**Definition.** Let  $p_t, N, \bar{p}$  be given parameters. Let us define

$$c_m^{BE} \tag{14}$$

as the value of  $c_m$  for which  $e = 100$ .

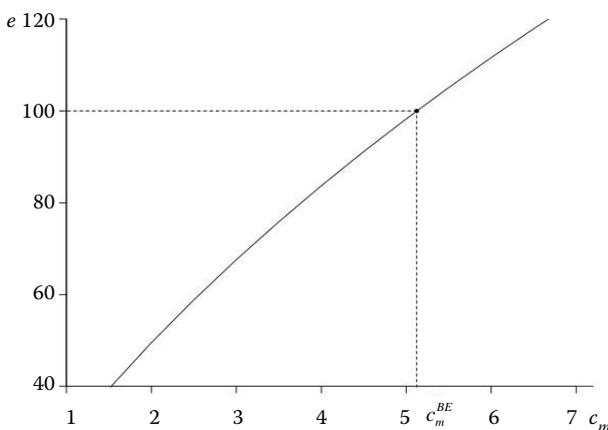


Figure 1. Graph of the function  $e = e(c_m)$  for  $p_t = 0.01$ ,  $N = 1000$ ,  $\bar{p} = 0.001$

According to (14)  $c_m^{BE}$  is such value of  $c_m$  for which mean inspection cost per lot of process average quality for inspection by variables and attributes is equal to mean inspection cost per lot of process average quality for inspection by attributes (Figure 1).

If  $c_m$  is statistically estimated and

$$c_m < c_m^{BE} \tag{15}$$

then the LTPD plans for inspection by variables and attributes are more economical than the corresponding Dodge-Romig LTPD plans. If

$$c_m > c_m^{BE} \tag{16}$$

then the Dodge-Romig LTPD plans for inspection by attributes are more economical than the corresponding LTPD plans for inspection by variables and attributes.

If value of  $c_m$  parameter is not known in some situation in practice, then  $c_m^{BE}$  (a break-even value of  $c_m$  parameter) may be calculated to provide some guidance in deciding if inspection by variables and attributes is worth considering. If  $c_m^{BE}$  is high, then using inspection by variables and attributes may be efficient (and one should try to estimate  $c_m$  to make some more precise evaluation), on the other hand if  $c_m^{BE}$  is near 1, then inspection by variables and attributes cannot be supposed to bring significant advantage over inspection by attributes. Calculation of  $c_m^{BE}$  value is implemented in LTPDvar package (Kasprikova 2012).

Parameter  $c_m^{BE}$  defined by (14) is a function of three variables  $p_t, N, \bar{p}$ , i.e.

$$c_m^{BE} = c_m^{BE}(p_t, N, \bar{p}) \tag{17}$$

Some values of function (17) are in Table 2 and Table 3.

In the first step we shall study dependence of  $c_m^{BE}$  (a break-even value of  $c_m$  parameter) on the lot size  $N$ . Let  $p_t, \bar{p}$  be given parameters. Function (17) for given  $p_t, \bar{p}$  is a function of one variable  $N$ , i.e.

$$c_m^{BE} = c_m^{BE}(p_t, \bar{p})(N) \tag{18}$$

From the results of numerical investigations it follows (see also Table 2 and Table 3) that function (18) has increasing trend in  $N$ .

In the second step we shall study dependence of  $c_m^{BE}$  (a break-even value of  $c_m$  parameter) on the process average fraction defective  $\bar{p}$ . Let  $p_t, N$  be given parameters. Function (17) for given  $p_t, N$  is a function of one variable  $\bar{p}$ , i.e.

Table 2. Values of the function  $c_m^{BE}$ , for  $\bar{p} = 0.005$ 

$p_t \setminus N$	10 000	15 000	20 000	25 000	30 000
0.0125	3.510	3.626	3.729	3.766	3.828
0.0150	3.466	3.563	3.623	3.653	3.670
0.0200	3.253	3.295	3.321	3.354	3.358
0.0250	3.071	3.081	3.129	3.115	3.122
0.0300	2.883	2.932	2.942	2.935	2.946

Source: Own construction

Table 3. Values of the function  $c_m^{BE}$  for  $p_t = 0.012$ 

$\bar{p} \setminus N$	10 000	15 000	20 000	25 000	30 000
0.002	4.525	4.580	4.675	4.666	4.684
0.003	4.173	4.267	4.315	4.371	4.394
0.004	3.820	3.938	4.010	4.072	4.096
0.005	3.514	3.666	3.748	3.809	3.857
0.006	3.219	3.391	3.500	3.566	3.618

Source: Own construction

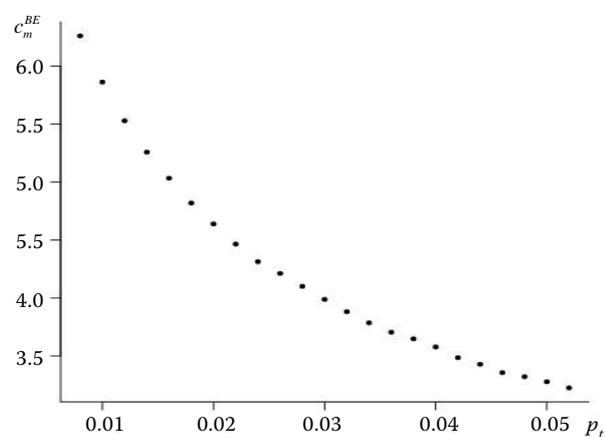
$$c_m^{BE} = c_{m,p_t,N}^{BE}(\bar{p}) \quad (19)$$

From the results of the numerical investigations, it follows (see also Table 3) that the function (19) has a decreasing trend in  $\bar{p}$ .

Finally we shall study the dependence of  $c_m^{BE}$  (a break-even value of  $c_m$  parameter) on the lot tolerance fraction defective  $p_t$ . Let  $\bar{p}$ ,  $N$  be the given parameters. Function (17) for given  $\bar{p}$ ,  $N$  is a function of one variable  $p_t$  i.e.

$$c_m^{BE} = c_{m,\bar{p},N}^{BE}(p_t) \quad (20)$$

From the results of the numerical investigations it follows (see also Table 2 and Figure 2) that the function (20) has a decreasing trend in  $p_t$ .

Figure 2. Graph of the function for  $N = 2000$ ,  $\bar{p} = 0.0005$ 

It means that the economic efficiency of the LTPD plans for inspection by variables and attributes roughly speaking increases when the lot size  $N$  is increasing and decreases when the process average fraction defective  $\bar{p}$  or the lot tolerance proportion defective  $p_t$  increases.

## CONCLUSIONS

From the results of the numerical investigations, it follows that under the same protection of consumer the LTPD plans for the inspection by variables and attributes are in many situations more economical than the corresponding Dodge-Romig LTPD attribute sampling plans. For the chosen value of the lot tolerance fraction defective  $p_t$ , this conclusion is valid especially when:

- the number of items in the lot  $N$  is large,
- the process average fraction defective  $\bar{p}$  is small,
- the cost of the inspection one item by variables is not much greater than the cost of the inspection one item by attributes, i.e.  $c_m$  is not large (see a break-even value  $c_m^{BE}$  defined in this paper).

Similar conclusions were obtained also for the comparison of the LTPD plans for the inspection by variables (special case of the LTPD plans for the inspection by variables and attributes) with the Dodge-Romig LTPD plans, but saving of the inspection cost

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is here lower than for the LTPD plans for the inspection by variables and attributes. It can be proved that under the assumption  $c_m > 1$ , the LTPD plans for the inspection by variables and attributes are always more economically efficient than the corresponding LTPD plans for the inspection by variables (for  $c_m \leq 1$  the LTPD plans for inspection by variables are evidently most economically efficient).

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