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Semi-empirical estimation of log taper using stem profile equations

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Abstract: In January 2019 the forest industry in Ukraine adopted European standards for measuring and grading of round wood based on mid-point diameters, which caused major discrepancies from traditionally used estimates of timber volume using top diameters. To compare methods of merchantable wood volume estimation, we investigated the stem form inside bark for two dominant tree species in Ukraine, i.e. Scots pine (*Pinus sylvestris* L.) and common oak (*Quercus robur* L.). We used tree stem measurements to fit stem profile equations, whereas simulation was applied to derive log taper. We found that Newnham's (1992) variable-exponent taper equation performed well for predicting stem taper for both tree species. Then, we simulated the structure of harvested wood, so that it replicated annual distribution of logs by their length and diameters. As a result, the average log taper was estimated at 0.836 ÷ 0.855 $cm \cdot m^{-1}$ and 1.180 ÷ 0.121 $cm \cdot m^{-1}$ for pine and oak, respectively. The study also indicated that log taper varied along stems. The higher rates of diameter decrease were found for butt logs, for which the taper was 2.5–3.5 times higher than its average for the whole stem. The results of our study ensure the stacked round wood volume conversion between estimates obtained using top and mid-point diameters.

Keywords: stem profile modelling; log taper; mid-point diameter; top diameter; Scots pine; common oak

In January 2019, the forest industry of Ukraine adopted European standards for round wood measurements and classification required by the European Union (EU)-Ukraine association agreement. Before that time, the log volume estimation in Ukraine was carried out using national standards (i.e., GOST 2708-75) which incorporated top diameter measurements instead of mid-point diameter (Svynchuk et al. 2014). Once all national standards developed prior to 1992 were suspended, the round wood volume estimation has been based on volume tables (DSTU 4020-2-2001, 2001) using log length and mid-point diameter measurements. Despite these national standards being adopted for ten main tree species of Ukraine (pine, spruce, fir, oak, beech, ash, hornbeam, aspen, birch, and alder), they were not widely used in practice. Foresters often encountered difficulties in obtaining mid-point log diameter measurements, especially if wood was stacked.

In 2019 the forest industry of Ukraine faced a problem to reclassify a significant merchantable wood volume by diameter classes that was harvested in 2018 and classified by size classes using top diameters. So, there was a need to make conversion between two different approaches to log classification, i.e. based on top diameters (before 2019) and mid-point diameter (since 2019). The literature review showed that the overwhelming majority of publications focused on stem taper estimation (Newberry, Burkhart 1986; Lee et al. 2003; Rojo et al. 2005; Fonweban et al. 2011; Burkhart, Tomé 2012; Menendéz-Miguélez et al. 2014) but a

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few of them are focused on log taper (Larsen 2017; Clutter 1980). It is also worth noting that the use of stem taper equations that describe a gradual decrease in diameters along the stem (West 2015; Beltran et al. 2017) can be complicated due to the need of diameter at breast height (*DBH*) and stem height measurements for each felled tree. The complexity of stem taper models limits their use during the round wood volume estimation, however modelling stem profiles provides necessary input data for the further calculation of log taper that are harvested from different parts of the stem. In general, log taper must be accounted merely between the top and mid-point diameter of logs. Nevertheless, to assess for example average value of log taper, a detailed analysis of stem profiles is required.

Numerous equations have been developed for estimation of stem taper. According to Burkhardt and Tomé (2012) taper functions are divided into three general classes, i.e. (1) simple equations (Kozak et al. 1969; Sharma, Oderwald 2001); (2) segmented polynomial regression models (Clark et al. 1991; Max, Burkhart 1976); and (3) variable-exponent or variable-form taper equations (Newnham 1992; Kozak 2004). Given the significant variation in stem form, some equations were developed to include crown variables (Larson 1963; Leites, Robinson 2004). Taper equations to the top merchantable diameter (Clark et al. 1991; Kozak 1998) were also widely applied to calculate the merchantable wood volume. A reliable taper equation ought to estimate diameter with a minimum variance as well as be flexible to adapt to a wide variety of species, thus provide accurate predictions of stem volume (Kozak, Smith 1993). However, the optimal taper equations must be simple in use, require relatively few tree measurements, and provide accurate volume predictions (Larsen 2017).

Scots pine (*Pinus sylvestris* L.) and common oak (*Quercus robur* L.) are the most important commercial tree species in Ukrainian forests. Pine and oak cover 2.2 million ha (34.6%) and 1.7 million ha (27.5%) of forest land, respectively (Anonymous 2012). In 2019, about 12 million m^3 of pine and 3 million m^3 of oak were harvested in Ukraine (SSSU 2019). Reported values correspond well with the annual harvest data for the period between 2018 and 2019.

The goal of this study was to assess log taper for two dominant tree species in Ukraine. To estimate log taper, we used a semi-empirical approach based on the modelling of stem shape followed by simulation of various timber harvesting scenarios. The specific objectives of the study were twofold: (1) to develop stem profile equations inside bark for pine and oak trees; and (2) to assess taper of harvested logs.

MATERIAL AND METHODS

Data collection. Tree stem measurements of sample trees were used to predict the stem form of pine and oak trees. The empirical data were collected on temporary sample plots established in the most relevant regions of Ukraine for the studied tree species. The pine data were collected in northernmost regions (Polissya climatic zone) representing nearly 65% of all pine forests of Ukraine. Data for oak preferentially came from central regions of Ukraine (forest-steppe climatic zone) where the share of oak stands reaches 43% of the total forested area. Because we aimed to investigate the taper of merchantable timber, sample plots were established in premature, mature, and overmature stands. Only trees with $DBH > 14$ cm were used in further stem form analysis. Thus, 7–12 trees were sampled on each plot depending on the range of tree diameters in the forest stand. To represent trees of different diameters the number of sample trees was proportional to the frequency of 4-cm diameter class which usually has better performance in tree volume estimation (David et al. 2016).

For either of the two species, two independent datasets were used for model development and accuracy assessment. The fitting dataset for pine included 105 sample trees collected on 22 temporary sample plots. For oak, the taper model was fitted using 149 sample trees measured on 23 sample plots (Table 1). To test the model performance, we collected additional data so that it represents the fitting dataset in terms of *DBH* and height distribution of sample trees (Table 2).

On each plot, sample trees were felled, and their height was measured from the ground level. In addition, the stems were divided into 2-meter sections starting from the ground level and measurements of diameters outside bark coupled with bark thickness measurements were taken at stump height, *DBH*, and at the mid-point of each 2-meter section. Diameters were calculated as an average of two diameter measurements taken perpendicularly to each other with a calliper. Bark thickness was

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Table 1. Summary statistics of sample tree attributes of fitting dataset

Tree species	No. of plots	No. of trees	*DBH* (cm)		Tree height (m)	
			mean	range	mean	range
Pine	22	105	31.8	16.0–65.0	25.4	12.0–35.7
Oak	23	149	33.4	20.0–56.0	22.9	15.7–30.2

DBH – diameter at breast height

Table 2. Summary statistics of sample tree attributes of validation dataset

Tree species	No. of plots	No. of trees	*DBH* (cm)		Tree height (m)	
			mean	range	mean	range
Pine	7	54	30.1	14.0–52.2	25.0	13.1–33.0
Oak	7	56	36.2	20.8–54.8	24.5	18.3–31.9

DBH – diameter at breast height

measured with a ruler to give the estimated diameters inside bark.

Modelling stem taper. We modelled the stem taper inside bark using Newnham's (1992) exponential equation applied to relative values of tree height and diameters:

$$\frac{d_i}{DBH} = x^{\left(a_0 + a_1 \times (z-1) + a_2 \times exp \times (a_3 \times z)\right)} \quad (1)$$

where:
d_i – diameter at distance;
DBH – diameter at breast height;
exp – exponent;
h – height of a tree;
h_i – height above the ground level (cm);
a_0, a_1, a_2, a_3 – parameters of the equation.

$$x = \frac{h - h_i}{h - 1.3}$$

$$z = \frac{h_i}{h}$$

Ordinal nonlinear least-squares regression was applied to get fitted values of the model parameters without consideration of random effects in stem form variation (Burkhardt, Tomé 2012; Gomes-Garcia 2013; Arias-Rodil et al. 2015). Then, the model fit was evaluated using an independent dataset of tree measurements. We calculated systematic bias (*SB*) and root mean squared error (RMSE) using observation from the validation dataset:

$$SB = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\sum_{j=1}^{m_i}\left(d_{ij} - \tilde{d}_{ij}\right)}{m_i}\right) \quad (2)$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\sum_{j=1}^{m_i}\left(d_{ij} - \tilde{d}_{ij}\right)^2}{m_i}\right)} \quad (3)$$

where:
d_{ij} – observed value of diameter inside bark of *i*-th sample tree at *j*-th point along the stem;
$\tilde{d}_{ij}$ – predicted value of diameters inside bark of *i*-th sample tree at *j*-th point along the stem;
m_i – number of diameter measurements for *i*-th sample tree;
n – number of sample trees for each tree species.

Simulating timber structure. We used in-situ measurements of tree stems to develop stem taper equations for pine and oak trees and simulated different harvesting scenarios to estimate the log taper. We used the annual report for 2018 to replicate the structure of harvested logs. According to it, we preferably focused on timber with top diameter inside bark of 14–30 cm for pine and 18–40 cm for oak, the share of which was revealed to be 75–80% of the total harvested merchantable wood volume. We also used information on the occurrence of logs of different length to simulate a dataset which meets specified above parameters regarding log length and diameters.

After stem profile equations were developed, we simulated different harvesting scenarios to evaluate the taper of logs regarding their distribution by length and diameters. We used an annual report provided by the State Enterprise "Forestry Innovation and Analytical Centre" that accumulates data on the harvested merchantable roundwood

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volume in Ukraine. These data have been collected in Ukraine under the state unified system of electronic registration of wood since 2013. Table 3 reports on the distribution of timber lengths harvested in pine and oak forests in 2018 which was used in the study.

The key assumption of our study was that timber dimensions, i.e. lengths and diameters, have an effect on average log taper. Both the potential occurrence of logs of the specific length (Table 3) and the distribution of trees by diameter classes (Table S1, S2 in Electronic Supplementary Material (ESM)) were considered in our simulation. Firstly, to simulate the structure of harvested trees in mature stands, we used previously published tree diameter distribution models (Anonymous 1987). Secondly, we used the normal distribution of tree heights within diameter classes (Monness 2015) to calculate the probability of the occurrence of trees within each *DBH* class. As input data for these models, we used the following parameters of mature stands according to the Reference Book of forest resources of Ukraine (Reference Book, 2012): average diameter for pine stands – 29.2 cm, for oak stands – 37.9 cm; average height for pine stands – 26 m, for oak stands – 26 m.

Based on the taper equations, the harvested timber structure was generated by successfully cutting trees using different lengths of timber as it is specified in Table 3. We developed an algorithm in R statistical software (Ver. 4.0.3, 2020) which used the taper model to estimate diameters inside bark along tree stems and specify mid-point and top diameters of each log. Then, we assigned the weight coefficient to each log which was calculated as the product of two probabilities, i.e. probability (occurrence) of the log of predefined length, and probability of the tree having predefined diameter and height (Table S1 and S2 in the ESM). Based on standard specifications on round timber (TUU 16.1-00994207-001:2018, 2019), in modelling we used only logs with the mid-point diameter without bark of 15 cm which satisfy minimal requirements for quality class D of merchantable wood for both pine and oak. Thus, we created the dataset that allowed us to deal with log taper.

To make the transition between the mid-point and upper cross-section diameter of the log, their taper was determined as the rate of diameter decrease from mid-point to top using the formula:

$$S = \frac{1}{2} \times \frac{d^m - d^t}{L} \quad (4)$$

where:
S – timber taper accounted from the mid-point diameter up to the upper cross-section (cm·m^{-1});
d^m – mid-point diameter of log without bark (cm);
d^t – upper cross-section diameter of log without bark (cm);
L – length of log (m).

Further, the log taper was modelled using weighted ordinary least-squares regression in R statistical package regarding log diameter and length as potential predictor variables. In this regard we tested the linear regression model as well as the following equation:

$$\tilde{S} = a_0 + \frac{a_1 \times \left(d^m\right)^{a_2}}{L^{a_3}} \quad (5)$$

where:
$\tilde{S}$ – predicted value of log taper (cm·m^{-1}).

RESULTS

Stem taper equations. Newnham's (1992) variable-exponent taper equation was used to predict diameter distribution along tree stems. Specifically,

Table 3. The distribution of merchantable logs by length harvested in Ukraine in 2018

Length of logs (m)	Pine		Oak	
	number of logs	occurrence (%)	number of logs	occurrence (%)
2.0	–	–	96 297	5.3
2.4	2 491 145	14.1	–	–
2.5	–	–	91 163	5.0
2.7	–	–	84 930	4.7
3.0	4 171 707	23.6	1 434 080	78.9
4.0	8 501 679	48.2	64 740	3.6
4.8	1 137 639	6.4	–	–
6.0	1 354 506	7.7	46 310	2.5

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a scatter plot of relative values of stem diameters (d_i/DBH) over relative height (h_i/h) with fitted taper models is shown in Figure 1. One can see from the plot that tree stems of pine and oak have different profiles that have an impact on the harvested log taper. The scatter plot also indicates a substantial variation of tree stem form at different relative height. The larger taper for both tree species is observed in the tree butt section in a range of relative heights of 0–0.1*h* (*h* – height of the tree, see Equation 1). On the branch-free stem length there is a section with a low rate of diameter decrease. We also found that due to the crown development in a range of relative heights of 0.5–1*h* the taper of oak stems increases. A significantly larger variation of tree form is also seen from Figure 1 which is a subject of different stand structure (canopy closure, relative stocking, etc.).

Table 4 presents parameters of Newnham's (1992) taper model for both tree species derived using ordinary least-squares regression. All parameters are significant at an alpha level of 0.05.

We also assessed the accuracy of diameter prediction along tree stems using independent validation datasets. Because of variation in diameters at different height we evaluated the performance of the models in four intervals of relative heights for which we estimated *SB* and RMSE (Table 5).

Newnham's taper equation performs well for both tree species. The value of *SB* estimated along the total height of stems was revealed to be equal to 0.00 cm for pine and 0.02 cm for oak, which indicates that prediction of diameters inside bark is unbiased in prediction of diameters inside bark. We have also found that developed models have negligible errors within different stem sections. For

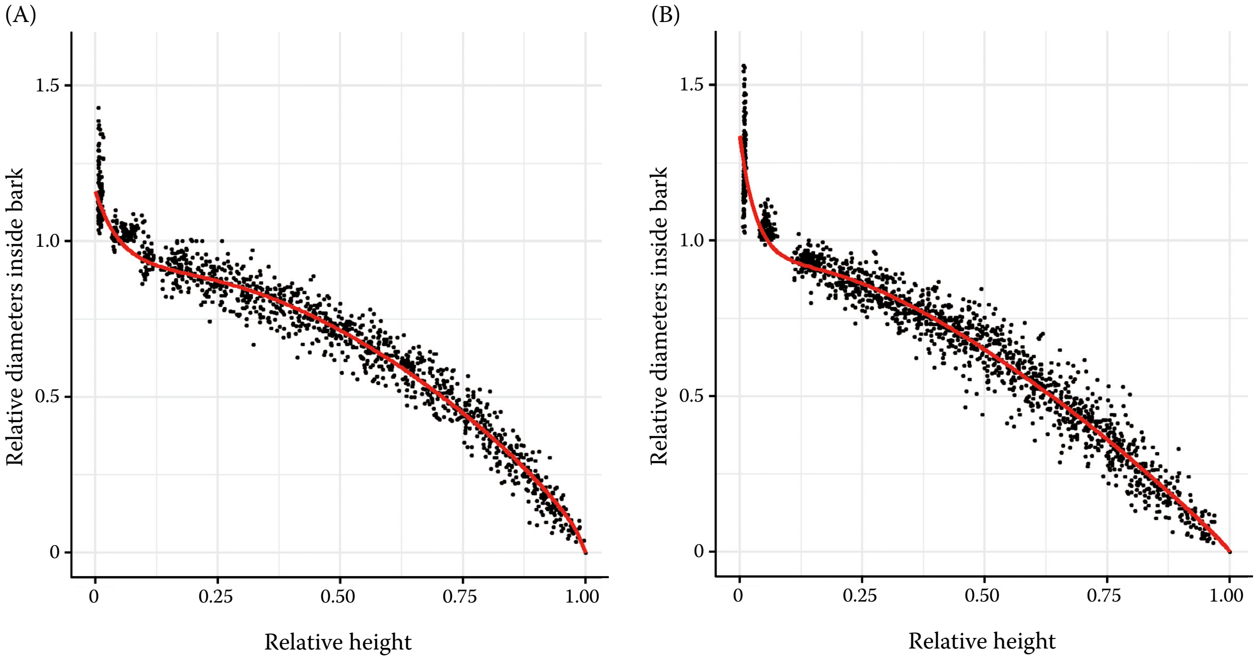

Figure 1. Scatter plots of the relative diameters inside bark over relative stem heights: (A) pine; (B) oak

Red line shows the fitted taper model

Table 4. Estimated parameters and corresponding standard errors (SE) of stem taper equations for pine and oak trees

Tree species	a_0		a_1		a_2		a_3	
	estimate	SE	estimate	SE	estimate	SE	estimate	SE
Pine	0.6826	0.0089	0.3307	0.0289	2.486	0.0951	–11.41	0.6852
Oak	0.8563	0.0115	0.3587	0.0294	4.498	0.0994	–17.62	1.070

a_0, a_1, a_2, a_3 – parameters

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Table 5. The performance of the models for diameter inside bark (cm) prediction

Tree species	Tree stem zone								Total	
	0.0–0.25*h*		0.25–0.5*h*		0.5–0.75*h*		0.75–1.0*h*			
	SB	RMSE	*SB*	RMSE	*SB*	RMSE	*SB*	RMSE	*SB*	RMSE
Pine	0.16	1.37	0.30	1.42	–0.38	1.32	–0.18	1.25	0.00	1.35
Oak	0.27	1.89	–0.26	1.52	–0.36	2.09	0.23	1.33	0.02	1.75

SB – systematic bias; RMSE – root mean squared error

example, the larger bias of stem diameter prediction reaches –0.38 cm for pine trees and –0.36 cm for oak trees. The RMSEs are nearly equal for both species and ranged from 1.3 to 2.1 cm.

The models showed an even residual distribution above and below the locally weighted smoothing line with no significant autocorrelation observed (Figure 2). The absolute values of residuals are higher for oak and increase with stem height. This effect is more significant for larger trees, the stem form of which is an object of the crown influence. However, we did not include any additional crown variables which could only slightly improve diameter prediction (Li, Weiskittel 2010).

Log taper. Log taper was calculated for logs of different lengths and diameters using stem profile equations inside bark. Firstly, we calculated the mean values of log taper for all simulated logs, which was important to implement it into the electronic registration system of wood. Given different occurrence of logs of the specified sizes, we used weighted mean values to avoid biased estimates of taper. As a result, we estimated basic descriptive statistics of pine and oak log taper as well as provided confidence intervals for mean estimates (Table 6).

The greater value of average taper of oak timber can be explained by higher rates of diameter decrease in the butt section and in the crown zone of oak trees as compared with pine trees. The difference is important to properly estimate log volumes using methods that are based on mid-point and top diameter measurements. For practical use, we also developed a linear regression model that predicts the log taper depending on the mid-point diameter of logs without bark (Table 7).

To evaluate the accuracy of the developed models, *SB* and RMSE of the prediction of the top diameter without bark were estimated. For this purpose, we used 157 observations with log taper measurements collected in the field for Scots pine, and 132 measurements for common oak. The above specified statistical errors were –±0.1% and 3.4% for pine logs,

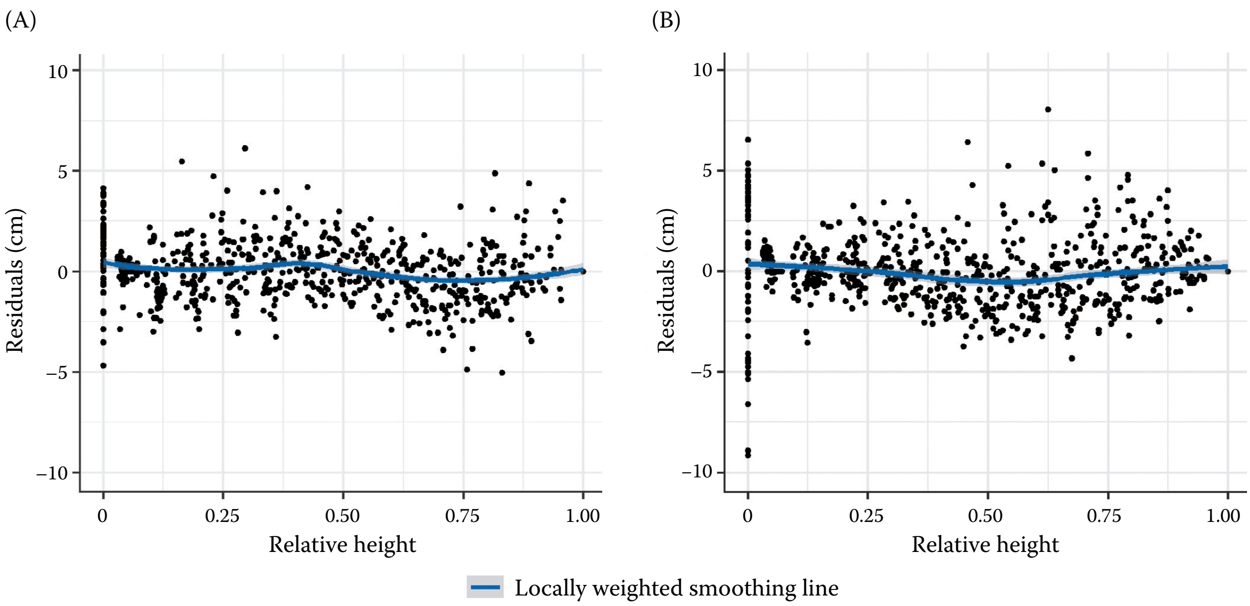

Figure 2. Residuals of fitted models of diameter inside bark: (A) pine; (B) oak

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Table 6. Descriptive statistics and 95% confidence intervals of the mean log taper

Tree species	Mean	RMSE	Standard error of the mean	Confidence interval of the mean
		cm·m^{-1}		
Pine	0.845	0.370	0.005	0.836 ÷ 0.855
Oak	1.19	0.425	0.006	1.18 ÷ 1.20

and ±0.05% and 2.6% for oak logs. We did not find a statistically significant relationship between log taper and log length. Nonlinear regression model (Equation 5) though has been developed to describe the taper of the first butt log (Table 8).

All estimated parameters are statistically significant at a level of 5%. The model resulted in taper between the top and the mid-point diameter that was substantially higher for butt logs than the entire log average (Table 5). Additionally, Figure 3 shows that the taper decreases with the log length.

The taper was significantly higher in oak vs. pine butt logs. The main differences in taper were observed in short logs (2–3 m), whereas in longer logs (4–6 m) both species showed nearly identical taper values. Thus, based on the mean values of taper derived in this study the mid-point diameter of butt logs will be underestimated.

Table 7. Parameters of the linear regression models of log taper

Tree species	Intercept		Slope	
	estimate	SE	estimate	SE
Pine	0.915	0.020	–0.00281	0.00079
Oak	1.02	0.024	0.00575	0.00076

SE – standard error

Table 8. Parameters of the regression taper models (Equation 5) for butt logs

Tree species	Regression coefficients							
	a_0		a_1		a_2		a_3	
	estimate	SE	estimate	SE	estimate	SE	estimate	SE
Pine	0.071	0.016	0.128	0.008	0.995	0.018	2.220	0.020
Oak	0.367	0.018	0.087	0.006	1.21	0.018	1.770	0.027

a_0, a_1, a_2, a_3 – parameters; SE – standard error

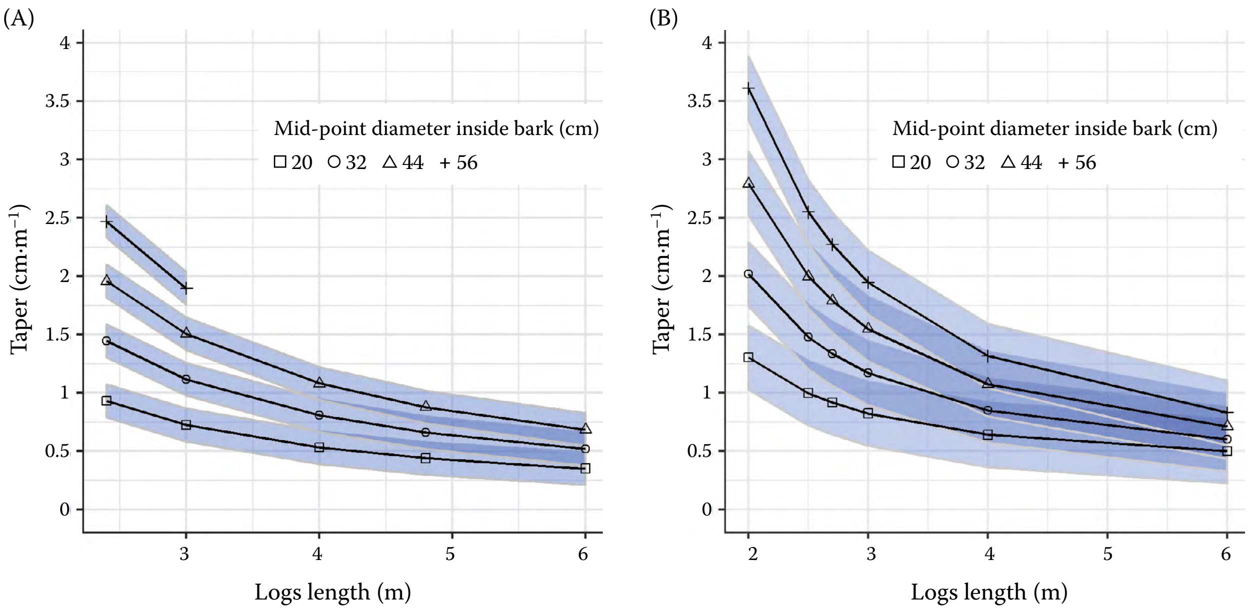

Figure 3. Taper of butt logs with confidence intervals: (A) pine; (B) oak

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DISCUSSION

Stem taper equations. This study presents a semi-empirical approach to estimate the taper of merchantable timber produced during harvesting operations. Specifically, the paper demonstrated how stem profile equations can be used to predict log taper. We used Newnham's (1992) variable-exponent stem taper equation to predict diameters inside bark along stem profiles. An advantage of this model is its ability to predict diameters along the tree stem using one relatively simple function with a changing exponent to describe the shape of neiloid, paraboloid and conus from ground to treetop. Newnham's (1992) taper function was previously used for different tree species, for example it provided higher predictive performance for Scots pine in Northern Britain (Fonweban et al. 2011).

One potential methodological problem of tree stem profile modelling is that the taper model is likely to exhibit multicollinearity. Due to multiple points where measurements of diameter are taken, data sets for modelling the tree stem taper contain repeated data. In that case, the statistical background for nonlinear least squares is violated, thus ordinary least-squares regression can result in the bias of parameter estimates. Many recent studies have proved the advantage of mixed-effect modelling to deal with the problem of highly correlated data (e.g., Yang et al. 2009; Beltran et al. 2017; Adamec et al. 2019). According to Burkhart and Tomé (2012), one of the solutions to overcome the problem of multicollinearity is to reduce the number of variables. In this regard used Newnham's (1992) equation potentially suffers less from the specified problem. It was also shown that multicollinearity and autocorrelation do not seriously affect the accuracy of variable-exponent models (Kozak 1997). Ultimately, we have demonstrated that modelling the stem taper using ordinary least-squares regression applied to relative values and relative diameters provides the reliable and accurate prediction of tree diameters. In addition, our results also provide evidence that Newnham's (1992) model is not biased at different relative heights of tree stems..

The taper models along with the analysis of tree stem measurement allowed depicting some features of pine and oak stem taper. Similarly to other studies (e.g. Rojo et al. 2005; Duan et al. 2016), our results indicate the lower model performance in a range of relative height values of 0–0.25h and 0.5–1h which for oak trees we refer to the crown development. However, improving the accuracy by introducing additional crown variables to the model (Li, Weiskittel 2010) was not a subject of the research. Along the butt section, the precision of diameter prediction was revealed to be higher for pine that also observed for many other coniferous tree species.

We believe that using *DBH* inside bark (instead of diameter outside bark) as a predictor variable in the taper equation provides more reliable estimates of diameters inside bark along tree stems and further calculation of timber taper. However, such approach is not applicable to predicting the merchantable volume of growing trees because of the need to measure bark thickness at breast height.

Timber taper. Tree stem profile equations can be useful for predicting diameters at any height, however their application to characterize a decrease of certain log diameters is limited because of variation in the taper along tree stems. Although taper equations are flexible, they cannot be used directly to estimate the mid-point diameter using top diameter measurement of logs unless the complete stem form analysis is performed. It is usually helpful for timber harvesters to have the average values of timber taper by tree species.

The distribution of different lengths and diameters of harvested timber contributes to the average log taper because the tree form is changing from the ground level to treetop. The structure of forest stands has an additional effect on timber taper since trees of different diameters at breast height have variable merchantability limits (i.e., merchantable height, stump height, log length etc.). We tried to combine all these factors using the most common logging scenarios to simulate the realistic structure of harvested wood and calculate the average timber taper. In contrast to similar studies (Larsen 2017) that used the simple taper equation our approach is more complex and provides necessary input data to further modelling.

We developed regression models that predict the taper of butt logs because its rates reach up to 2.5–3.5 times higher values than total means. Similar results were also previously shown for other tree species (Larsen 2017). In general, using average stem taper values and Huber's stem volume formula (Kershaw et al. 2016) based on log top diameter measurements in an electronic timber accounting system would potentially underestimate the round wood volume for butt logs.

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CONCLUSION

The aim of this study was to statistically justify the timber taper used to easily convert the round wood volume classification using mid-point or top diameter. Here we demonstrated an approach that uses tree stem profile modelling based on field measurements and simulation of the most likely scenarios of wood harvesting to estimate timber taper. The variable-exponent stem profile equation of Newnham (1992) performed well to predict diameters inside bark for given *DBH* and tree height. We also concluded that modelling the stem taper inside bark using relative values and height had a positive effect on the accuracy of stem diameter prediction. One should expect sufficient variation in timber taper because harvested wood comes from different stem zones which have a specific form. Thus, in a large volume of empirical data there is a need to get reliable estimates of average log taper. From this point of view, our approach is more effective since it uses the tree stem profile equation to derive all required information on timber characteristics and predict the taper.

Based on our study we concluded that the oak logs are characterized by slightly higher rates of taper than the pine ones. We also found that log taper varies along the stem zone. The higher values are observed for logs produced from the butt section of stems which can reach up to 3.5 times higher values than the mean values for tree species. Although we developed models to predict taper, for practical use we recommend the average values of taper.

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