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The economic analysis of instrument variables estimation in dynamic optimal models with an application to the water consumption

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Abstract: This study examines one of the most important issues in water economic research, namely, the nexus between water consumption and economic growth. Water consumption is determined by the intersection of endogenous growth function and water consumption function, neither function can be consistently identified by comparing average quantities of water consumed at different values of observed real *per capita* output. The contribution of this study is an investigation of the endogenous nexus between economic output and water consumption. Water consumption function is derived using an optimal dynamic equilibrium model. Two instrument variable models are proposed with real *per capita* economic output specified as a function of institutional reform and urbanization, which are used to examine the nexus among water consumption, reform, urbanization, and economic growth in Guangzhou, China.

Keywords: agricultural population urbanization; China; generalized method of moments; endogenous economic growth; two-stage least squares

Econometrician's interest in the identification of supply and demand functions led to the development of instrument variable approaches for local average treatment effects (Angrist and Krueger 2001). The results of this research programme, starting with Angrist and Krueger (1991), are now part of econometric textbooks (Angrist and Pischke 2008). In natural resource economics, water consumption is determined by the intersection of endogenous growth function and water consumption function, neither function can be consistently identified by comparing average quantities of water consumed at different values of observed real *per capita* output. Specification requires the presence of separate instruments that shift either water consumption function or endogenous growth function but not both (Gao and He 2017). These results are typically presented in the context of linear regression models. Attempting to relax the assumptions that have no firm grounding in economic theory, empiricists have considered estimation in more specific models (Imbens and Angrist 1994). For example, Angrist and Imbens (1995) show that two stage least squares (TSLS) can also be used to estimate the average causal effect of variable treatments such as drug dosage, hours of exam preparation, cigarette smoking, and years of schooling. By providing context to the current applications, a better understanding of the applicability of these methods may arise (Imbens 2014).

Barbier (2004) provides strong support for the inverted-U relationship between economic growth and the rate of water use across countries. Barbier and Chaudhry (2014) show that higher water use and

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population growth are associated with *per capita* income increases in urban countries in the United States. Cazcarro et al. (2013) finds *per capita* income growth as the main factor driving increases in water consumption. In support of the growth hypothesis, Ngoran et al. (2016) suggests that economic growth in 38 Sub-Saharan African countries for the period 1980–2011 is driven mainly by water and labor.

In contrast to those efforts, Gleick (2003) reports support for the neutrality hypothesis with no relationship between national water consumption and *per capita* personal income. While some studies quantify links between water consumption GDP, most do not examine the causality between growth and water consumption, especially for metropolitan economies. Clear patterns may be elusive. The nexus between electricity consumption and economic output has been studied extensively, but the evidence is contradictory and inconclusive (Stern et al. 2018).

There has been some research that analyzes the nexus between water consumption and economic growth. Empirical evidence has been obtained indicating that national *per capita* water consumptive use follows an inverted-U shaped path, with respect to *per capita* income. That pattern is consistent with the well-known Kuznets curve type of water usage progression. Limited support has also been uncovered in favor of an Environmental Kuznets Curve (EKC). One example is provided by Katz (2015) wherein international cross-sectional data, plus panel data for OECD nations and U.S. states, indicates that water consumption initially rises and later declines as incomes increase. Recently, Sheng et al. (2020) measured costs of the two paths by shadow prices of water use and wastewater emissions, and then we built a non-parametric input–output model to produce the estimates in China.

The contribution of this study is an investigation of the endogenous nexus between economic output and water consumption. Water consumption function is derived using an optimal dynamic equilibrium model. Two basic questions are examined: (*i*) What is the relationship between water consumption and economic output? (*ii*) If water usage increases, does that lead to greater economic output? It is examined for Guangzhou, China, the second largest metropolitan economy in China.

To shed light on these questions, two instrument variable (IV) models are proposed with real *per capita* economic output specified as a function of institutional reform and urbanization. Based on that model, two stage least squares (2SLS) analysis, as well as generalized method of moments (GMM), is used to examine the nexus among water consumption, reform, urbanization, and economic growth in Guangzhou, China. This contrasts with most of the literature on water-output models, which casts important requirements in terms of unobservable functional-form-specific residuals. Our approach is similar to the causal inference to evaluation programs originating in the econometric literature on IV (Angrist et al. 2000).

METHODS

Optimal water consumption function. In this section, our objective is to derive the optimal water consumption function that must be satisfied for water allocation to be optimal, in the sense that the allocation maximizes a representative household's inter-temporal utility function (ITUF). We now assume that ITUF is utilitarian in form. Writing the ITUF in this form assumes that it is meaningful to refer to an aggregate level of utility for the representative household in each period. Then his inter-temporal utility is a function of these aggregates. Note that given that we are going to work with the inter-temporal utility function that aggregates utilities at different periods, it follows that we are assuming that utilities are cardinally measurable. The utilitarian inter-temporal ITUF can be expressed as below:

$$ITUF = \sum_{t=0}^{T} U\left(C_t\right)\left(\frac{1}{1+\theta}\right)^t \quad (1)$$

where:
θ – the utility discount rate (time discounting, for $\theta > 0$ as generally assumed means that future utility counts for less than, is "discounted" with respect to, the same quantity of present utility in obtaining a measure of total welfare over time);
C_t – real numeraire consumption at year *t*.

Now let us assume that utility in each period is a concave function of the level of consumption in that period. Notice that the utility function itself is not dependent upon time, so that the relationship between consumption and utility is the same in all periods. We assume the utility function is iso-elastic form with a constant parameter γ:

$$U\left(C_t\right) = \frac{C_t^{1-\gamma}}{1-\gamma} \text{for } \gamma > 0 \text{ and } \gamma \neq 1 \quad (2)$$

This form for the utility function is known as an iso-elastic utility function because the elasticity

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of marginal utility with respect to real numeraire consumption C_t is a constant, equal to the parameter γ.

For Equation (2):

$$U' = \frac{dU}{dC_t} = C_t^{-\gamma} \text{ and } U'' = \frac{d^2U}{dC_t^2} = -\gamma C_t^{-\gamma-1}.$$

And substituting in the definition for the elasticity of marginal utility as $-U''C_t / U'$ gives:

$$\gamma = -\frac{-\gamma C_t^{-\gamma-1} C_t}{C_t^{-\gamma}}.$$

For γ = 1, Equation (2) would give U as infinity for all C_t, which is why the restriction γ ≠ 1 appears.

For any dynasty, the optimization problem is expressed as below:

$$\max_{C_t} \sum_{t=0}^{T} \frac{C_t^{1-\gamma}}{1-\gamma}\left(\frac{1}{1+\theta}\right)^t \quad (3)$$

According to the choice variables, we should turn to pay attention to the constraints.

First, constraint on the numeraire consumption is based on the national income accounting identity:

$$Y_t = C_t + I_t \quad (4)$$

where:
Y_t – real total output;
I_t – real net investment at year *t*.

Actually, real net investment is the change in the stock of capital (K) or total capital expenditure minus depreciation of capital:

$$I_t = K_{t+1} - (1-\delta)K_t \quad (5)$$

Hence, plug Equation (5) into Equation (4), we obtain:

$$Y_t = C_t + K_{t+1} - K_t + \delta K_t \quad (6)$$

Writing this identity in continuous-time form we have:

$$I_t = K_{t+1} - K_t = Y_t - C_t - \delta K_t$$

where:
δ – depreciation rate.

Water is a critically important input for many types of industrial, commercial, and household production processes. Based on fairly recent studies involving resource economics (He et al. 2017), water consumption is included as a key input in the aggregate production function shown in Equation (7).

$$Y_t = Q(K_t, W_t, L_t, z_t) = AK_t^a W_t^b L_t^d \quad (7)$$

where:
A – technology advancement;
K_t – capital at year *t*;
L_t – labor at year *t*;
W_t – water consumption at year *t*;
$0 < a < 1, 0 < b < 1$, and $0 < d < 1$.

Again, aggregate C–D production function here is to help ensure well behaved solutions. Based on the optimal growth model, the real total output here represents the sum of market values of all final goods and services, which have been adjusted to inflation.

Therefore, the constraint for real numeraire consumption is: $C_t = Y_t - (K_{t+1} - K_t) - \delta K_t$. So, the change of capital stock can be derived from the Equation (6) and Equation (7):

$$K_{t+1} - K_t = AK_t^a W_t^b L_t^d e^{z_t} - C_t - \delta K_t \quad (8)$$

As we know, a second constraint comes from the capacity of water. The differential equation which describes the water table as a function of time is obtained by equating "race in" and "rate out" with the impact on the water table, namely:

$$S_{t+1} - S_t = R_t + (m-1)W_t \quad (9)$$

where:
R_t – natural recharge;
m – return flow coefficient.

Therefore, we have the following problem:

$$\max_{C_t, W_t} \sum_{t=0}^{T} \frac{C_t^{1-\gamma}}{1-\gamma}\left(\frac{1}{1+\theta}\right)^t$$

s.t. $K_{t+1} - K_t = AK_t^a W_t^b L_t^d e^{z_t} - C_t - \delta K_t$

$S_{t+1} - S_t = R_t + (m-1)W_t$

$K_0 = K^*$ and K_T is free

$S_0 = S^*$ and S_T is free

$W_t \geq 0$.

Hence, we form the current Hamiltonian function:

$$\mathcal{H}_C = \frac{C_t^{1-\gamma}}{1-\gamma} + \left(\frac{1}{1+\theta}\right)\varphi_{t+1}\left[AK_t^a W_t^b L_t^d e^{z_t} - C_t - \delta K_t\right] + \left(\frac{1}{1+\theta}\right)\tau_{t+1}\left[R_t + (m-1)W_t\right] \quad (10)$$

The first order conditions are as below:

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$$\frac{\partial\mathcal{H}_C}{\partial C_t} = C_t^{-\gamma} - \left(\frac{1}{1+\theta}\right)\varphi_{t+1} = 0 \quad (11)$$

$$\frac{\partial\mathcal{H}_C}{\partial W_t} = \left(\frac{1}{1+\theta}\right)\varphi_{t+1} bAK_t^a e^{z_t} W_t^{b-1} L_t^d + \\ + \left(\frac{1}{1+\theta}\right)\tau_{t+1}(m-1) = 0 \quad (12)$$

$$\frac{\partial\mathcal{H}_C}{\partial S_t} = 0 = -\left[\left(\frac{1}{1+\theta}\right)\tau_{t+1} - \tau_t\right] \quad (13)$$

$$\frac{\partial\mathcal{H}_C}{\partial K_t} = \left(\frac{1}{1+\theta}\right)\varphi_{t+1}\left[aAK_t^{a-1}W_t{}^b L_t^d e^{z_t} - \delta\right] = \\ = -\left[\left(\frac{1}{1+\theta}\right)\varphi_{t+1} - \varphi_t\right] \quad (14)$$

The solutions of the optimal control model above is as follow:

$$C_t = \left(\frac{1+\theta}{\varphi_{t+1}}\right)^{\frac{1}{\gamma}} \quad (15)$$

$$W_t^* = \frac{b\varphi_{t+1}^*}{\tau_{t+1}^*(1-m)} Y_t^* \quad (16)$$

$$\tau_t = (1+\theta)e^t \quad (17)$$

$$\varphi_t = \left(\frac{1+\theta}{1+\theta-\delta}\right)e^t \quad (18)$$

Plug Equation (15), and Equations (17–18) into Equation (16), we obtain the optimal water function:

$$W_t^* = \frac{b}{(1+\theta-\delta)(1-m)} Y_t^* \quad (19)$$

Following He et al. (2017), both sides of the aggregate production function are divided by labor force so that output, resource consumption, and the capital stock are expressed in *per capita* terms. Equation (20) shows the basic model that results from taking that step.

$$w_t(y) = Dy_t = f(y) \quad (20)$$

where:

$D = \frac{b}{(1+\theta-\delta)(1-m)}$;

y – real *per capita* gross product;

w – *per capita* water usage;

t – annual, time period.

In order to address the endogenous problem between w and y, we introduce institution (*reform*) and urbanization (u) as instrument variable. According to the urban growth theory (Black and Henderson 1999) and endogenous institutional growth theory (Pande and Udry 2009), we assume that:

$$y_t = g(reform, u) \quad (21)$$

So, in each of T periods, indexed by $t = 1, \ldots, T$, we postulate the existence of the water consumption function $w_t(y)$ and endogenous economic growth function $y_t(reform,\ u)$. The former describes the water quantity consumed in year t for all possible real *per capita* output y and the latter describes the real average output in year t for all possible values of generic instruments *reform* and u. Without essential loss of generality we could write both functions above in year t as a common function of real average output, instruments and stochastic, time-specific residuals ε_t^w and ε_t^y, such that $w_t(y) = w(y, \varepsilon_t^w)$ and $y_t(reform, u) = y(reform, u, \varepsilon_t^y)$.

So, there should be an optimal value for real *per capita* production (y_t^e) between these two functions, which implicitly defined that $w_t(y) = w(y, \varepsilon_t^w) = w(y_t^e) = w_t^e$ and $y_t(reform, u) = y(reform, u, \varepsilon_t^y) = y_t^e(reform, u)$.

Let us observe the quadruple:

$(reform_t, u_t, y_t^e, w_t^e, pr_t)$.

where:

$reform_t$ and u_t – the values of the instruments;

$y_t^e \equiv y_t^e(reform_t, u_t)$ – the optimal *per capita* output at the realized values of the instruments;

$w_t^e \equiv w_t^e(y_t^e(reform_t, u_t))$ – the water quantity used;

pr_t – the water popularization rate as a covariate, which affect both water consumption function and endogenous growth function.

Those two functions are presumed to satisfy the condition of continuous differentiability and are allowed to change freely from one year to the next according to some stochastic process. We assume their expected values, at fixed real *per capita* output and instruments and conditional on covariate, are defined:

$w(y \mid pr) \equiv E\left[w_t(y \mid pr_t = pr)\right]$, and

$y(reform, u \mid pr) \equiv E\left[y_t(reform, u \mid pr_t = pr)\right]$.

We also assume that these expected values are consistently estimable by averaging those functions at specific values for the instruments over time with identical values of the covariate. Similarly, population-average optimal water consumption and endogenous growth functions are defined as:

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$w^e(y \mid pr) \equiv E\left[w_t^e\left(ry \mid pr_t = pr\right)\right]$, and

$y^e(reform,u \mid pr) \equiv E\left[y_t^e\left(reform,u \mid pr_t = pr\right)\right]$.

Conditions for instruments

We begin by explain the reasons why reform and urbanization can be qualified as instruments. The first requirement for effective instrument variables is random assignment. It means that the water consumption function evaluated at y should be independent of the reform and urbanization and assumes that although quantity of water consumed may indirectly depend on the real *per capita* output and the institutional background, as well as the level of urbanization, the water consumption over the range of possible real *per capita* output and the status of institution and urbanization is independent of the value of the status of institution and urbanization actually realized. In other words, the reform and urbanization are as good as randomly assigned given covariate (water popularization rate). So, an implication of this requirement [**Requirement (1)**] is that:

$$reform_t, u_t \perp \left\{w_t^e(\mathrm{y}), \mathrm{y}_t^e(\mathrm{reform,u})\right\} \mid pr_t,$$

for all y, *reform* and u.

Because the potential values of the optimal real *per capita* output y_t^e are independent of the realized value of the instruments $reform_t$, u_t, the reduced form regression, that is, the average value of the optimal real *per capita* output as a function of reform and urbanization, can be estimated by averaging over time with $reform_t = reform$, $u_t = u$, and $pr_t = pr$:

$$\begin{aligned} y^e\left(reform,u|pr\right) &= E\left[y_t^e\left(reform,u\right)|pr_t = pr\right] = \\ &= E\left[y_t^e\left(reform,u\right)|reform_t = reform, u_t = u, pr_t = pr\right] = \\ &= E\left[y_t^e \mid reform_t = reform, u_t = u, pr_t = pr\right]. \end{aligned}$$

A similar result holds for equilibrium water consumption:

$$\begin{aligned} w^e\left(y \mid pr\right) &= E\left[w_t^e\left(y\right) \mid pr_t = pr\right] = \\ &= E\left[w_t^e\left(y\right) \mid reform_t = reform, u_t = u, pr_t = pr\right] = \\ &= E\left[w_t^e \mid reform_t = reform, u_t = u, pr_t = pr\right]. \end{aligned}$$

From this point on, we focus on the identification of the endogenous growth function. The argument for identification of the water consumption function is analogous. Therefore, the second requirement for instruments is the effect on real *per capita* output [**Requirement (2)**]:

$$\begin{aligned} &E\left[y \mid reform_t = 1, u_t = u, pr_t = pr\right] - \\ &- E\left[y \mid reform_t = 0, u_t = u, pr_t = pr\right] \neq 0. \end{aligned}$$

It implies that reform does not shift water consumption, any change in optimal real *per capita* output must come from an effect of reform on the endogenous growth function.

The final requirement for the instruments is the monotonicity of the optimal real *per capita* output in the endogenous growth function [**Requirement (3)**]:

$$\begin{aligned} &y_t^e\left(reform_t = 1, u_t = u, pr_t = pr\right) \geq \\ &\geq y_t^e\left(reform_t = 0, u_t = u, pr_t = pr\right). \end{aligned}$$

So, optimal water consumption function $\mathrm{w}_t^e\left(y, pr\right)$ is increasing in real *per capita* output, and for all water popularization rates:

$$\begin{aligned} &w_t^e\left(y_t^e\left(reform_t = 1, u_t = u\right), pr\right) \geq \\ &\geq w_t^e\left(y_t^e\left(reform_t = 0, u_t = u\right), pr\right). \end{aligned}$$

Specification of population average water consumption

Next, we use Requirements (1–3) to explain differences in average water quantities used and real *per capita* output at different values of the instruments. The following proposition presents the core idea.

Proposition (1). Assume Requirements (1–3) hold with $y_t^e\left(reform'\right) \geq y_t^e\left(reform\right)$. In addition let $g_t\left(\cdot\right)$ be any continuously differentiable function. Then the Wald estimator is:

$$\begin{aligned} &\rho^*\left(pr\right) = \\ &= \frac{E\left[w_t^e \middle| reform_t = 1, u, pr\right] - E\left[w_t^e \middle| reform_t = 0, u, pr\right]}{E\left[y_t^e \middle| reform_t = 1, u, pr\right] - E\left[y_t^e \middle| reform_t = 0, u, pr\right]} = \\ &= D \end{aligned} \quad (22)$$

Since the expected difference:

$$\begin{aligned} &E\left[g_t\left(y_t^e\left(reform'\right)\right) - g_t\left(y_t^e\left(reform\right)\right) \mid pr_t = pr\right] = \\ &= E\left[\int_{y_t^e(reform)}^{y_t^e(reform')} \frac{\partial g_t}{\partial y}\left(y\right) dy \mid pr_t = pr\right] = \\ &= E\left[\int_0^{\infty} \frac{\partial g_t}{\partial y}\left(y\right)\left\{y_t^e\left(reform\right) < y < y_t^e\left(reform'\right)\right\} dy \middle| pr_t = pr\right] = \end{aligned}$$

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$$=\int_0^\infty E[\frac{\partial g_t}{\partial y}(\mathrm{y})\{\mathrm{y}_t^e(reform)<y\left\langle \mathrm{y}_t^e(reform')\right| pr_t=pr\}dy=$$

$$=\int_0^\infty E[\frac{\partial g_t}{\partial y}(\mathrm{y})\,|\,\mathrm{y}_t^e(reform)<y<\mathrm{y}_t^e(reform'),pr_t=pr]\times \Pr[\mathrm{y}_t^e(reform)<y\left\langle \mathrm{y}_t^e(reform')\right| pr_t=pr]dy$$

So, if $g_t(y) = y$, we get:

$$y^e(reform',pr)-y^e(reform,pr)=E\left[y_t^e(reform')-y_t^e(reform)\,|\,pr_t=pr\right]=$$

$$=\int_0^\infty \Pr[y_t^e(reform)<y\left\langle y_t^e(reform')\right| pr_t=pr]dy$$

Again, if $g_t(y) = w_t(y)$, we get:

$$E\left[w_t(y_t^e(reform'))-w_t\left(y_t^e(reform)\right)\,|\,pr_t=pr\right]=$$

$$=\int_0^\infty E[\frac{\partial w_t}{\partial y}(y)\,|\,y_t^e(reform)<y<y_t^e(reform'),pr_t=pr]\times \Pr\left[y_t^e(reform)<y<y_t^e(reform')\,|\,pr_t=pr\right]dy$$

$$=\int_0^\infty E\left[D\,|\,y_t^e(reform)<y<y_t^e(reform'),pr_t=pr\right]\times \Pr\left[y_t^e(reform)<y<y_t^e(reform')\,|\,pr_t=pr\right]dy$$

$$=D\int_0^\infty \Pr[y_t^e(reform)<y\left\langle y_t^e(reform')\right| pr_t=pr]dy$$

Therefore, taking the ratio of these two differences gives:

$$\frac{E\left[\mathrm{w}_t^e\middle|reform_t=1,u,pr\right]-E\left[\mathrm{w}_t^e\middle|reform_t=0,u,pr\right]}{E\left[\mathrm{y}_t^e\middle|reform_t=1,u,pr\right]-E\left[\mathrm{y}_t^e\middle|reform_t=0,u,pr\right]}=D$$

This proposition says that institutional reform is as good as randomly assigned, affect the water consumption through a single known channel, has a first stage, and affect the causal channel of interest only in one direction can be used to estimate the average causal effect on the real *per capita* output. Thus, instrument estimand of institutional effect by using reform capture the impact on water quantities which were used because they were consumed under reform period but which would not otherwise have been consumed. This excludes water quantities which were exempted from pre-reform period. And the indirect effect of reform on water consumption holds constant.

Another important aspect of Propisition 1 is that it does not require additive residuals, and is therefore not tied to a specific functional form. To illustrate this point, consider the corresponding result in logarithms.

Let $g_t(y) = \ln w_t(y)$, we get:

$$E\left[\ln w_t(y_t^e(reform'))-\ln w_t\left(y_t^e(reform)\right)\,|\,pr_t=pr\right]=$$

$$=\int_0^\infty E[\frac{\partial \ln w_t}{\partial y}(y)\,|\,y_t^e(reform)<y<y_t^e(reform'),pr_t=pr]\times \Pr\left[y_t^e(reform)<y<y_t^e(reform')\,|\,pr_t=pr\right]dy$$

$$=\int_0^\infty E\left[\frac{1}{w_t}\frac{\partial w_t}{\partial y}\,|\,y_t^e(reform)<y<y_t^e(reform'),pr_t=pr\right]\times \Pr\left[y_t^e(reform)<y<y_t^e(reform')\,|\,pr_t=pr\right]dy$$

$$=\frac{1}{w_t}\frac{\partial w_t}{\partial y}\int_0^\infty \Pr[y_t^e(reform)<y\left\langle y_t^e(reform')\right| pr_t=pr]dy$$

And if $g_t(y) = \ln(y)$, we get:

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$$\ln y^e\left(reform',pr\right)-\ln y^e\left(reform,pr\right)=E\left[\ln y_t^e\left(reform'\right)-\ln y_t^e\left(reform\right)|\,pr_t=pr\right]=$$

$$=\int_0^\infty E[\frac{\partial g_t}{\partial y}\left(y\right)|\ln y_t^e\left(reform\right)<\ln y<\ln y_t^e\left(reform'\right),pr_t=pr]\times\Pr\left[\ln y_t^e\left(reform\right)<\ln y<\ln y_t^e\left(reform'\right)|\,pr_t=pr\right]dy$$

$$=\int_0^\infty E\left[\frac{1}{y}|\ln y_t^e\left(reform\right)<\ln y<\ln y_t^e\left(reform'\right),pr_t=pr\right]\times\Pr\left[\ln y_t^e\left(reform\right)<\ln y<\ln y_t^e\left(reform'\right)|\,pr_t=pr\right]dy$$

$$=\frac{1}{y}\int_0^\infty \Pr[\ln y_t^e\left(reform\right)<\ln y\left\langle\ln y_t^e\left(reform'\right)\right|pr_t=pr]dy$$

Then we have.

Proposition (2). Assume Requirements (1–3) hold. The Wald estimator in logarithms is the water consumption elasticity:

$$\tilde{\rho}\left(pr\right)=\frac{E\left[\ln w_t^e\,|\,reform_t=1,\ln u,pr\right]-E\left[\ln w_t^e\,|\,reform_t=0,\ln u,pr\right]}{E\left[\ln y_t^e\middle|reform_t=1,\ln u,pr\right]-E\left[\ln y_t^e\middle|reform_t=0,\ln u,pr\right]}=1 \tag{23}$$

It means if the water marginal/average consumption propensity is constant over time, then the water consumption elasticity is identical to one. So, Propisition 2 also provides a clear causal channel for the standard linear instrument estimand using a binary instrument.

Now, suppose that water consumption by the non-linear relationship, with the source of time-variation coming from an additive error term $\left(\varepsilon_t^w\right)$: $\ln w_t\left(y\right)=b_0+\rho\ln y+b_1\ln pr_t+\varepsilon_t^w$. In order to consider the case with discrete instruments, we transfer urbanization into natural logarithm form and then let {0, ..., *m*, ..., *n*, ..., 1} denote the set of possible values for the instruments $z_t=\left(reform_t,\ln u_t\right)$. Define the instrument variable estimator for each pair of instrument values as:

$$\rho_j=\frac{E\left[\ln w_t\middle|z_{tj},\ln pr\right]-E\left[\ln w_t\middle|z_{tj},\ln pr\right]}{E\left[\ln y_t\middle|z_{tj},\ln pr\right]-E\left[\ln y_t\middle|z_{tj},\ln pr\right]};\quad j=reform,\ln u.$$

The fitted values for endogenous growth function are:

$$\ln\hat{y}_t=a_0+\rho_{reform}reform_t+\rho_{\ln u}\ln u_t+a_1\ln pr_t+\varepsilon_t^y.$$

By the identification of instrument variable, the Wald estimator for $\ln w_t\left(y\right)=b_0+\rho\ln y+b_1\ln pr_t+\varepsilon_t^w$ is:

$$\rho=\frac{E\left[\ln w_t\,|\ln\hat{y}_t,\ln pr_t\right]-E\left[\ln w_t\,|\ln\hat{y}_t,\ln pr_t\right]}{E\left[\ln y_t\middle|\ln\hat{y}_t,\ln pr_t\right]-E\left[\ln y_t\middle|\ln\hat{y}_t,\ln pr_t\right]}=$$

$$=\rho_{reform}\frac{E\left[\ln w_t\,|\,reform_t=1,\ln pr_t\right]-E\left[\ln w_t\,|\,reform_t=0,\ln pr_t\right]}{E\left[\ln y_t\middle|\ln\hat{y}_t,\ln pr_t\right]-E\left[\ln y_t\middle|\ln\hat{y}_t,\ln pr_t\right]}+$$
$$+\rho_{\ln u}\frac{E\left[\ln w_t\,|\ln u_t=m,\ln pr_t\right]-E\left[\ln w_t\,|\ln u_t=n,\ln pr_t\right]}{E\left[\ln y_t\middle|\ln\hat{y}_t,\ln pr_t\right]-E\left[\ln y_t\middle|\ln\hat{y}_t,\ln pr_t\right]}$$

$$=\left\{\rho_{reform}\frac{E\left[\ln y_t\,|\,reform_t=1,\ln pr_t\right]-E\left[\ln y_t\,|\,reform_t=0,\ln pr_t\right]}{E\left[\ln y_t\middle|\ln\hat{y}_t,\ln pr_t\right]-E\left[\ln y_t\middle|\ln\hat{y}_t,\ln pr_t\right]}\right\}\times$$
$$\times\left\{\frac{E\left[\ln w_t\,|\,reform_t=1,\ln pr_t\right]-E\left[\ln w_t\,|\,reform_t=0,\ln pr_t\right]}{E\left[\ln y_t\,|\,reform_t=1,\ln pr_t\right]-E\left[\ln y_t\,|\,reform_t=0,\ln pr_t\right]}\right\}+$$

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$$+\left\{\rho_{\ln u}\frac{E\left[\ln y_t \mid \ln u_t = m, \ln pr_t\right] - E\left[\ln y_t \mid \ln u_t = n, \ln pr_t\right]}{E\left[\ln y_t \middle| \ln \hat{y}_t, \ln pr_t\right] - E\left[\ln y_t \middle| \ln \hat{y}_t, \ln pr_t\right]}\right\}\times$$
$$\times\left\{\frac{E\left[\ln w_t \mid \ln u_t = m, \ln pr_t\right] - E\left[\ln w_t \mid \ln u_t = n, \ln pr_t\right]}{E\left[\ln y_t \mid \ln u_t = m, \ln pr_t\right] - E\left[\ln y_t \mid \ln u_t = n, \ln pr_t\right]}\right\}.$$

Propisition 3. Assume Requirements (1–3) hold. The instrument estimand is:

$$\rho = \mu\rho_{reform} + (1-\mu)\rho_{\ln u} \tag{24}$$

where:

$$\mu = \frac{\rho_{reform}\left\{E\left[\ln y_t \mid reform_t = 1, \ln pr_t\right] - E\left[\ln y_t \mid reform_t = 0, \ln pr_t\right]\right\}}{\rho_{reform}\left\{E\left[\ln y_t \mid reform_t = 1, \ln pr_t\right] - E\left[\ln y_t \mid reform_t = 0, \ln pr_t\right]\right\} + \rho_{\ln u}\left\{E\left[\ln y_t \mid \ln u_t = m, \ln pr_t\right] - E\left[\ln y_t \mid \ln u_t = n, \ln pr_t\right]\right\}}.$$

We have shown that multiple instrument variables estimator of water consumption, is a weighted average of causal effects for reform and urbanization.

RESULTS AND DISCUSSION

In order to apply the propositions above into the estimation of water consumption function, we utilized time series data from Guangzhou, China. Annual data from 1949 to 2015 was obtained from the Guangzhou Statistical Yearbook (2016), within the Guangzhou Municipal Statistics Bureau. We conduct empirical research by Stata 15.

There is growing concern about the consumption of water worldwide, as demand grows and as supplies

Table 1. Variable definitions and summary statistics (T = 67)

Variable	ln*y*	*reform*	ln*w*	ln*u*	ln*pr*
Mean	9.7777	0.5373	3.6188	4.1266	4.4811
Maximum	12.2411	1.0000	5.1836	6.2347	4.6051
Minimum	7.8363	0. 0000	1.4365	3.8505	3.6296
Standard deviation	1.2022	0.5023	1.115993	0.3359	0.1909

T – observations; ln*w* – natural logarithmic of water consumption *per capita*; ln*u* – natural logarithmic of urbanization; ln*y* – natural logarithmic of real gross metropolitan product *per capita*; ln*pr* – natural logarithmic of water popularization rate; *reform* – economic institutional change after 1978, so it is identical one from 1979 to 2015; otherwise, it is zero
Source: : Own calculation using STATA

Table 2. Results of 2SLS and GMM estimates of annual water consumption function with reform and urbanization as instruments (dependent variable – ln*w*)

Variable	2SLS (1)	2SLS (2)	GMM (3)	GMM (4)
ln*y*	1.0500*** (0.0997)	0.8410*** (0.1298)	1.1187*** (0.1058)	0.9664*** (0.1376)
ln*pr*	–	1.3052** (0.6249)	–	0.8516 (0.5433)
Adj. *R*-squared	0.5991	0.7377	0.5473	0.6656

****P* < 0.01, ** *P* < 0.05, * *P* < 0.1; standard errors in parentheses; ln*y* – natural logarithmic of real gross metropolitan product *per capita*; ln*pr* – natural logarithmic of water popularization rate; adj. *R*-squared – a modified version of *R*-squ ared that has been adjusted for the number of predictors in the model; ln*w* – natural logarithmic of water consumption *per capita* 2SLS – two stage least squares analysis; GMM – generalized method of moments
Source: : Own calculation using STATA

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become more uncertain due to the potential effects of institutional change and urbanization. With rising *per capita* incomes and growing water popularization rate, human consumption of water is rising, just as demands for water for agriculture, recreation, and environmental habitats are increasing. At the same time, institutional change and urbanization are predicted to make precipitation more variable with the possibility of long-run economic growth periods. Table 1 lists the variables, and respective definitions, that included in the sample. Figures (1–4) show the increasing trends of (log) water consumption, (log) urbanization, (log) real GDP, and (log) water popularization rate in Guangzhou of China, from 1949 to 2015.

Table 2 reports two stage least squares (2SLS) and generalized method of moments (GMM) estimates of the relationship between log water consumption and *per capita* output with water popularization rate with reform and urbanization as instruments. These are estimates of the water consumption function and endogenous growth function for two instruments and water popularization rate as covariate. The binary instrument, *reform*, indicates the market system has been adopted by the entire society. A second instrument, ln*u*, indicates the ratio of population in urban area over total population.

The results of 2SLS (1) show that real *per capita* output is a statistically significant determinant of the water consumption *per capita* in Guangzhou, China. Real *per capita* output increases the quantity of water used. 2SLS (2) shows two-stage-least-squares estimates of ρ(*reform*, ln*u* , ln*pr*) for the same combinations of *re-*

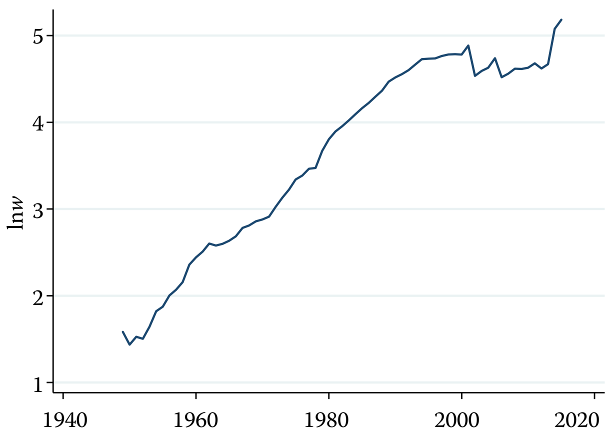

Figure 1. (Log) water consumption trend in Guangzhou (1949–2015)

ln*w* – natural logarithmic of water consumption *per capita*
Source: Own calculation using the graphics of STATA

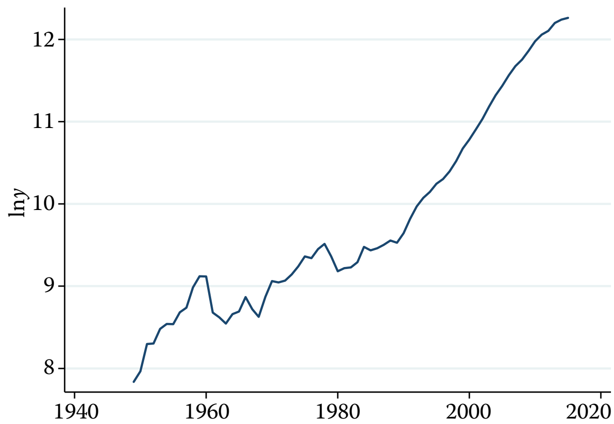

Figure 3. (Log) real GDP trend in Guangzhou (1949–2015)

ln*y* – natural logarithmic of real gross metropolitan product *per capita*
Source: Own calculation using the graphics of STATA

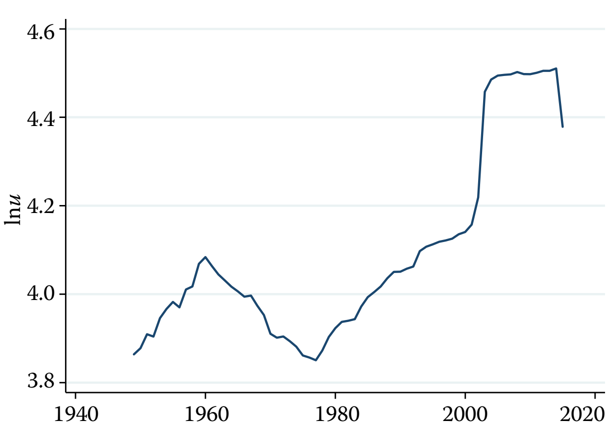

Figure 2. (Log) agricultural population urbanization trend in Guangzhou (1949–2015)

ln*u* – natural logarithmic of urbanization
Source: Own calculation using the graphics of STATA

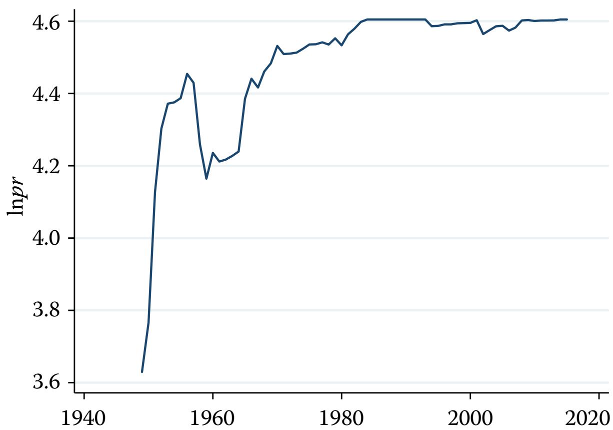

Figure 4. (Log) water popularization rate trend in Guangzhou (1949–2015)

ln*pr* – natural logarithmic of water popularization rate
Source: Own calculation using the graphics of STATA

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form and ln*u* as before. The water popularization rate being covariate also increases water consumption *per capita*, significantly.

GMM (1) presents generalized method of moments estimates using both the reform and urbanization instruments, without covariate. These estimates are close to those based in 2SLS (1), with slightly lower magnitude of coefficient. GMM (2) provides some evidence on the robustness of the results, as well. The estimated elasticity of the water consumption function in this case is appropriately one.

CONCLUSION

This paper investigates the relationship among water consumption, real output, reform, and urbanization in instrument estimation models. We show that under three requirements, instrumental variables methods can estimate weighted average causal effects of the water consumption function. Local average causal inference is over time and along the nonlinear water consumption function and endogenous growth function. The exact virtue of the average treatment effects depends on the nexus between the optimal output and institutional variable, as well as urbanization. The estimated weighted average treatment effect therefore depends on the instruments utilized. On the other hand, much can be learned about the range of real *per capita* output variation underlying each identification, and the popularization rate of water at each real *per capita* output. These results are obtained by formulating the optimal dynamic model in terms of water consumption at different outputs and instruments, rather than in terms of functional-form specific residuals. We illustrate the requirements and propositions by identifying the water consumption function from Guangzhou, China.

The key mechanism developed in the paper leads to potentially large differences in agricultural population urbanization in response to differences in water consumption. I also derived identifications of this mechanism for differences in degrees of water consumption during the process of agricultural population urbanization with different level of economic growth and the instrumental variable identification for endogenous economic growth. These specifications are consistent with a range of recent inferences for IV in the literature. It is also useful to note that while we have corrected the traditional model of economic growth, our results depend on the parameter of agricultural population urbanization.

This result is also helpful for metropolitan policy making in balancing the relationship between agricultural population urbanization, institutional change, water consumption, and economic growth. Especially, the experience of economic growth in Guangzhou provides evidence that GDP and water popularization generate more water consumption. So, the policy implication of that is if to improve the water consumption efficiency, the water consumption pattern should be changed into a pattern of low water intensity. The policy decision makers should: (*i*) set up R&D funds to drive up distributed water resource technology; (*ii*) implement subsides policy to develop water-saving business models; (*iii*) designing an efficient system of prices and regulated charges for water services; and (*iv*) enlarge the scale of investment on water utilities and development of water plants.

A number of areas are left for future research. These include, but are not limited to, the following. The model assumes that all activities are symmetric; an important extension is to see whether similar results hold with a more general water consumption function. Another area for future study is an investigation of the distance between consumer and water plant and the location of water plants. Finally, it is important to investigate whether the relationship between water consumption, agricultural population urbanization, and the economic growth is fundamentally different when we use alternative approaches to the causal inference, such as the geographic regression discontinuity approach.

REFERENCES

Angrist J.D., Pischke J.S. (2008): Mostly Harmless Econometrics : An Empiricist's Companion. New Jersey, United States, Princeton University Press: 113–218.

Angrist J.D., Graddy K., Imbens G.W. (2000): The interpretation of instrumental variables estimators in simultaneous equations models with an application to the demand for fish. Review of Economic Studies, 67: 499–527.

Angrist J.D., Imbens G.W. (1995): Two-stage least squares estimation of average causal effects in models with variable treatment intensity. Journal of the American Statistical Association, 90: 431–442.

Angrist J.D., Krueger A.B. (1991): Does compulsory school attendance affect schooling and earnings? The Quarterly Journal of Economics, 106: 979–1014.

Angrist J.D., Krueger A.B. (2001): Instrumental variables and the search for identification: From supply and demand to natural experiments. Journal of Economic Perspectives, 15: 69–85.

https://doi.org/10.17221/270/2020-AGRICECON

Barbier E.B. (2004): Water and economic growth. Economic Record, 80: 1–16.

Barbier E.B., Chaudhry A.M. (2014): Urban growth and water. Water Resources and Economics, 6: 1–17.

Black D., Henderson V. (1999): A theory of urban growth. Journal of Political Economy, 107: 252–84.

Cazcarro I., Duarte R., Sánchez-Chóliz J. (2013): Economic growth and the evolution of water consumption in Spain: A structural decomposition analysis. Ecological Economics, 96: 51–61.

Gao S., He Y. (2017): The effect of urbanization and economic performance on metropolitan water consumption : Theoretic model and evidence from Guangzhou of China. Applied Economics and Finance, 4: 163–71.

Gleick P.H. (2003): Water use. Annual Review of Environment and Resources, 28: 275–314.

Guangzhou Statistical Yearbook (2016): Guangzhou Statistical Yearbook. Guangzhou Municipal Statistics Bureau, China Statistics Press, Beijing, China: 10–201.

He Y., Fullerton T.M., Walke A.G. (2017): Electricity consumption and metropolitan economic performance in Guangzhou : 1950–2013. Energy Economics, 63: 154–160.

Imbens G.W. (2014): Instrumental variables: An econometrician's perspective. Statistical Science, 29: 323–58.

Imbens G.W., Angrist J.D. (1994): Identification and estimation of local average treatment effects. Econometrica, 62: 467–75.

Katz D. (2015): Water use and economic growth: Reconsidering the environmental Kuznets curve relationship. Journal of Cleaner Production, 88: 205–213.

Ngoran S.D., Xiong Z.X., Wesseh P.K. (2016): Signatures of water resources consumption on sustainable economic growth in Sub-Saharan African countries. International Journal of Sustainable Built Environment, 5: 114–122.

Sheng P., Dong Y., Vochozka M. (2020): Analysis of cost-effective methods to reduce industrial wastewater emissions in China. Water, 12: 1–13.

Stern D.I., Burkes P.J., Bruns S.B., Burke P.J. (2018): The impact of electricity on economic development: A macroeconomic perspective. International Review of Environmental and Resource Economics, 12: 85–127.

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