A mathematical model for predicting the cracking efficiency of vertical-shaft centrifugal palm nut cracker

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Abstract

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A mathematical model for predicting the cracking efficiency of vertical-shaft palm nut cracker was presented using dimensional analysis based on the Buckingham's π theorem. A high coefficient of determination of 94.3% between the predicted and measured values showed that the method is good. The model was validated with data from existing palm nut cracker and there was no significant difference between the experimental cracking efficiency with the predicted values at 5% level of significance.

Keywords: cracking efficiency; prediction equation; feed rate; throughput capacity; shaft speed; dimensional analysis

The modern crackers are of two types; the hammer impact and the centrifugal impact types. The hammer impact type breaks or cracks the nut on impact when the hammer falls on it; while centrifugal impact nut cracker uses centrifugal action to crack the nut (Ndukwu, Asoegwu 2010). In the centrifugal impact type; the nut is fed into the hopper and it falls into the housing where a plate attached to the rotor is rotating; which flings the nut on the cracking ring; thereby breaking the nut. Cracking therefore is an energy-involving process. According to some researchers (Asoegwu 1995; Ndukwu 1998) shelling or cracking has always posed a major problem in the processing of bio material and they attributed this to the shape and the brittleness of the kernel of the nut; rendering them susceptible to damageduring cracking. Presently most of the research work is tailored into modelling of the variables which determine the functionality of processing machines. Most of these models are specific and related to a particular design of a machine. Some researchers (Degrimencioglu, Srivastava 1996; Shefii et al. 1996; Mohammed 2002; Ndirika 2006) used the dimensional analysis based on the Buckingham's π theorem as veritable instrument in establishing a prediction equation of various systems. Therefore the present study is undertaken to establish a mathematical model for predicting the cracking efficiency of vertical-shaft centrifugal palm nut cracker using the dimensional analysis.

MATERIAL AND METHODS

Prototype of palm kernel cracker machine

A centrifugal palm nut cracker prototype testing machine described in Ndukwu and Asoegwu (2010) was used in validating the model. The palm kernel cracker is powered by 1,600 kW electric motor and operates with centrifugal action. It consists

Table 1. Dimensions of variables influencing cracking efficiency

Variables	Symbol	Dimensions
Cracking efficiency (%)	CE	$M^0L^0T^0$
Nut moisture content (%)	Ø	$M^0L^0T^0$
Bulk density of the nut (kg/m^3)	δ_1	$M^1L^{-3}T^0$
Nut particle density (kg/m^3)	δ_2	$M^1L^{-3}T^0$
Feed rate (kg/s)	γ_r	$M^1L^0T^1$
Throughput capacity (kg/s)	T_c	$M^1L^0T^1$
Cracking speed (m/s)	v	$M^0L^1T^1$
Diameter of the cracking chamber (m)	D	$M^0L^1T^0$

of a conical shaped hopper that opens up into a cylindrical cracking chamber with a force-fitted mild steel cracking ring. A vertical-shaft is fitted into the cracking chamber from the bottom and is attached to a channel for directing the palm nut falling on it. The centrifugal action of the shaft flings the nut on the cracking ring with the nut cracking on impact. The palm kernel used in the experimental analysis is described in (Ndukwu, Asoegwu 2010) and is made up of a mixture of dura and ternara species of palm kernel. The diameters and thickness were determined with a vernier calliper reading up to 0.01 mm while the moisture content was determined in an oven.

Model development: palm kernel cracking and separation

Cracking involves all action from the hopper orifice through the cracking chamber to the collector chute. The physical quantity affecting the cracking process includes both crop physical properties and machine parameters (Ndukwu 1998; Simonyan et al. 2006; Asoegwu et al. 2010).

1. Crop properties include: crop species; age; nut moisture content; bulk density; nut geometric mean diameter.
2. The machine properties: feed rate; diameter of the cracking chamber; shaft speed; and throughput capacity.

The following assumptions were made in developing the model:

1. the moisture content of the shell and kernel is the same,
2. the nut dimension is constant at the same moisture content,
3. the thickness of the shell is the same at the same moisture constant,
4. diameter of the cracking ring is fixed,
5. distance between the channel and cracking ring is fixed,
6. the age of the nut is the same,
7. the individual weight and volume of the nut is constant at a particular moisture content,
8. cracking speed is the same as the shaft speed,
9. the shaft speed is fixed.

Based on the above assumptions the major variables of importance are: the nut moisture content; bulk density of the nut; nut particle density; feed rate; throughput capacity and cracking speed (Ndukwu 1998). The cracking efficiency which is the fraction of cracked and undamaged kernel recovered from the collector chute can be expressed as follows:

$$CE = f\left(\varnothing; \delta_1; \delta_2; \gamma_r; v; D; T_c\right) \quad (1)$$

where:
CE – cracking efficiency (%)
Ø – nut moisture content (%)
δ_1 – bulk density of the nut (kg/m^3)
δ_2 – nut particle density (kg/m^3)
γ_r – feed rate (kg/s)
T_c – throughput capacity (kg/s)
v – cracking speed (m/s)
D – diameter of the cracking chamber (m)

The dimensions of the variables is presented in Table 1.

The number of variables of importance that determines the cracking efficiency (CE) is 7 and the

Table 2. The dimensional matrix of variables is given as follows

	Ø	δ_1	δ_2	γ_r	T_c	v	D
M	0	1	1	1	1	0	0
L	0	–3	–3	0	0	1	1
T	0	0	0	1	1	1	0

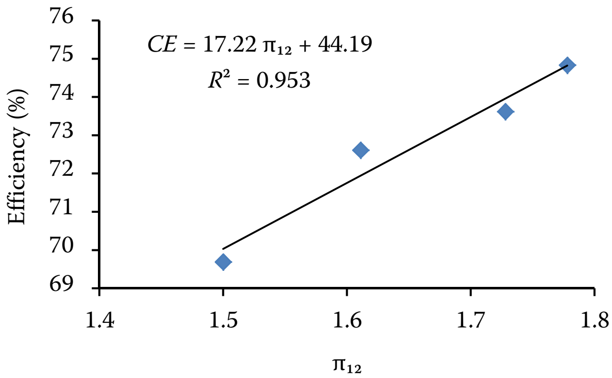

Fig. 1. Plot of the cracking efficiency against dimensionless π_{12} with π_{34} constant at average value of 6.896

number of fundamental units is 3; therefore the number of π terms is 4. It follows that π_1; π_2; π_3 and π_4 will be formed. The dimensional matrix of the variables is shown in Table 2. From the above matrix; Ø is dimensionless and therefore excluded from the dimensionless terms determination and is added when other dimensionless terms are determined (Simonyan et al. 2006).

$$CE = f\left(\delta_1; \delta_2; \gamma_r; v; T_c; D\right) \tag{2}$$

The dimensional equation is as follows:

$$f(\delta_1;\ \delta_2;\ \gamma_r;\ v;\ T_c;\ D) = 0 \tag{3}$$

The variables D; γ_r and v are chosen as recurring set since there combination cannot form a dimensionless group.

The dimensions of these variables are

$$D = L \tag{4}$$

$$v = \frac{L}{T} \tag{5}$$

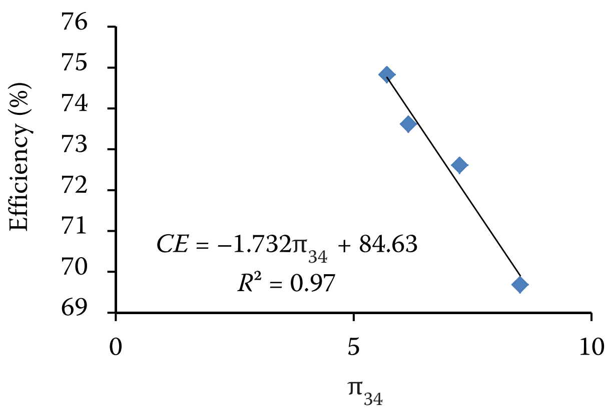

Fig. 2. Plot of the cracking efficiency against dimensionless π_{34} with π_{12} constant at average value of 1.654

$$\gamma_r = \frac{M}{T} \tag{6}$$

Rewriting the dimensions in terms of the variables chosen:

$$[L] = D \tag{7}$$

$$[T] = \frac{D}{v} \tag{8}$$

$$[M] = \frac{\gamma_r}{v} \tag{9}$$

The dimensionless groups based on the Buckingham's π theorem are formed by taking each of the remaining variables T_c, δ_1 and δ_2 in turn:

$$\pi_1 = \frac{T_c}{\gamma_1} \tag{10}$$

$$\pi_2 = \frac{v\delta_1 D^2}{\gamma_r} \tag{11}$$

$$\pi_3 = \frac{v\delta_2 D^2}{\gamma_r} \tag{12}$$

$$\pi_4 = Ø \tag{13}$$

$$CE = f(\pi_1;\pi_2;\pi_3;\pi_4) \tag{14}$$

$$CE = \left(\frac{T_c}{\gamma_r};\frac{v\delta_1 D^2}{\gamma_r};\frac{v\delta_2 D^2}{\gamma_r};Ø\right) \tag{15}$$

Combining the dimension terms to reduce it to a manageable level (Shefii et al. 1996) by multiplication and division:

$$\pi_1 \times \pi_2^{-1} = \frac{T_c}{v\delta_1 D^2} \tag{16}$$

$$\pi_3 \times \pi_4 = \frac{v\delta_2 Ø D^2}{\gamma_r} \tag{17}$$

$$CE = f(\pi_{12};\pi_{34}) \tag{18}$$

Substituting Eqs (16) and (17) into Eq. (18):

$$CE = f\left\{\frac{T_c}{v\delta_1 D^2};\frac{v\delta_2 Ø D^2}{\gamma_r}\right\} \tag{19}$$

Prediction equation

The prediction equation is established by allowing one π term to vary at a time while keeping the other constant and observing the resulting changes in the function (Shefii et al. 1996). This is achieved by plotting the values of CE against π_{12}; keeping; π_{34} constant and CE against π_{34} keeping π_{12} constant as shown in Figs 1 and 2. The linear equation is presented as shown in the Eqs (20) and (21) below with R^2 = 0.9532 and 0.97; respectively.

$$CE = 17.227\pi_{12} + 44.19 \quad (20)$$

$$CE = -1.732\pi_{34} + 84.62 \quad (21)$$

The plot of the π terms (Figs 1 and 2) forms a plane surface in linear space and according to Mohammed (2002) it implies that their combination favours summation or subtraction. Therefore the component equation is combined by summation or subtraction. The component equation is formed by the combination of the values of Eqs (20) and (21) (Shefii et al. 1996)

$$CE = f_1(\pi_{12};\pi_{34}) - f_2(\pi_{12};\pi_{34}) + K \quad (22)$$

Note:
at f_1; π_{34} was kept constant while π_{12} varied,
at f_2; π_{12} was kept constant while π_{34} varies.

$$CE = 17.227\pi_{12} + 44.19 - (-1.732\pi_{34} + 84.62) \quad (23)$$

Therefore the predicting equation becomes

$$CE = 17.227\pi_{12} + 1.732\pi_{34} + 40.43 \quad (24)$$

Substituting the values of the dimensionless π terms gives the equation for cracking efficiency:

$$CE = 17.227\left(\frac{T_c}{v\delta_1 D^2}\right) + \left(\frac{1.732v\delta_2 ØD^2}{\gamma_r}\right) + 40.43 \quad (25)$$

Determination of validation parameters

Bulk density: The bulk density was calculated with the method described by Ndirika and Oyeleke (2006); this was done by packing some seeds in a measuring cylinder. The seed was taped gently to allow the seed to settle into the spaces. The volume occupied by the seed in the cylinder is used to calculate the bulk density as follows

$$B_d = \frac{\text{weight of packed seed}}{\text{volume occupied by the seed}} \quad (26)$$

Moisture content: The validation of the model was done at four moisture contents. The moisture content was determined in an oven at a temperature of 105°C for 18 h (Ndukwu 2009). To obtain the desired moisture content; the samples were conditioned by soaking in a calculated quantity of water and mixing thoroughly. The mixed samples were sealed in polyethylene bags at 5°C in a refrigerated cold room for 15 days to allow the moisture to distribute evenly throughout the sample (Ndukwu 2009). The moisture content was calculated on dry basis.

Feed rate (kg/h): The time to completely empty the nut into the cracking chamber was determined with a stop watch. The feed rate was calculated as the mass of the palm kernel per unit time taken to empty the palm nut into the cracking chamber:

$$\text{feed rate} = WT/t \quad (27)$$

where:
WT – weight of the palm nut (kg)
t – time taken to empty the whole palm nut into the cracking chamber (h)

Cracking speed: The linear velocity for a rotating shaft is calculated as follows

$$V = \frac{2\pi rn}{60} \quad (28)$$

where:
n – rotational speed of the shaft (rad/s)
r – radius of the pulley (m)
V – linear velocity (m/s)

Throughput capacity (kg/h): This is the weight of the nut leaving the machine per unit time. It is calculated as:

$$\text{Throughput capacity} = \frac{WT}{T} \quad (29)$$

where:
WT – total weight of the palm nut fed into the hopper (kg)
T – total time taken by the cracked mixture to leave the chute (h)

Cracking efficiency (%): This is the ratio of the mass of completely cracked and undamaged nut to the total mass of the nut fed into the hopper. It is calculated as:

$$CE = \frac{WT - X}{WT} \times 100 \quad (30)$$

where:
WT – total weight of the palm nut fed into the hopper (kg)
X – weight of partially cracked and uncracked palm nut (kg)

Experimental procedures: Total sample of 240 kg of palm nut (mixture of ternera and dura

Table 3. Evaluation parameters (Nᴅᴜᴋᴡᴜ 1998; Ndukwu, Aꜱᴏᴇɢᴡᴜ 2010)

Parameters	Values				Standard deviation
Palm nut moisture content (Ø, db %)	10.94	11.74	13.48	15.18	1.90
Bulk density (δ_1, kg/m^3)	832.5	843.11	843.45	851.09	7.60
Particle density (δ_2, kg/m^3)	1,129.04	1,134.23	1,162.80	1,213.67	38.80
Feed rate (γ_r, kg/h)	714	714	714	714	–
Throughput capacity (T_c, kg/h)	662	646	644	600	26.60
Cracking speed (v, m/s)	3.92	3.92	3.92	3.92	–
Diameter of cracking ring (D, m)	0.29	0.29	0.29	0.29	–

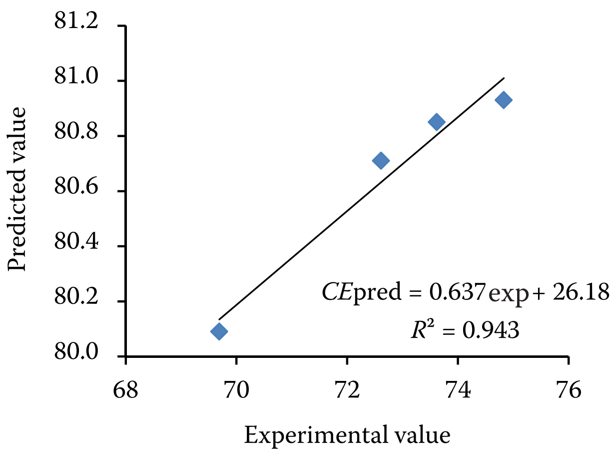

Fig. 3. Measured and experimental cracking efficiency

sp.) was divided into 5 kg each and fed into the hopper for each test run and cracked at different speed; feed rate and moisture contents. The quantities of cracked and uncracked palm nut; damaged and undamaged kernel were sorted out and weighed. This was done at different feed rate and at different moisture content. The cracking efficiency and throughput capacity were calculated based on the equations above. This was done in triplicate and the average was recorded and used for the analysis.

RESULT AND DISCUSSION

Model validation

The mathematical model was validated using data generated from an existing palm nut cracker presented by Nᴅᴜᴋᴡᴜ and Aꜱᴏɢᴡᴜ (2010). The model validation was done at four levels of moisture content and constant feed rate as shown in Table 3. The evaluation parameters are also presented in Table 3. Microsoft Excel 2007 statistical package for Windows Vista was used for the statistical analysis based on general linear model (GLM). The predicted and experimental cracking efficiency is presented in Table 4 with a standard deviation of 2.19 and 0.38; respectively. From Fig. 3; it can be observed that the measured value and experimental value has a very high correlation with R^2 value of 94.3% with a standard error of 0.42 between the experimented and predicted value which is less than 1% of the average value of the experimental cracking efficiency. When the mean of predicted and experimental value is compared using the least significance difference (LSD); at 1% and 5% level of significance; there is no statistical difference since the calculated "t" value is less than the Table "t" value. Also the validity of the model equation was

Table 4. Experimental and predicted cracking efficiency

Moisture content (% db)	Efficiency (%)	
	experimental (*CE*meas)	predicted (*CE*pred)
10.94	74.83	80.93
11.74	73.62	80.85
13.48	72.61	80.71
15.18	69.69	80.09
Standard deviation	2.19	0.38

examined by testing if the intercept and the slope were statistically significantly different from 0 and 1.0 respectively in the 1:1 model equation (Simonyan et al. 2010). The slope was found to be not significant at 5%. The regression equation obtained by the least square method is:

$$CE\text{pred} = 0.637CE\text{meas.} + 26.18 \quad (31)$$

where:
*CE*pred – predicted cracking efficiency
*CE*meas – measured cracking efficiency

At lower moisture content between 10–13% the predicted values is lower than the actual or experimental value.

CONCLUSION

A mathematical model was presented using dimensional analysis based on the Buckingham's π theorem. A functional relationship between some machine and crop parameters was established. The model was validated with data from existing palm nut cracker. The results showed a high coefficient of determination (R^2= 0.943) which implies good agreement. There was no significant difference between the experimental and predicted cracking efficiency at 5% level of significance.

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