Field Evaluation of Various Mathematical Models for Furrow and Border Irrigation Systems

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Abstract: In this study, three mathematical models in the SIRMOD package including the hydrodynamic (HD), zero inertia (ZI), and kinematic wave (KW) models were tested using the data from several field experiments for both border and furrow irrigation systems. Five data sets for borders and seven data sets for furrows were used in this assessment. The results indicated that the performance of all models was satisfactory for the prediction of the advance and recession times. There was no difference in the prediction of the advance and recession times and infiltrated and runoff volumes between the hydrodynamic and zero-inertia approaches of the SIRMOD software. The HD, ZI, and KW models predicted the recession times better than the advance times for both the experimental borders and furrows. The predicted advance and recession times were estimated by these models more accurately than the infiltrated and runoff volumes. Also the accuracy of these models for the prediction of the advance and recession times was better for the experimental furrows in comparison with the experimental borders.

Keywords: border irrigation; furrow irrigation; mathematical models; SIRMOD

Iran has an arid and semi-arid climate with average annual rainfall of 240 mm. It has been reported that only 35% of the total water that is utilised for agriculture in Iran is used efficiently. The poor design, implementation, and management are generally responsible for insufficient irrigation, leading to the wastage of water, waterlogging, salinisation and pollution of surface water and groundwater resources. Considering surface irrigation that covers about 90% of the total irrigated land in Iran, an accurate and suitable design of the surface irrigation systems can save more water and increase the irrigated land area.

Surface-irrigation mathematical models are important for the evaluation and design purposes. Those models are classified into four main groups: (1) full hydrodynamic models; (2) zero-inertia models; (3) kinematic-wave models, and (4) volume balance models. The fully hydrodynamic model is the most complex and the most accurate. It is based on the complete Saint-Venant equations for the conservation of mass and momentum. The zero-inertia model is a slightly simplified version of the complete Saint-Venant equations that leaves out the acceleration or inertia terms in the momentum equation. The kinematic wave model uses further simplifications and uniform flow assumptions. The simplest model, i.e., one that involves the largest number of assumptions, is the volume balance model. It is based on the analytical or numerical solution of the temporally and spatially-lumped mass conservation, commonly referred to as the "volume balance" approach (Jurriens *et al.* 2001).

The data from the mathematical models have allowed engineers to improve systematically irrigation system design and operation which, for many years, have been mainly based on the rule of thumb, rough empirical guidelines, and approximations (Jurriens *et al.* 2001). Mathematical models for the design, operation, and evaluation of various surface irrigation methods have been used in user-friendly computer programs such as the SRFR (Strelkoff *et al.* 1998); SURDEV (Jurriens *et al.* 2001), and SIRMOD (Walker 1998). The SIRMOD software simulates the hydraulics of surface irrigation (border, basin, and furrow) at the field level. The simulation routine used in SIRMOD is based on the numerical solution of the Saint-Venant equations for the conservation of mass and momentum as described by Walker and Skogerboe (1987). The SIRMOD software includes the hydrodynamic, zero-inertia, and kinematic-wave models.

The objective of this study was to test and compare the three mathematical models in the SIRMOD package including the hydrodynamic, zero inertia, and kinematic wave models with field data. The ultimate goal was to determine the accuracy of these models for border irrigation as compared with furrow irrigation.

Models of surface irrigation

The mathematical models are based on the equations that describe the processes governing the overland flow and infiltration in surface irrigation. The hydrodynamic equations used in the mathematical models for describing the overland flow in surface irrigation are the equations of mass and momentum conservation, known as the Saint-Venant equations (Chow 1959; Strelkoff 1969). These equations, after the modification to include infiltration, are

$$\frac{\delta A}{\delta t} + \frac{\delta Q}{\delta x} + \frac{\delta Z}{\delta \tau} = 0 \tag{1}$$

$$\frac{1}{Ag}\frac{\delta Q}{\delta t} + \frac{2Q}{A^2 g}\frac{\delta Q}{\delta x} + \left(1 - \frac{Q^2 T}{A^3 g}\right)\frac{\delta y}{\delta x} - S_0 + S_\mathrm{f} = 0 \tag{2}$$

where:

y – depth of flow (m)
t – time from the beginning of irrigation (s)
τ – intake opportunity time (s)
Q – discharge (m^3/s)
x – distance along the field length (m)
Z – infiltration rate (m/s)
g – acceleration due to gravity (m/s^2)
S_0 – longitudinal slope of the field (m/m)
S_f – slope of energy grade line, also called friction slope (m/m)
A – cross-sectional area (m^2)
T – top width of flow (m)

The mathematical models differ mainly in terms of their solution techniques and assumptions used. The momentum equation is often simplified or in some cases ignored completely to reduce the computational difficulties. Depending upon the simplifying assumptions used, the models can be grouped in decreasing order of complexity into four subclasses: (i) hydrodynamic, (ii) zero-inertia, (iii) kinematic-wave, and (iv) volume balance.

Hydrodynamic models

The most complex and accurate is the full hydrodynamic numerical simulation model, which uses the full form of the Saint-Venant equations, i.e. both equations of mass and momentum conservation. These models, if properly implemented, should provide simulations that are more accurate over a wide range of field conditions when compared to the other mathematical models. Due to their accuracy, they are often used for thr calibration and evaluation of simpler models. The examples of hydrodynamic models include those developed by Haie (1984), Walker and Gichuki (1985), Strelkoff *et al.* (1998), and Bautista and Wallender (1992).

Zero-inertia models

The zero inertia models are a simplified form of the full hydrodynamic model without the acceleration and inertia terms. Strelkoff and Katopodes (1977) simplified the full hydrodynamic equations by neglecting the inertial terms in the Saint-Venant equations. If the inertia terms are neglected, Eq. (2) becomes

$$\frac{\delta y}{\delta x} = S_0 - S_\mathrm{f} \tag{3}$$

Equations (1) and (3) are parabolic, rather than hyperbolic, and the numerical solutions of the equations for these models are less complex than the full hydrodynamic models. Therefore, they require less computer time to simulate an irrigation event than does the hydrodynamic model. This approach was first used to model to flow in surface irrigation by Strelkoff (1972) and later by Katapodes (1974). However, the first operational model was reported by Strelkoff and Katopodes (1977) for borders and was later followed by thatfor furrow irrigation by Elliott *et al.* (1982), Oweis (1983), Rayej and Wallender (1985), among others.

Kinematic-wave models

The depth gradient of the flow ($\delta y/\delta x$) and inertial terms of the momentum equation (Eq. (2)) are often small in comparison with those of the bottom and friction slopes. Therefore, Eq. (2) can be further simplified by assuming that the depth gradient and inertial terms are negligible and thus becomes

$$S_0 = S_f \tag{4}$$

This assumption shows that the depth of flow at a point along the field is uniform. This approximation greatly simplifies the mathematical solution of the momentum equation. The approximation limits the application of the kinematic-wave models to freely draining sloping field conditions. In general, the kinematic-wave models are simpler and take less computer time than the hydrodynamic and zero-inertia models. The kinematic-wave approach has been used by a number of investigators to develop models of border and furrow irrigation systems (Walker & Humpherys 1983; Rayej & Wallender 1988; Esfandiari & Maheshvari 2001; Abbasi *et al.* 2003).

Volume-balance models

The volume balance is applied primarily onto the advance phase, and can be written for the border, basin, or furrow conditions. As the solution of full hydrodynamic equations is possible only with the numerical techniques using computers, some early studies on surface irrigation modelling focused on providing analytical solutions of the flow problem. The momentum equation was therefore completely neglected. The models based on this simplification were called volume-balance models and are based on the principle of mass conservation and on the assumption of normal flow depth at the upstream end. Water-front advance can be predicted by the volume balance approach in border and furrow using the following equation:

$$Q_0 t_x = \int_0^x A(x,t)dx + \int_0^x Z(x,t)dx \tag{5}$$

where:

Q_0 – flow rate at the inlet boundary

t_x – time of advance

$A\ (x, t)$ – cross-sectional area of the surface flow, variable with distance (x) and time (t)

$Z\ (x, t)$ – cross-sectional area of the infiltrated water, variable with distance and time

Both kinematic wave and volume-balance approaches are limited to sloping and free-draining systems.

SIRMOD software

The SIRMOD model uses three approaches, viz., the full hydrodynamic (HD), zero-inertia (ZI) and kinematic-wave (KW) to simulate the hydraulics of surface irrigation (border, furrow and basin) on the field scale and helps in the evaluation of alternative field layouts, i.e. field length, slope and management practices like water application rates and cut-off times (Walker 1998). It presents a simplified field design module and a "two-point" solution for the calculation of the infiltration parameters from the irrigation advance data. The software allows the user to specify furrow, border, or basin configurations with free-draining or blocked downstream boundary conditions under continuous or surged flow regimes and cutback options. The input data requirements for the simulation component include the field length, slope, infiltration characteristics, and advance data, target application depth, water application rate, Manning's resistance, and furrow geometry. The output includes a detailed advance–recession trajectory, the distribution of infiltrated water, volume balance, runoff hydrograph, depth of water flow at the end of the field, application and requirement efficiencies, and distribution uniformities.

Model verification

Input data. Generally, the numerical models are verified by comparing the model predictions with field measured data. In this study, the results of the various models in SIRMOD package were compared with the observed data filed. Five data sets for bor-

Table 1. Model input parameters of the experimental borders used for assessment of the performance of the various simulation models

Parameters	Data series				
	B1	B2	B3	B4	B5
Inflow rate, q_o (l/s)	1.28	1.35	1.50	1.37	0.98
Field length, L (m)	180	180	110	147	147
Border width (m)	8	8	8	8	8
Field slope, S_o (m/m)	0.00820	0.00820	0.00086	0.00156	0.00156
Manning's n ($m^{1/6}$)	0.040	0.040	0.063	0.049	0.049
Time of cut-off, T_{co} (min)	152	390	178	137	205
Kostiakov-Lewis parameters					
k (m/min)	0.0091	0.0098	0.0090	0.0016	0.0063
a (–)	0.211	0.302	0.210	0.143	0.336
f_o (m/min)	0.00022	0.00022	0.00013	0.00013	0.00013

ders and seven data sets for furrows were used in this assessment. All border data were derived from the studies of Abbasi (1994), whereas for the furrow data sets, three data sets (F2, F3 and F4) and four data sets (F1, F5, F6 and F7) were derived by Abbasi *et al.* (1999, 2008), respectively. The input parameters of SIRMOD are summarised in Tables 1 and 2 for the experimental borders and furrows, respectively.

All border experiments were conducted on bare soil (silty clay loam) under the free-draining condition. The F1, F2, F3, F4, and F5 data series were

Table 2. Model input parameters of the experimental furrows used for assessment of the performance of the various simulation models

Parameters	Data series						
	F1	F2	F3	F4	F5	F6[e]	F7[e]
Inflow rate, q_o (l/s)	1.30	1.30	1.25	0.77 [a]	1.00 [b]	1.15 [c]	1.04[d]
Field length, L (m)	200	160	160	250	200	200	200
Field slope, S_o (m/m)	0.0175	0.0080	0.0061	0.0064	0.0175	0.0176	0.0177
Manning's n ($m^{1/6}$)	0.03	0.03	0.03	0.04	0.03	0.03	0.03
Furrow spacing (m)	0.75	0.75	0.75	0.75	0.75	0.75	0.75
Furrow section parameters							
ρ_1 ($m_2^{3.33-2\rho}$)[f]	0.42	0.48	0.44	0.31	0.41	0.37	0.43
ρ_2 (–)[f]	2.79	2.84	2.77	2.74	2.79	2.78	2.80
Time of cut-off, T_{co} (min)	120	120	120	250	120	120	120
Kostiakov-Lewis parameters							
k (m^3/min/m)	0.0051	0.0081	0.0037	0.0029	0.0037	0.0024	0.0036
a (–)	0.455	0.136	0.171	0.219	0.254	0.421	0.309
f_o (m^3/min/m)	0.000075	0.000096	0.000090	0.000090	0.000075	0.000075	0.000075

[a, b, c, d] After completing the advance phase, decreased to 0.37, 0.64, 0.51 and 0.48 l/s, respectively; [e] blocked end furrow; [f] the furrow section parameters as $A^2R^{4/3} = \rho_1 A_2^{\rho}$

under the free- draining condition, whereas F6 and F7 data series were conducted under blocked end condition. The soil types of the three data sets (F2, F3, and F4) and the four data sets (F1, F5, F6, and F7) were silty clay loam and clay silty loam, respectively. The experimental borders and furrows were marked and water advance and recession times were taken. The inflow and outflow rates (for each border and furrow separately) were measured by Washington State College (WSC) flumes type IV installed at the inlet and outlet of the fields. In this study, for both borders and furrows, the average basic infiltration rate, f_o, was determined by the inflow-outflow method (Elliott & Walker 1982). The two-point method (Elliott & Walker 1982) was also used to determine the Kostiakov-Lewis parameters, coefficients *a* and *k*. The Manning's *n* was assumed to be 0.04 for B1, B2 and F4 data sets, 0.03 for F1, F2, F3, F5, and F6 data sets, according to the Soil Conservation Service (SCS) recommendations for the bare soils and it was also measured for B3, B4 and B5 data sets in the field (Abbasi 1994, 1998; Abbasi *et al.* 2008).

Evaluation of the models performance

In this study, hydrodynamic, zero inertia and kinematic wave models (in the SIRMOD package) were run with the input data for both border and furrow irrigation systems. The outputs of the models included the advance and recession curves and total infiltrated and runoff volumes were compared with the field data.

To evaluate the suitability of the surface irrigation models, three criteria were chosen to analyse the degree of the goodness of fit. These criteria can be defined as follows:

(1) The coefficient of determination (R^2)

$$R^2 = \frac{\left[\sum_{i=1}^{n}(O_i - \overline{O})(P_i - \overline{P})\right]^2}{\sum_{i=1}^{n}(O_i - \overline{O})^2 \times \sum_{i=1}^{n}(P_i - \overline{P})^2} \qquad (6)$$

The value of R^2 ranges from 0.0 to 1.0, indicating a better agreement for the values close to 1.0.

(2) Root Mean Square Error (RMSE)

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(O_i - P_i)^2}{n}} \qquad (7)$$

The RMSE has minimum value of 0.0, with a better agreement close to 0.

(3) Standard error (SE)

$$\text{SE} = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[O_i - P_i\right]^2}}{\overline{P}} \qquad (8)$$

where:

n – number of observations
O_i – i^{th} value of the observed measurement
P_{i} – i^{th} value of the predicted measurement
$\overline{O}$ – mean of the observed values
$\overline{P}$ – mean of the predicted values

The better is the fit, the closer is SE to zero.

These indices were used for the advance and recession data.

RESULTS AND DISCUSSION

The results predicted by the various models in the SIRMOD were compared with the field measured data and are presented in Figures 1 and 2 for the experimental borders and furrows, respectively. The hydrodynamic, zero inertia, and kinematic wave models in the SIRMOD are presented in those figures as HD, ZI, and KW, respectively.

The KW model cannot simulate the blocked-end condition. Due to this, the results of KW model for F6 and F7 data series were not presented in Figure 2. The HD and ZI models simulated the blocked-end condition well but these models predicted the recession times in the final ten percents of the furrow length very badly, so the predicted recession times were several times longer than the recession times measured (for example, predicted by HD model, the recession time in the downstream end of the furrow length was equal to 1008 minutes whereas its measured value was equal to 136 minutes for F7 data series). Thus the recession times were presented only for the first ninety percent of the furrow lengths for F6 and F7 data series in Figure 2.

Figure 1 indicates a good fit of the observed and predicted values of the advance and recession times over the entire length of the borders. An excellent agreement exists between both the measured data and the simulated advance and recession times in the length of furrows (Figure 2). In a few cases, the models slightly overestimated or underestimated the recession times. For instance, there are overes-

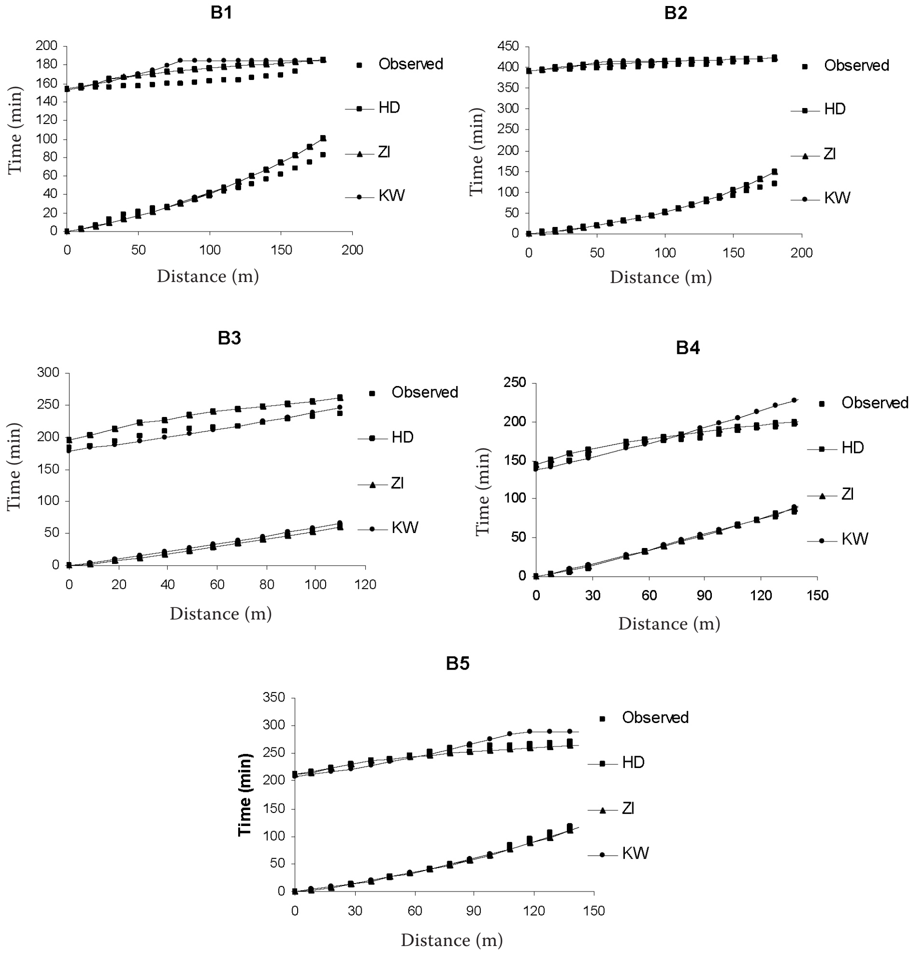

Figure 1. Predicted and observed advance and recession trajectories for different experimental borders

timations and underestimations for B1 and F3 data series, respectively. This was likely because of the difficulties in accurately observing the recession curve and the variability of the surface roughness and infiltration properties.

The plots of the observed and predicted advance and recession times for all the data series for HD, ZI, and KW models performances are given in Figures 3 and 4 for the experimental borders and furrows, respectively. The regression lines are fitted in these figures. The slope of linear equations ($y = ax$) is close to one that is like 1:1 line indicating a good performance for each of the three models (Figures 3 and 4). All the models predicted the advance and recession times well. The values of R^2, RMSE, and SE for the various models indicate a very good fit of the observed and predicted values of advance and recession times (Figures 3 and 4).

Table 3. Comparison of various models in terms of estimating infiltrated water and runoff volumes for different experimental borders (m^3/m)

Data	Measured			Hydrodynamic		Zero inertia		Kinematic wave	
	inflow	runoff	infiltration	runoff	infiltration	runoff	infiltration	runoff	infiltration
B1	11.67	5.64	6.03	1.83	9.84	1.83	9.84	1.84	9.83
B2	31.59	8.56	23.03	7.08	24.51	7.08	24.51	7.09	24.50
B3	16.02	8.42	7.60	10.00	6.02	10.00	6.02	9.37	6.65
B4	10.45	3.24	7.21	3.78	6.67	3.78	6.67	3.89	6.56
B5	12.05	3.48	8.57	1.41	10.64	1.41	10.64	1.49	10.57

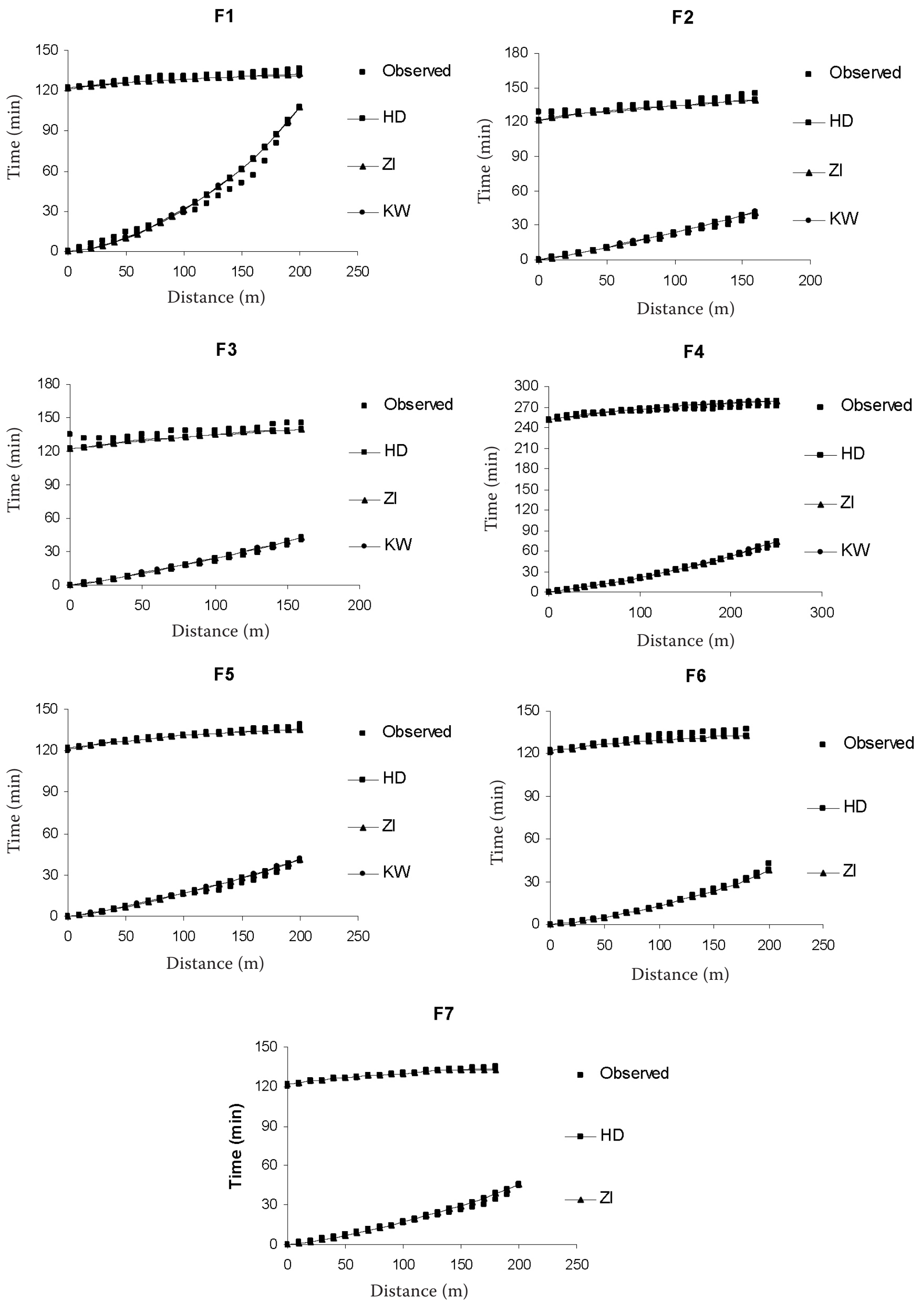

Figure 2. Predicted and observed advance and recession trajectories for different experimental furrows

The coefficients of determination of the three models are almost the same and equal to 0.97 and 0.99 for the prediction of advance and recession times for the experimental borders, respectively. These values for the experimental furrows are 0.99 and 1.00. Being high, the coefficient of determination shows a very good correlation between the predicted and measured values of the advance and recession times.

To predict the advance times, the values of RMSE for the HD, ZI, and KW models were 6.67, 6.70, and 6.98 min for the borders and 3.02, 3.04 and 3.53 min for the furrows, SE 0.148, 0.149, and 0.152 for the borders and 0.130, 0.130, and 0.135 for the furrows, respectively. With respect to SE values, the accuracy of the various models for predicting the advance time is more satisfactory for the furrows as compared with the borders.

For predicting the recession time, the values of RMSE for the HD, ZI, and KW models were 11.48, 11.49, and 12.71 min for the borders and 3.49, 3.51

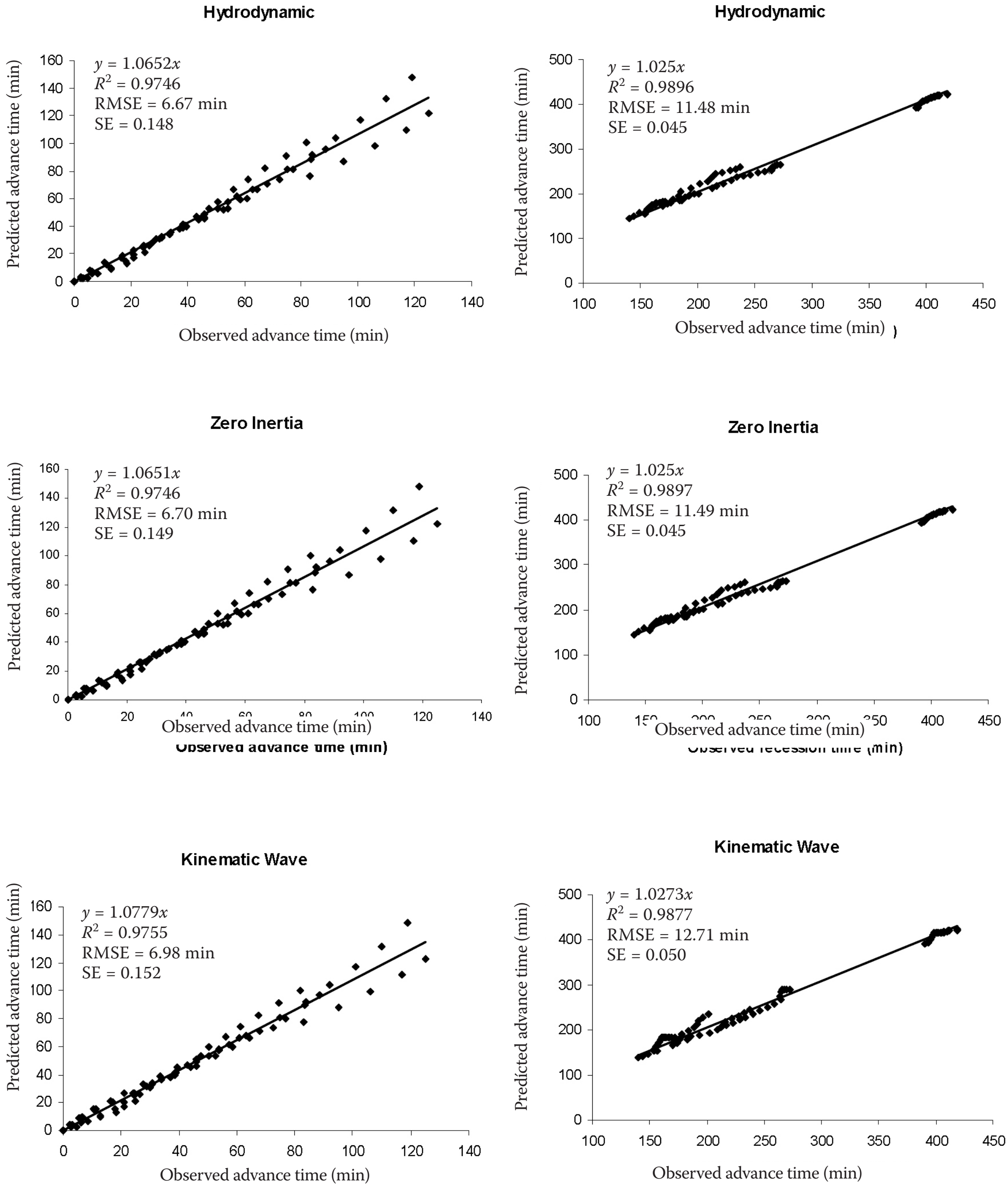

Figure 3. Observed versus predicted advance and recession times for the total data of borders with the Hydrodynamic, Zero inertia, and Kinematic wave models

and 4.22 min for the furrows, SE 0.045, 0.045, and 0.050 for the borders and 0.022, 0.023, and 0.025 for the furrows, respectively. This indicates a very good fit of the observed and predicted values of the recession time. As for the advance phase, there was no difference in the prediction of the recession time with the HD and ZI models of the SIRMOD software. All the models predicted the recession times for the furrows better than for the borders.

In the past, studies on border irrigation by Maheshwari and McMahon (1993) suggested that various models of surface irrigation predict the advance time better than the recession time. On the contrary, Esfandiari and Maheshwari (2001) showed that the models predicted the recession time better than the advance time in furrow irrigation. They claimed that the recession time in furrow irrigation is relatively easy to detect due to a short width of the furrow cross-section (< 1.5 m) compared to a large border width (up to 80 m). In the present study, HD, ZI, and KW models predicted the recession times better than

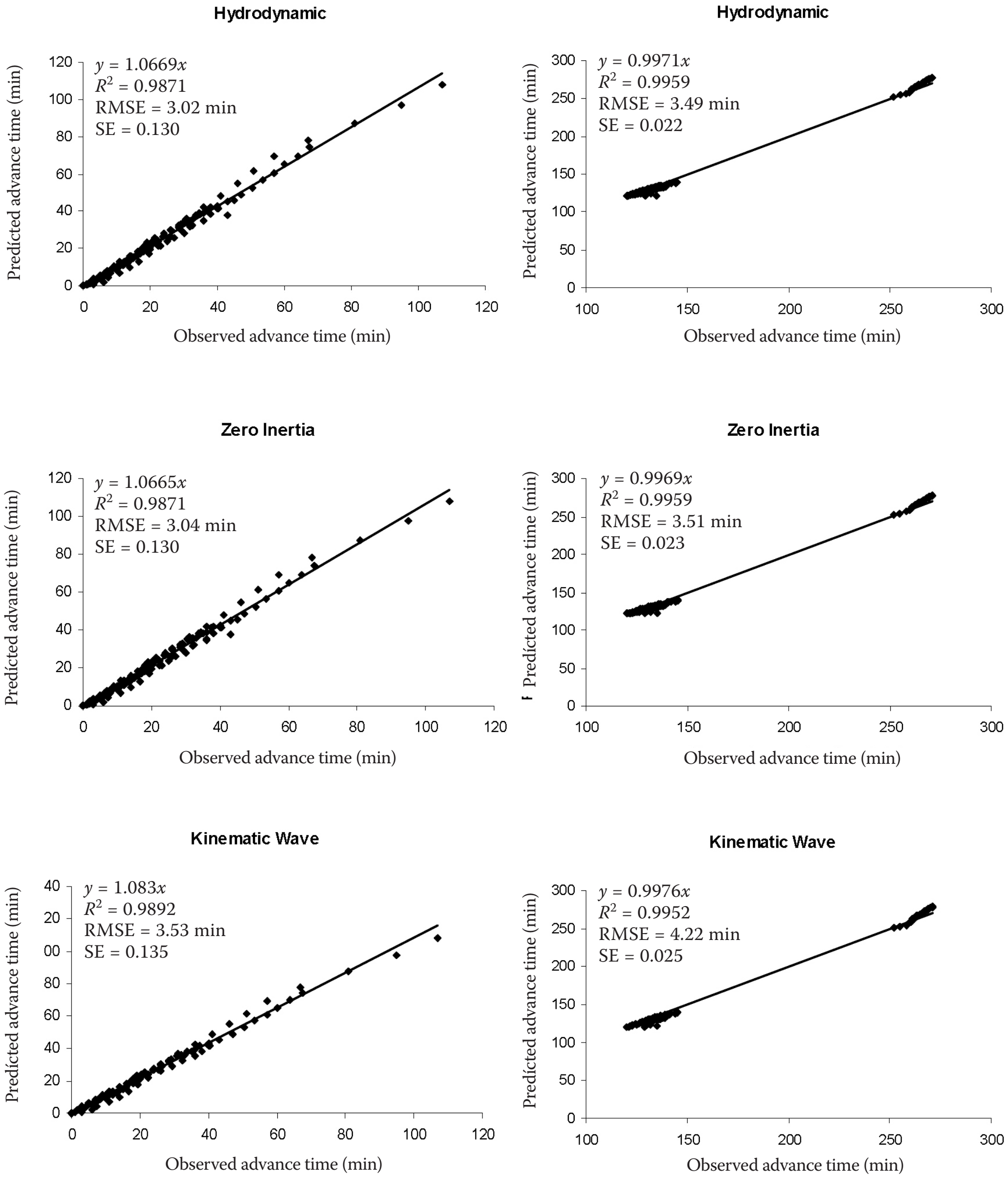

Figure 4. Observed versus predicted advance and recession times for the total data of furrows with the Hydrodynamic, Zero inertia, and Kinematic wave models

the advance times for both border and furrow irrigation. Also, the accuracy of these models for the predicting of both the advance and recession times was better for the experimental furrows in comparison with the experimental borders.

Overall, the KW model exhibits a larger error in the prediction of the advance and recession times, but the model prediction was generally satisfactory. There is no difference in the prediction between HD and ZI models of the SIRMOD software. This may be related to the negligible effects of inertial terms in the momentum equation in the study. This agrees with Maheshwari and McMahon (1993) who found that there was no difference in the prediction of the advance and recession times between the HD and ZI approaches of the Walker model in their border irrigation study. Abbasi *et al.* (2003) also reported such this results for both borders and furrows.

The total values of the predicted infiltrated and runoff volumes with the various models are given in Tables 3 and 4 for the experimental borders and furrows, respectively. The comparison of the predicted infiltrated and runoff volumes with the measured values showed that all the models estimated relatively satisfactorily the infiltrated and

Table 4. Comparison of various models in terms of infiltrated water and runoff volumes estimation for different experimental furrows (m^3)

Data	Measured			Hydrodynamic		Zero inertia		Kinematic wave[a]	
	inflow	runoff	infiltration[b]	runoff	infiltration	runoff	infiltration	runoff	infiltration
F1	9.40	0.19	9.22	0.27	9.13	0.27	9.13	0.27	9.13
F2	8.65	4.89	3.76	4.46	4.19	4.46	4.20	4.46	4.19
F3	8.91	4.75	4.16	4.52	4.40	4.51	4.40	4.53	4.39
F4	7.17	1.67	5.50	0.82	6.35	0.82	6.35	0.82	6.35
F5	5.86	1.96	3.90	0.70	5.15	0.70	5.15	0.70	5.16
F6	5.60	–	5.60	–	5.60	–	5.60	–	–
F7	5.10	–	5.10	–	5.10	–	5.10	–	–

[a]Kinematic wave model cannot simulate blocked-end condition; [b]calculated as measured inflow-measured runoff

runoff volumes. But the predicted advance and recession times were estimated by these models more accurately than the infiltrated and runoff volumes. The differences between the infiltrated and runoff values measured and those predicted by the models were likely related to the assumed steady-state conditions at the downstream boundary (using Manning's equation) inappropriate estimation of the infiltration parameters and precision of the WSC flumes used for measuring inflow-outflow hydrographs (Abbasi *et al.* 2003; Clemmmens 2009). The models in SIRMOD provided almost the same results for most of the experimental borders and furrows (Tables 3 and 4).

CONCLUSION

In the present study, three mathematical models in the SIRMOD package including hydrodynamic (HD), zero inertia (ZI), and kinematic wave (KW) were tested by using the data from several field experiments for both border and furrow irrigation systems. The results indicated that the performance of all models was satisfactory for the prediction of the advance and recession times. There was no difference in the prediction of the advance and recession times between the HD and ZI approaches of the SIRMOD software. Therefore, the assumption of negligible effects by inertial terms in the momentum equation used for modelling flows in furrow and border irrigations was satisfactory. The ZI model was superior to the HD because this model gives the same results as compared with the complex and complete HD model. Furthermore, this model does not require much computer memory, high computational cost, or complex programming (Strelkoff & Katapodes 1977; Abbasi *et al.* 2003).

In this study, the HD, ZI, and KW models predicted the recession time better than the advance time for both the experimental borders and furrows. The predicted advance and recession times were estimated by these models more accurately than the infiltrated and runoff volumes. Also, the accuracy of these models for the predicting of the advance and recession times was better for the experimental furrows in comparison with the experimental borders.

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