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Determining the optimal selling time of cattle: A stochastic dynamic programming approach

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Abstract: The world meat market demands competitiveness, and optimal livestock replacement decisions can help to achieve this goal. In the article, there is introduced a novel discrete stochastic dynamic programming framework to support a manager's decision-making process of whether to sell or to keep fattening animals in the beef sector. In particular, the presented proposal uses a non-convex value function, combining both economic and biological variables, and involving uncertainty with regard to price fluctuations. The methodology is very general, so the practitioners can apply it in different regions around the world. There is illustrated the model convenience with an empirical application, finding that the methodology generates better results than actions based on the empirical experience.

Keywords: decision analysis, farm management, simulation

We introduce a discrete stochastic dynamic programming framework suited to supporting the optimal livestock replacement decisions. Specifically, we propose a stochastic non-convex value function, which implicitly depends on a profit function that involves both economic and biological variables and their interactions, and incorporates the selling price uncertainty. The main motivation in establishing this methodology is the scarce literature regarding formal procedures to address an important issue in the beef production, namely the optimal livestock replacement decisions (Frasier and Pfeiffer 1994), this being one of the most important factors affecting the farm profitability (Kalantari et al. 2010). Unfortunately, many livestock decisions are not based on the economic or financial data, but on the cattlemen's intuition (Glen 1987; Takahashi et al. 1997) as it was shown by Mourits et al. (2000), when they performed a study to determine the extent to which farmers use the pre-set targets and data monitoring to evaluate their results, finding that farmers are not familiar with applying the basic support tools as the record keeping and data monitoring.

Livestock should be replaced when the performance deteriorates. Performance is affected by age, growth rate, production, costs, prices, and conditions of nature, among other aspects. Evaluating the optimal factors in replacing a productive asset such as livestock involves understanding of the sequential nature of replacement decisions (Glen 1987), the biological and economic factors that affect these decisions, and the uncertainty that affects the future selling price realizations. Stochastic dynamic programming is an excellent technique that accommodates all these issues, however, despite the considerable potential of its application and the extensive literature around the general replacement problem, it has been little used for evaluating the livestock replacement, since the animal replacement problem differs in three main aspects from the general problem: (i) *uniformity,* since the traits of an animal are difficult to measure, (ii) *herd restrictions,* which are independent of the animal such as the limited capacity, availability and reproductive cycles, and (iii) *dimensionality of the problem,* because the higher is the number of traits, the higher is the complexity to compute the solution (Kristensen 1994).

Literature on optimal livestock actions can be divided into research focusing on optimizing the fattening strategies, the research looking for an economic basis on which to determine the optimal policies, and studies aiming to define the optimal fattening/replacement time. For optimizing the fattening strategies, Meyer and Newett (1970) proposed a deterministic methodology, based on a dynamic programming structure, to define the optimal food ration and selling time that would maximize profits for any type of cattle. Apland (1985) and García, Rodríguez, and Ruiz (1998) used linear programming to describe the impact of interest rates and diet on a herd's productivity, respectively, and Mourits, Huirne, Dijkhuizen, Kristensen and Galligan (1999a) developed a stochastic dynamic

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programming model using the hierarchic Markov process technique to model a wide variety of calves and to determine the optimal policy for dairy heifers.

Looking for an economic basis to determine the optimal policies, Bentley et al. (1976) used an expression to calculate the net expected revenue for specific periods of time using prices and costs, including probabilistic uncertainty concerning the asset's productivity due to mortality or infertility. Randela (2003) proposed a method to compute the average total value of an adult cow, which could be understood as the opportunity cost of replacing an animal, allowing farmers to determine the impact of mortality.

Different methodologies have been used to define the optimal times for the livestock replacement. Clark and Kumar (1978) proposed a deterministic dynamic programming model to define the optimal time for selling and buying beef cattle using prices and live weight, both variables depending on time and breed. Muftuoglu et al. (1980) and Göncü and Özkütük (2008) employed the least squares analysis to find the optimum culling age and weight. Frasier and Pfeiffer (1994) exploited the Markovian decision analysis with dynamic programming to find the optimal replacement time for cattle breeding according to the nutritional path. Kristensen (1994) presents a survey of dynamic programming applications applied to several animal replacement problems involving the lack of uniformity, diverse herd restrictions and a large state space, concluding that the Bayesian techniques can be used for solving the lack of uniformity, as well as reducing the state space without losing information minimizing the problem of dimensionality. Also Kristensen (1996) compared numerical methods for solving different versions of the animal replacement problem: the value function iteration, the policy function iteration, linear programming and the hierarchic Markov process, finding that the value function iteration only reaches the optimal solution under certain conditions, otherwise the results are a good approximation. The policy function is very good for small problems while the hierarchic Markov process works well in problems with a large state space. Takahashi et al. (1997) presented a new optimization method based on dynamic programming to establish the optimal policy for the herd shaping. Mourits et al. (1999b) performed a dynamic programming model to determine the optimal rearing policy of dairy heifers including the replacement decision and the sensibility analysis to deal with the scarcity of information. Arnade and Jones (2003) used the seemingly unrelated regression (SUR) together with dynamic programming to establish the cattle cycle. Kalantari et al. (2010) used the stochastic dynamic programming to define the optimal replacement policy for dairy herds using milk production, parity, and pregnancy status as the state variables to solve the problem. Yerturk et al. (2011) developed the analysis of variance (ANOVA) to describe the fattening performance.

Cattle raising is an old economic activity, disseminated worldwide, which consists of animal handling for productive purposes such as the milk and beef production. As meat has been considered the main source of protein for human nutrition (FAO 2012a), the livestock sector plays an important role in many economies in terms of producing food supplies, and generating employment and investment in different segments of the beef industry value chain (Ramírez 2013; Randela 2003). However, the world beef industry has shown decreasing rates in the last few decades (FAO 2012a; Schroeder and Graff 2000). Researchers hypothesize about the restructuring of the global meat consumption patterns (Galvis 2000). In fact, net returns for the beef cattle feeding have been volatile since the mid-1970s (Hertzler, 1988), and a significant decay in sales and the loss of the meat market share to poultry and pork has been demonstrated (Katz and Boland 2000). Nowadays, the world meat consumption configuration is 42% pork, 35% poultry, and 23% cattle (FAO 2012b).

The worldwide beef market suffers many pitfalls. First, the supply fluctuations, the volatility in prices (Glen 1987; Kalantari et al. 2010), and the foodborne illnesses attributed to the red meat (Katz and Boland 2000) caused that the consumers' preferences have shifted to other meat types (Galvis, 2000). Second, there is a separation between production and the processing processes in contrast to the substitute industries that are strongly integrated (Katz and Boland 2000). In particular, the asymmetry in the supply chain (Lafaurie 2011), the lack of coordination between production and commercialization (Schroeder and Graff 2000), and poor vertical integration (Galvis 2000) are crucial factors that must be addressed in the beef sector.

Third, cattlemen avoid the changes necessary to improve competitiveness due to the rigidity in regulations (Katz and Boland 2000), input prices, cost structures, volatile selling prices, and poor economic incentives (Kalantari et al. 2010). All these factors reduce their capacity to develop technical changes to increase

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efficiency (Galvis 2000). In addition, it is clear that the industry's dependence on natural conditions, the influence of the climate change, the interdependence with other human activities, and increasing requirements to become a global competitor, as well as the health requirements for the exportation of meat (Takahashi et al. 1997), demand a strong reorientation to achieve competitiveness (Crespi and Sexton 2005), to improve the flow of information (Schroeder and Graff 2000), to valorise whilst taking into account the value-generating factors (Scoones 1992) and to increase productivity.

In this dynamic and challenging competitive environment, proposing methodological approaches that can help to improve the performance of the beef sector is a valuable contribution from the economic and financial perspective.

THEORETICAL FRAMEWORK

Dynamic programming is a versatile optimization method developed by Bellman (1957), which uses the principle of optimality to reduce the number of calculations required to determine the optimal decision path (Kirk 1970). Bellman's principle of optimality postulates that:

> "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (Bellman 1957, p. 83).

The principle of optimality applies to problems characterized by an optimal substructure, that is, when a problem solution can be defined as a function of optimal solutions to minimize the size of sub-problems or problems with overlapping sub-problems, so the same problem is solved several times when a recursive solution arises. The idea behind the method is to find a functional form for each problem through the principle of optimality, thereby establishing a recurrence that generates an algorithm solving the problem. The recursive expression essentially converts a T-period problem into a two-period problem with the appropriate rewriting of the objective function. This expression is known as the value function and the mapping from the state to actions is summarized in the policy function.

For the purposes of the dynamic programming problem, it does not matter how the decision sequence was taken from the initial period; all that is important is that the agents are rational and act optimally in each period of time (Guerequeta and Vallecillo 1998). Indeed, the state variables summarize all the information from the past that is required to make a decision. The main features of the dynamic programming method are its sequential approach (Kristensen 1996), its versatility in modelling both continuous and discrete variables, and its capability to introduce uncertainty; this is the only general approach for the sequential optimization under randomness (Bertsekas 2005). As the livestock replacement problem can be represented as a multi-stage decision process involving uncertainty (Frasier and Pfeiffer 1994), dynamic programming is a natural modelling tool for solving it (Glen 1987).

Because complexities in finding a closed form solution are common in dynamic programming problems, numerical methods such as the value function iteration procedure, the policy function iteration method, and projection methods are used to solve them. The value function iteration procedure starts from the Bellman's equation and computes the value function by iterations on an initial guess; albeit slower than methods that operate on the policy function rather than the value function, it is trustworthy as it has been proved that under certain conditions – a continuous, bounded real-valued payoff and a continuous, compact non-empty constraint – there is a unique value function that solves the problem. Thus, the solution of the Bellman equation can be reached by iterating the value function starting from an arbitrary initial value (Stokey and Lucas 1989; Adda and Cooper 2003).

To compute the value function using this procedure, we must define functional forms and discretize state variables. In the case of the stochastic dynamic programming problems, the formulation of which includes expected values for the future, we can approximate an order one autoregressive random shock, which comes from a continuous distribution, to a discrete Markov chain using the technique presented by Tauchen (1986). This method simplifies computation of the expected values in the value function iteration framework and has the advantage that we can discretize before implementing the numerical method, avoiding the calculation of a cumbersome integral in each iteration.

Formulation of the model

Determining the optimal selling time for livestock is a basic problem that the farmers face. We define

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this as the time at which the farm managers maximize the net expected present value of financial profits associated with livestock management, $\Pi(q_t, p_t)$, where the state variables are q_t, the animal's weight (kilograms), and p_t, the price per kilogram (US dollars).

Specifically, at each point in time, the agent chooses whether to sell or to wait for another period. Given that this problem fits within the family of problems called the optimal stopping problems (Chow et al. 1971), we can describe it as a dynamic stochastic discrete choice problem, which can be expressed as a two-period problem using the Bellman's equation.

Formally, let $V(q_t, p_t)$ represent the value function of having an animal in state (q_t, p_t). We can express this as the maximum value between keeping the animal and selling it, and thus:

$$V(q_t, p_t) = \max\{V^k(q_t, p_t), V^s(q_t, p_t)\} \tag{1}$$

where, ($V^k(q_t, p_t)$ and $V^s(q_t, p_t)$ represent the value functions of *keeping* and *selling* the animal in state (q_t, p_t), respectively.

This problem has a non-convex value function, which is common in the economic applications but is unusual in the dynamic programming applications given the complexity of introducing it in the dynamic programming framework.

We define δ as the probability of death, $E[V(.|I_t)]$ as the expected value function conditioned by the information available in period I_t, and Π(.) as the present value of profit from selling the animal. Then, the value of keeping the animal is the expected value function of the next period conditioned on the available information at time *t*, multiplied by the survival probability. The value of selling the animal is the present value of the profit. Thus:

$$V^k(q_t, p_t) = (1 - \delta)E[V(q_{t+1}, p_{t+1}| I_t)] \tag{2}$$

$$V^s(q_t, p_t) = \Pi(q_t, p_t) \tag{3}$$

The net present value of profit at time *t* is the present value of income, discounted at the rate *r*, minus the initial inversion made when the producer bought the animal at $t = 0$, and the present value of the costs per kilogram earned in each keeping period. Hence:

$$\Pi(q_t, p_t) = \beta^t q_t p_t - q_0 p_0 - \sum_{s=1}^{t} \beta^s \tilde{c}(q_s - q_{s-1}) \tag{4}$$

where $\beta = (1 + r)^{-1}$ and $\tilde{c}$ *is* the average cost per kilogram.

Let a_t represent the age of the cattle; a_t is implicitly a control variable as it maintains a straight relation with the state variable weight, q_t, and the real control variable, which is the time an investor should keep the animal.

We assume that the weight of the cattle, q_t, is a function of the age and a Gaussian stochastic perturbation. We also introduce the square age to gather the concavity in the weight evolution. The empirical evidence suggests that animals gain more weight when they are calves.

In addition, we model the price per kilogram, p_t, as the product between two components. The first component is the expected price conditioned on the weight. The second component (u_t), is an autoregressive Gaussian process; this represents changes around the expected price. Modelling prices in a multiplicative form, rather than an additive form, simplifies the interpretation and the analysis of the price shocks. For instance, $u_t = 1$ implies a neutral situation. We introduce these shocks because prices are a source of uncertainty that affects the business profitability.

The functional forms that define the state variables q_t and p_t are:

$$q_t = \eta_1 a_t + \eta_2 a_t^2 + \varepsilon_t \tag{5}$$

$$p_t = E[\bar{p}_t | q_t] u_t \tag{6}$$

$$\bar{p}_t = \gamma_0 + \gamma_1 q_t + \gamma_2 q_t^2 + \epsilon_t \tag{7}$$

$$u_t = \mu(1 - \phi) + \phi u_{t-1} + \xi_t \tag{8}$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, $\epsilon_t \sim N(0, \sigma_\epsilon^2)$, and $\xi_t \sim N(0, \sigma_\xi^2)$.

EMPIRICAL APPLICATION

Estimation

To apply our methodological approach, we estimate Equation (5) using 24 representative fattening cattle that were weighed at different ages since they were weaned at the age of 10 months. This dataset comes from an extensive cattle farm, providing a sample size of 162 observations, meaning that the farmer weighed each animal approximately seven times. Also, we found that farm managers sold these animals at the weight of 440 kg in average. In addition, we use the average weight and market prices between October 2010 and May 2013 to estimate Equations (7) and (8).

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Table 1. Parameter estimates: age versus weight

Weight $q_t = \eta_1 a_t + \eta_2 a_t^2 + \varepsilon_t$		
Observations	162	
R^2	0.681	
Parameter	Value	Standard error[a]
η_1	26.43***	0.878
η_2	−0.34***	0.046

***Significant at the 0.01 level; [a]Robust standard errors

Table 1 shows the estimation results of Equation (5). The coefficients have the expected signs, gathering the concavity in age (we show the regression diagnostics in Appendix 1). Figure 1 shows the relation between age and weight for the representative animal; as we can see, the weight increases at a declining rate.

We obtain the parameters of price in two phases: in the first stage, we estimate Equation (7); then, we calculate u_t using Equation (6) to estimate an autoregressive model with drift (Equation (8)). Table 2 displays the estimation results. The coefficients are significant at the 0.05 level and correspond to those expected based on the theory (we show the regression diagnostics in Appendix 1).

Figure 2 exhibits the price prediction conditioned on weight. As we can see, the price per kilogram decreases at decreasing rates: as the animal weighs more, the marginal value for gaining 1 kilogram is lower; that is, the relative price per 1 kilogram is higher when the animal is younger.

We set the mortality rate at 2%, which is consistent with the empirical evidence for the livestock sector in the region (FEDEGAN 2006). The average cost per 1 kilogram of cattle weight in this farm is US $0.5. The monthly interest rate is equal to 1%, corresponding to the annual interest rate of 12.7%, which is the average annual interest rate for a credit loan in the country.

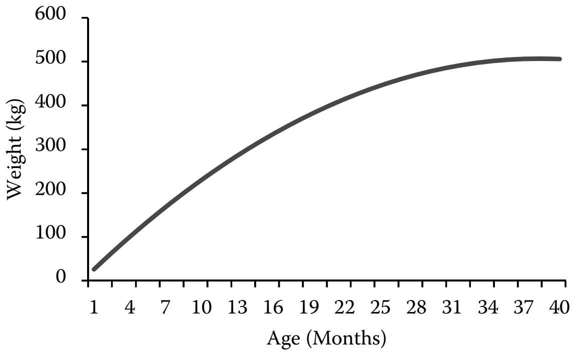

Figure 1. Average relation between age and weight

Table 2. Parameter estimates: price equations (US$/kg)

Price $p_t = E[\bar{p}_t \vert q_t] u_t$		
First stage $\bar{p}_t = \gamma_0 + \gamma_1 q_t + \gamma_2 q_t^2 + \epsilon_t$		
Observations	180	
R^2	0.250	
Parameter	Value	Standard error[a]
γ_0	1.7799***	0.0514
γ_1	−0.0014***	0.0003
γ_2	1.32×10^{-6}***	4.35×10^{-7}
Second stage $\frac{p_t}{E[\bar{p}_t \vert q_t]} = u_t = \mu(1-\phi) + \phi u_{t-1} + \xi_t$		
Observations	95	
R^2	0.122	
Parameter	Value	Standard error[a]
μ[b]	1.002***	0.007
ϕ	0.354***	0.099

***Significant at the 0.01 level; [a]Robust standard errors; [b]Do not reject the null hypothesis of $\mu = 1$ at the 0.05 level

Dynamic programming

We must use a numerical technique to approximate the solution because the problem presented in section Formulation of the model does not have a closed solution. This is a valid mechanism, as the problem fulfils the conditions to ensure that the value function can be achieved by iteration (that is, the operator T mapping from a guess concerning the value function to another value function, is a contracting mapping). Therefore, we implement the value function iteration procedure to compute the value function from the initial guess. To solve the dynamic problem using the value function iteration method, we follow four steps: first, the specification of functional forms; second, the discretization of both control and state variables; third, the computation of iterations and definition

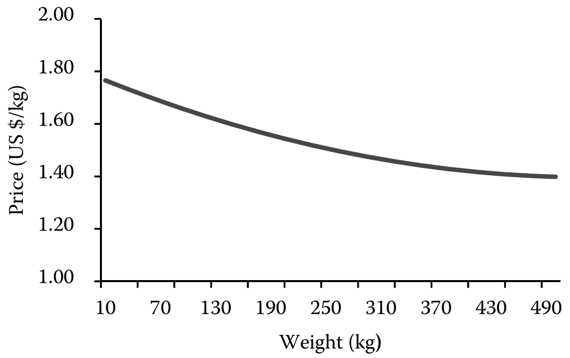

Figure 2. Average relation between price and weight

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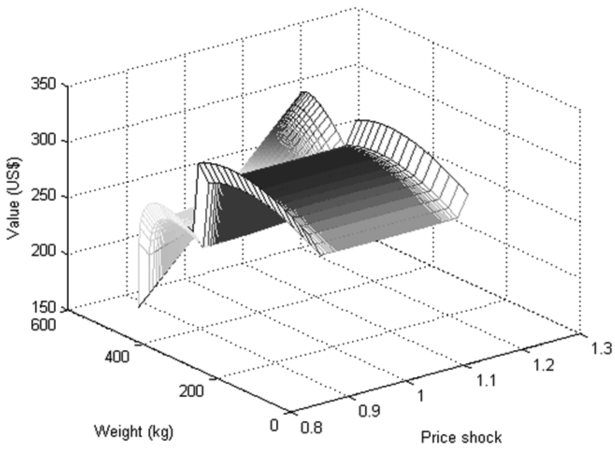


Figure 3. Value function

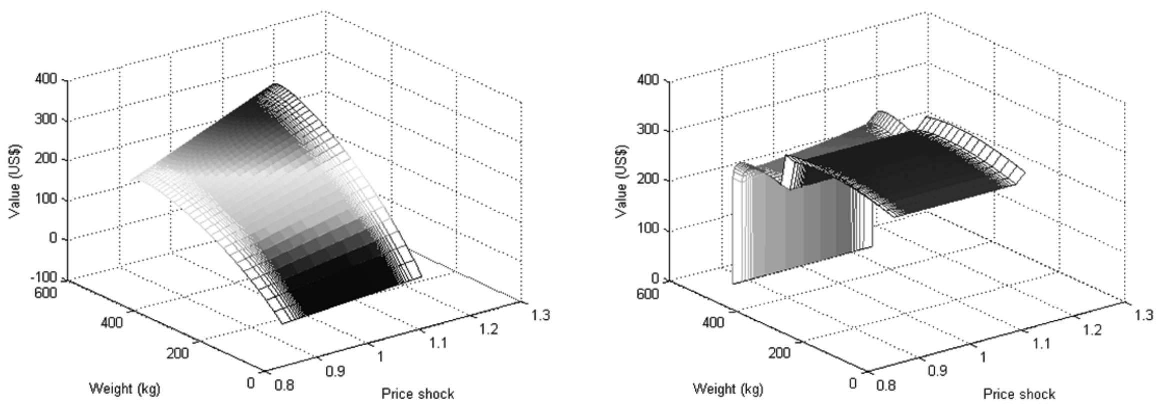
of the tolerance parameters; finally, the evaluation of the value and the policy functions.

We performed the first step in section Formulation of the model, in which we specified all the functional forms, including the payoff functions for selling and keeping the animal. To complete the second step, we discretize the control variable age a_t into 36 points, with each point representing 1 month; thus, the time horizon is set over three years, which is the maximum time that the animals stay on the farm in our study case. Taking the age discretization, we can discretize the weight and expected price through Equations (5) and (7). As the multiplicative random shocks of the price come from a continuous distribution that follows the Gaussian autoregressive process of order one with parameters (μ, ϕ, σ_ξ), we implement the Tauchen's (1986) procedure to avoid the calculation of an integral for the expected value function in each iteration. This method approximates the autoregressive process of order one using the Markov chain to create a discrete state space of the shock process, discretizing it into N optimal points and defining the transition matrix $\pi_{ij} = P[u_t = u_i | u_{t-1} = u_j]$ by calculating the transition probabilities between points. Therefore, the Markov chain mimics the autoregressive process (Tauchen 1986; Tauchen and Hussey 1991; Adda and Cooper 2003). We show the pseudo-code in Appendix 2.

We use the parameters given in this section to run the code. In addition, we discretize age and price shocks into 36 and 500 points, respectively. The simulation exercises show that the autoregressive process is well approximated and that 500 points are sufficient to reach the equilibrium point in the resulting value function. The method takes 21 iterations to converge to the value function V, which we present in Figure 3.

Figure 4 presents the selling and keeping value functions V^s and V^k. In the panel (a), we can see that when the animal weighs less, that is, when it is younger, the selling function is lower, even negative, meaning that the farm managers should wait for another period to sell. On the other hand, when there is a positive price shock ($u_t > 1$), the farmer should sell. We observe in the panel (b) the keeping value function. In particular, we observe that when the animal is younger, the keeping value function is higher, so the farmer should wait to sell.

The policy function defines whether the farmer should sell or wait at the time according to the cattle weight and the selling price features. Specifically, the policy function takes the value one if the selling value function is higher than the keeping value function. Figure 5 shows the policy function, from which we deduce that the investor should wait for a positive price shock and a weight of around 300 kg. However,



(a) Selling value function (b) Keeping value function

Figure 4. Selling and keeping value functions

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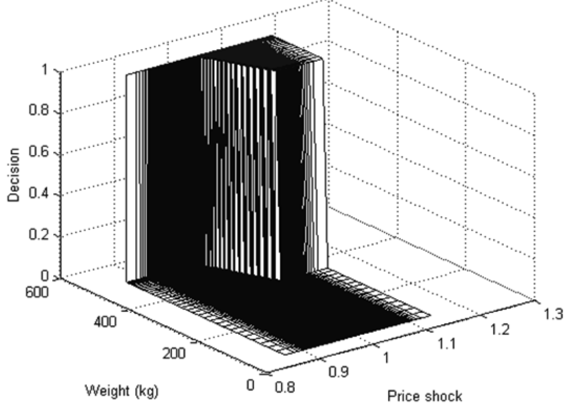

Figure 5. Policy function

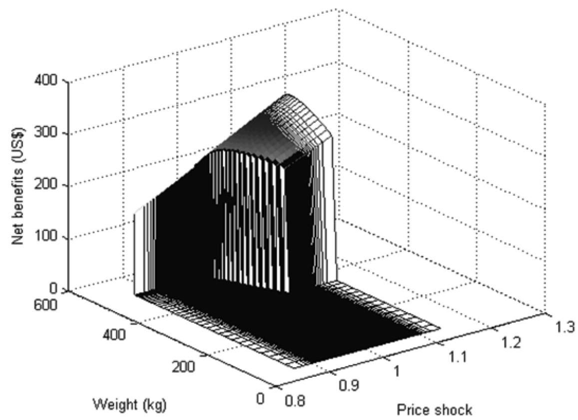

Figure 6. Value function if the animal is sold

if the animal weighs more than 500 kg, it is not necessary to wait for a favourable price shock to sell.

The value function is formed by blending both the selling and keeping value functions, taking the maximum of these at each point of the grid; that is, the value function represents the potential farmer's profit for each configuration of the state variables. However, it is important not to interpret the value function as the present value cash profits as there are some configurations of the state variables for which the value function denotes the expected profits of waiting for another period. The policy function allows us to determine where the value function actually displays the selling profits. Figure 6 displays the net present value of the farmer's profit, that is, the value function of selling the cattle.

Variable u_t is an unknown price shock that the investors cannot predict, so for the decision-making process, the managers will always expect that the shocks take the value of one, which is the mean or neutral situation. Table 3 summarizes the maximum value for each function, when $u_t = 1$. It is remarkable that the maximum found for the value function equals the maximum of the keeping value function although the maximum in the selling function is lower. This is explained by the fact that prices have a stochastic component and the calculation when the animal is younger generates expected values that are slightly higher than the real values once the animal gains weight.

In addition, we can see in this table that the present value of cash profits (US $238.98) is lower than the maximum obtained in other functions. This happens because the configuration that generates the highest value in the selling value function produces a higher value in the keeping value function. Thus, it is better for the owner to wait for another period in the hope of a positive price shock in the future, which will represent higher profits, but risking a negative price shock, which represents lower profits.

To summarize, a neutral price situation would imply that the managers should sell animals with the weight of 497.6 kg. This generates the maximum attainable present value of profit per 1 animal, i.e., US $238.98.

As stated above, the farm managers sell animals weighing 440 kg in our study case. In a neutral price scenario, this weight represents a net present value of US $235. This is close to the optimal strategy proposed in our framework (US $238.98), although we obtain a 1.7% higher net return using our proposal.

Let us analyse this 1.7% net return excess: It takes 32 months to achieve an animal weighing 497.6 kg,

Table 3. Maximum values and variable configuration: neutral price situation

Function	Maximum value (US $)	Variable configuration		
		age a_t (months)	weight q_t (kg)	price p_t (US $)
Selling – V_s	241.64	29	480.53	1.44
Keeping – V_k	295.29	12	268.20	1.51
Value – V	295.29	12	268.20	1.51
Value*	238.98	32	497.60	1.44

*Value function if the animal is sold

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while it takes 24.4 months to have an animal weighing 440 kg, that is, there is a difference of 7.6 months. This implies an annual net return excess equal to 2.69% ($(1 + 1.70\%)^{12/7.6}$). The total factor productivity growth for the last few years in the entire economy and the agricultural sector has been estimated at 1.4% and 1.1%, respectively (DNP 2011). Thus, we find that our methodological approach can generate significant improvements in competitiveness.

Stochastic discrete problems, such as the one that we present, have the feature that a threshold function, representing the point at which the decision of whether to sell or not is indifferent, can be computed. In the model, we can define the threshold p^* as the price at which the choice to sell or keep the animal is indifferent. Thus, if $p > p^*$, the policy function d takes the value of one, that is, the investor should sell.

We can calculate the threshold by equating V^s and V^k, and solving for p^* (Equation 9).

Figure 7 depicts the price threshold in a neutral situation. If the price is higher than the threshold given a weight qt, the investor should sell. For instance, if the price is higher than US$2.1 per kg for fattening animals that weigh 250 kg, the farm manager should sell those animals.

Finally, an important feature of the dynamic programming framework is its facility to simulate models using the policy function to determine the optimal choice for each period. Furthermore, when we can describe the problem as a stochastic discrete model, the simulations are simplified as the policy function is mapped using the threshold function. As a consequence, we can use simulations to describe the multiple agents' behaviour and the market's configuration patterns through time.

To perform model simulations representing a stock of S animals, we have to define a price shock for each animal at each point in time simulating the S autoregressive process. Then, we can calculate the selling price at each point in time by multiplying the shock and the expected price at that point. Thus, if the price is higher than the threshold, the farm managers should sell the animals of that specific weight. We use this framework to find the percentage of cattle at age a_t in the herd that farm managers should sell in a rational environment. Appendix 3 shows the pseudo-code.

Figure 8 illustrates our simulation exercise using a herd composed of S = 10 000 animals. We observe in this figure the percentage of sales according to weight. For example, our model predicts that in a rational market, 12% of the animals that weigh 351 kg or 30% of the animals that weigh 417 kg are sold in the market. In addition, we observe that the farm managers should sell 100% of the cattle weighing more than 510 kg. Finally, a clear consequence of our framework is that the farm managers should sell 50% of the livestock weighing 497.6 kg.

$$V^s(q_t, p_t) = V^k(q_t, p_t)$$

$$\Pi(q_t, p_t) = (1-\delta)E[V(q_{t+1}, p_{t+1}|I_t]$$

$$\beta^t q_t p_t - q_0 p_0 - \sum_{s=1}^{t} \beta^s \tilde{c}(q_s - q_{s-1}) = (1-\delta)E[V(q_{t+1}, p_{t+1}|I_t]$$

$$p^* = \frac{(1-\delta)E[V(q_{t+1}, p_{t+1}|I_t] + q_0 p_0 + \sum_{s=1}^{t} \beta^s \tilde{c}(q_s - q_{s-1})}{\beta^t q_t} \quad (9)$$

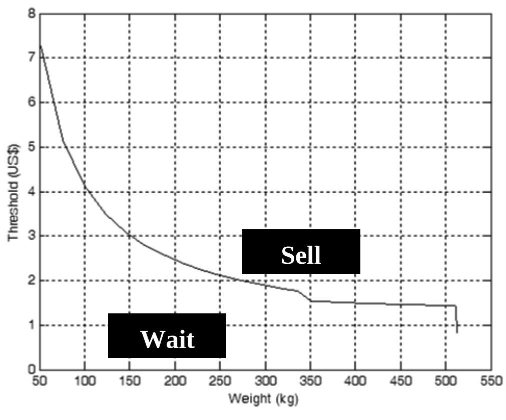

Figure 7. Price threshold

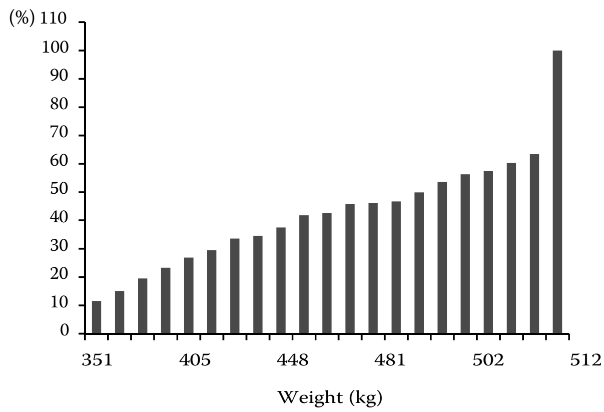

Figure 8. Simulated sales according to age

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CONCLUSIONS

We introduce a flexible stochastic dynamic program that allows the investor to support decisions concerning the best time to sell fattening cattle. Our proposal contains both economic and biological variables, and involves uncertainty derived from the future price realizations. This dynamic program makes it possible to find the optimal time by comparing financial outcomes rather than other biological or technical measurements that are common in the literature; thus it is easier to interpret the results as the financial profit, which is a classic figure that the investors use to evaluate investments. In addition, our proposal allows us to perform different simulation exercises to identify the livestock life cycles in the market, and the practitioners can use it in different regions by using the appropriated parameter estimates. Moreover, its economic and financial foundations, as well as its mathematical, statistical, and computational framework, can be used as a basis to model other economic sectors. Finally, we find in our study case that although the common sense and empirical experience are priceless assets, the techniques based on scientific principles can help to improve the level of competitiveness of the livestock sector.

The future work lies in improving our estimation strategy. In particular, we would like to estimate our model using the structure of our stochastic dynamic program. However, we require an excellent micro dataset, as well as a macro dataset, to achieve this objective. In particular, we need a detailed information of representative samples of herds in different regions of the country, and an aggregated information of the livestock's flow and stock at different ages. Unfortunately, we have not yet found such a resource.

Appendix 1. Statistical tests

	Equation	Jarque-Bera Normality Test	White's Heteroskedasticity Test
Weight	$q_t = \eta_1 a_t + \eta_2 a_t^2 + \varepsilon_t$	1.1 (0.578)*	3.77 (0.012)
Price	First component: $\bar{p}_t = \gamma_0 + \gamma_1 q_t + \gamma_2 q_t^2 + \epsilon_t$	320.74 (0.00)	3.51 (0.0319)
	Stochastic component: $u_t = \frac{p_t}{E[\bar{p}_t \mid q_t]} = \mu(1-\phi) + \phi u_{t-1} + \xi_t$	17.20 (0.00)	0.69 (0.504)*

p-value appears in parenthesis; *Do not reject null hypothesis

Appendix 2. Pseudo-code for the value function iteration method applied to the optimal selling time problem

```
optimalSellingTime()
  Define animal information
  Read p_0, a_t, t
  a_0 ← a_t + t
  Define parameters
  Read δ, r, c̃
  β ← (1 + r)^(-1)
  Initialize η_1, η_2, γ_0, γ_1, γ_2, N, μ, ϕ
  Discretize Variables
  Discretize AR u ← Tauchen procedure (N,μ, ϕ)
  Save probability transition matrix π
  Discretize Age a ← a_0: 1: a_0 + 36
  q ← η_1 a + η_2 a^2
  q_0 ← q(1)
  E[p̄|q] ← γ_0 + γ_1 q + γ_2 q^2
  p ← uE[p̄|q]
  Iterate Value Function
  Define maxIter, tol
  for i_q = 1 to size(a) − 1
      for i_p = 1 to size(u)
          t ← i_q
          Initialize V(i_q, i_p) ← β^t q(t)p(i_p, t) − q_0 p_0
      end for
  end for
  for i = 1 to maxIter
      for i_q = 1 to size(a) − 1
          for i_p = 1 to size(u)
              t ← i_q
              δ_q = q(t + 1) − q(t)
              c(t) ← β^t c̃
              sum_c(t) ← Σ_{s=1}^{t} c(s)
              V_s ← β^t q(t)p(i_p, t) − q_0 p_0 − sum_c(t)
              V_k(i_q, i_p) ← (1 − δ)π(i_p, :)V(i_q + 1, :)
              V_aux ← max(V_s, V_k)
          end for
      end for
      error ← max((V_aux − V)/V);
      if error < tol then break else V ← V_aux end if
  end for
  Calculate Policy Function
  Policy function d ← V_s > V_k
end optimalSellingTime
```

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Appendix 3. Pseudo-code for simulating sales behaviour applied to the optimal selling time problem

```
Simulations()
    Define information
    Define number of periods a
    Read threshold function given u = 1 T_{a×1}
    Read expected price E_{p_{a×1}}
    Define parameters
    Initialize number of simulations S
    Initialize AR Parameters μ, ϕ, σ_u
    Simulate AR
    Define Burn-in iterations B
    e_{(B+a)xS} ← generate shocks ~ N(0, σ_u^2)
    Initialize u(1,:) ← μ(1 − ϕ) + e(1,:)
    for t = 2:(B + a)
        for s = 1 to S
            u(t,s) ← μ(1 − ϕ) + ϕu(t − 1,s) + e(t,s)
        end for
    end for
    Drop B first simulations of u
    Simulate agent's behaviour
    for t = 1:a
        for s = 1 to S
            p(t,s) ← u(t,s)E_p(t)
            if p(t,s) ≥ T(t) → sell(t,s) = 1 else
sell(t,s) = 0 end if
            if sell(t,s) = 1 → Csell(t,s) = 1 else
Csell(t,s) = 0 end if
            if t > 1
              if Csell(t − 1,s) = 1 → Csell(t,s) = 1
end if
            end if
        end for
    end for
end Simulations
```


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