How to combine precious metals with corn in a risk-minimizing two-asset portfolio?

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Abstract: This paper tries to find out which precious metal futures are the best hedging tools for corn spot commodity, taking into account three different risk measures – variance (Var), value at risk (VaR), and conditional value at risk (CVaR). For computation purposes, we use an optimal dynamic conditional correlation (DCC) specification for every considered pair. Our findings indicate that portfolio with gold outperforms the other three precious metals (silver, platinum, and palladium) with respect to all three risk metrics. The reason for such findings is two-fold. First, gold has the lowest average dynamic correlation with corn (below 11%), and gold also has the lowest average risk of all precious metals. The second-best combination is corn-platinum, whereas the corn-silver pair gives the worst hedging results. This happens because silver has the highest average dynamic correlation with corn (14.5%), but more importantly, silver is the riskiest commodity, which makes this asset unsuitable for combining with corn. According to the results, the ratio between corn and gold in a two-asset portfolio should be about 27:73.

Keywords: different risk measures; dynamic correlations; dynamic weights

Agricultural products are essential food components for people, but they are also raw materials in a number of industrial processes. Due to very high importance of agricultural commodities for everyday life, unstable agricultural prices may cause serious social problems worldwide, such as poverty, food trade restrictions, and bioenergy disputes, as Fakari et al. (2013) asserted. Therefore, exploring the behaviours of agricultural commodity prices is of great interest for wide range of market participants – agricultural producers, commodity traders, and portfolio managers – from the aspect of asset price valuation, investment allocation, and risk management. It can be argued that the major problem with agricultural commodities stems from their highly volatile prices that can generate widespread concern and discussion among policy makers and academics. In that respect, it is well known that agricultural commodities have experienced huge oscillations over the past two decades. Corn, as one of the most important and globally traded agricultural products, was affected by numerous global events in recent years, such as global financial crises, changes in global demand and supply, rapidly growing interest in biofuels, and financialization in futures markets. The issue of corn price risk is important to consider because more volatile corn prices imply more difficult and costly risk management for agricultural producers and traders, which leads to non-optimal production and investment/hedging decisions, as Wu et al. (2011) contended. Another reason is that increased corn volatility makes consumers in poor countries more vulnerable to price spikes and fears of scarcity (Mensi et al. 2017). Therefore, the volatile nature of corn prices has been and continues to be, a cause for concern among governments, traders, producers, and consumers.

In order to illustrate high corn price swings in last 15 years, we report that corn spot price was below
USD 220 cents per bushel in January 2006, in June 2008, it was well over USD 700 cents per bushel, while in November 2008, the price was significantly below USD 400 cents per bushel. At the beginning of March 2011, corn cash prices reached again very high values over USD 700 cents per bushel, whereas a couple of years later, the price of corn futures plummeted for the second time in less than 5 years period, going slightly over USD 400 cents per bushel. All these price movements of corn cash commodities can be viewed in Figure 1.

According to the aforementioned, the goal of this paper is to determine with which auxiliary asset corn cash commodity has to be combined in a two-asset portfolio in order to minimize risk of such portfolio. To the best of our knowledge, this type of research has never been attempted so far. More specifically, we combine corn cash agricultural commodity with four precious metal futures – gold, silver, platinum, and palladium, and calculate dynamic optimal in-sample portfolio weights via Kroner and Ng (1998) equation, which produces a minimum-variance (Var) portfolio by default. In addition, in order to be more thorough in the analysis, we also calculate two downside risk measures – value at risk (VaR) and conditional value at risk (CVaR), since various market participants have different risk-minimizing goals. From a theoretical point of view, commodities in a portfolio could bring numerous gains for investors, such as diversification benefits and risk reductions. In addition, Bessler and Wolff (2015) claimed that commodities have a low correlation with other types of assets because commodity price changes are connected with different risk factors, e.g. weather, geopolitical events, and global supply and demand conditions. Besides, different commodities vary significantly across business cycles and inflation episodes, which makes them a suitable hedge against inflation.

As for precious metals (pm), the main motivation why we use these commodities stems primarily from two reasons. First, precious metals are regarded as safe-haven assets since their values are considered to be more stable than those of stocks, industrial commodities, and other assets (Mansor 2011). Another reason is the fact that prices of precious metals and corn are driven by different fundamental factors that are related to their own supply and demand structure (Mensi et al. 2020). In other words, precious metals are used in jewelry, automotive manufacturing, and in electronic and chemical industries, while corn is primarily used as an animal feed and alternatively as a feedstock in the production of green energy (ethanol). Therefore, it implies that structurally different supply and demand models are expected for them, which means, theoretically, that low correlation exists between corn and precious metals.

Numerous papers reported that the combination between corn and different assets produces lower risk. In order to be concise as much as possible, we present in Table 1 the recent papers that coupled corn with different assets, aiming at lowering the portfolio risk.

From the methodological point of view, designing an optimal portfolio first requires good modeling of the selected time-series that will recognize their stylized facts, such as volatility asymmetry and clustering properly. In that manner, we use dynamic conditional correlation (DCC) model of Engle (2002). Due to the fact that cross-market correlation coefficients are conditional on market volatility, we use several different generalized autoregressive conditional heteroscedasticity (GARCH) specifications in DCC framework: symmetric GARCH and three asymmetric GARCH counterparts – Glosten, Jagannathan and Runkle GARCH (GJR-GARCH), exponential GARCH (EGARCH) and asymmetric power autoregressive conditional heteroscedasticity (APARCH) models. This approach is used because if market volatility is not adjusted for heteroscedasticity, the estimated correlation coefficients could be biased. In that process, we can determine optimal time-varying correlations between the observed corn-precious metal pairs, but we can also obtain dynamic conditional volatilities for each commodity. All these dynamic time-series are used subsequently as inputs in the construction of dynamic risk-minimizing portfolios via Kroner and Ng (1998) equation. Generally speaking, DCC-GARCH model is a suitable tool for this type of research, because it al-

![Corn](https://example.com/corn-graph.png)

Figure 1. The dynamics of corn cash (spot) prices in the last 15 years period

Y-axis denotes corn spot prices expressed in USD cents per bushel
Source: Authors’ own calculations based on Stooq (2020)
allows the correlations to change over time, while at the same time it utilizes the flexibility of the univariate GARCH model, but without the perplexity of conventional multivariate GARCH model (Dajčman and Alenka 2011; Kučerová and Poměnková 2015; Bala and Takimoto 2017).

**RESEARCH METHODOLOGIES**

**Dynamic conditional correlation model.** This section explains the DCC-GARCH methodology used to calculate dynamic conditional volatilities and dynamic conditional correlations of the selected commodities. We use daily data, which is well-known for the presence of clustering phenomenon and leverage effect in the volatility. In order to recognize these stylized facts in the best way, we use several univariate GARCH specifications in DCC framework – simple GARCH, GJRGARCH, EGARCH, and APARCH for every corn-precious metal pair. The optimal model is chosen based on the lowest Akaike information criterion (AIC). The mean equation of all univariate GARCH models is in the form of autoregressive equation (1), which provides enough lag-order to resolve the autocorrelation problem in our case. Mathematical formulation of the mean equation and four different GARCH specifications (simple GARCH, GJRGARCH, EGARCH, and APARCH) are given in Equations (1–5) respectively.

\[
I_{t-1} = \begin{cases} 
1 & \text{if } e_{t-1} < 0 \\
0 & \text{if } e_{t-1} > 0
\end{cases}
\]

\[
\ln(\sigma_t^2) = C + \alpha \frac{e_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{e_{t-1}}{\sqrt{\sigma_{t-1}^2}}
\]  

\[
(4)
\]

\[
\sigma_t^2 = c + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \epsilon_{t-1} I_{t-1} 
\]

\[
(5)
\]

where: \(C, c – \) constants in the mean and variance equations; \(\phi –\) autoregressive parameter; \(t –\) time; \(y_t = 2 \times 1 \) vector of corn and precious metal returns, \(y_t = \begin{bmatrix} y_{t,\text{corn}} & y_{t,\text{precious metal}} \end{bmatrix}^\top; \(\epsilon_t = 2 \times 1 \) vector of error terms, \(\epsilon_t = \begin{bmatrix} \epsilon_{t,\text{corn}} & \epsilon_{t,\text{precious metal}} \end{bmatrix}^\top; \(z_t – \) independently and identically distributed process, i.e. \(z_t \sim N(0,1)\); \(\beta – \) persistence of volatility; \(\alpha – \) ARCH effect; \(\gamma – \) presence of an asymmetric effect, if \(\gamma > 0\), then negative shocks affect volatility more than positive shocks and vice versa; \(\delta – \) power term parameter, and it takes finite positive values.

The multivariate DCC model of Engle (2002) implies a two-stage estimation procedure. First, for each pair of the selected time-series a univariate GARCH model is fitted, and estimates of \(\sigma_t^2\) are acquired. In the second step, asset-return residuals are standardized, i.e. \(\nu_t = \epsilon_t / \sqrt{\sigma_t^2}\), whereby \(\nu_t\) is used subsequently to estimate the parameters of the conditional correlation. Accordingly, the multivariate conditional variance is specified as: \(\Sigma_t = D_t \Sigma \Sigma_t D_t^\top\), where \(D_t = \text{diag}(\sqrt{\sigma_{1,t}^2}, \sqrt{\sigma_{2,t}^2})\) and \(\sigma_{ij,t}\) represents the conditional variance obtained from some form of univariate GARCH model in the first stage. The evolution of correlation in the DCC model is presented as in Equation (6):

<table>
<thead>
<tr>
<th>Authors</th>
<th>Methodology</th>
<th>Portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td>Park and Jei (2010)</td>
<td>DCC-GARCH</td>
<td>corn and soybean futures</td>
</tr>
<tr>
<td>Wu et al. (2011)</td>
<td>volatility spillover model</td>
<td>corn and oil futures</td>
</tr>
<tr>
<td>Cheng and Anderson (2017)</td>
<td>a two-stage stochastic program in form of a mixed integer linear program (MILP)</td>
<td>corn and ethanol</td>
</tr>
<tr>
<td>Ulusoy and Onbirler (2017)</td>
<td>DCC-GARCH</td>
<td>corn and coffee, cotton, wheat and sugar</td>
</tr>
<tr>
<td>Hernandez et al. (2019)</td>
<td>cross-quantilogram</td>
<td>corn and oil futures</td>
</tr>
<tr>
<td>Dahlgran and Gupta (2019)</td>
<td>survey based methodology</td>
<td>corn and ethanol</td>
</tr>
<tr>
<td>Nguyen et al. (2020)</td>
<td>ARMA filter-based correlation and rotational dynamic conditional correlation</td>
<td>corn and S&amp;P 500 and MSCI World</td>
</tr>
</tbody>
</table>

ARMA – autoregressive moving average; DCC – dynamic conditional correlation GARCH – generalized autoregressive conditional heteroscedasticity; MSCI – Morgan Stanley Capital International
\[ Q_t = (1-a-b)Q_t + \alpha v_{t-1}v_{t-1} + \beta Q_{t-1} \quad (6) \]

where: \( a, b \) – nonnegative scalar parameters under condition \( a + b < 1 \); \( Q_t = \{ q_{ij,t} \} - n \times n \) time-varying covariance matrix of residuals; \( Q = E\{ v_t v_t \} - n \times n \) time-invariant variance matrix of \( v_t \). Due to the fact that \( Q_t \) does not have unit elements on the diagonal, it is scaled to obtain a proper correlation matrix \( (C_t) \) according to the following form: \( C_t = (\text{diag}(Q_t))^{-1/2} Q_t (\text{diag}(Q_t))^{-1/2} \). Accordingly, the element of \( C_t \) is as in Equation (7).

All DCC models were estimated by a quasi-maximum likelihood estimation (QMLE) technique.

**Portfolio construction and risk measurement.**

In the construction of the two-asset portfolio, we use the equation of Kroner and Ng (1998), in which the conditional correlations and conditional variances are used as inputs. This particular equation minimizes unsystematic risk without affecting the potential of expected returns. The optimal dynamic portfolio weight of an auxiliary asset \( (W_{pm}^t) \), with the following restrictions, is computed as in Equations (8–9):

\[
W_t^{pm} = \frac{\sigma^2_{t}^{(pm)} - \sigma^2_{t}^{(pm, pm)}}{\sigma^2_t^{(pm, corn)} - 2\sigma^2_t^{(corn,pm)} + \sigma^2_t^{(pm, pm)}}
\]

\[
W_t^{pm} = \left\{ \begin{array}{ll}
0, & \text{if } W_t^{pm} < 0 \\
W_t^{pm}, & \text{if } 0 < W_t^{pm} < 1 \\
1, & \text{if } W_t^{pm} > 1
\end{array} \right.
\quad (9)
\]

where: \( W_t^{pm} \) – weight of the particular precious metal in USD 1 portfolio of two-asset holding at time \( t \); \( \sigma^2_t^{(corn)} \), \( \sigma^2_t^{(pm)} \) – conditional variances of corn and selected precious metals \( (pm) \), respectively; \( \sigma^2_t^{(corn, pm)} \) – conditional covariance between the corn and precious metals at time \( t \). The weight of corn in two-asset portfolio is calculated as \( 1 - W_t^{pm} \).

We evaluate risk-reduction performances of the portfolios by three hedge effectiveness indices \( \text{(HEI)} \) – minimum-variance, \( \text{VaR} \), and \( \text{CVaR} \). Minimum-variance metrics incorporate both upside and downside risk, assigning an equal weight to positive and negative returns. However, some investors prefer to know the down-side risk of the hedged portfolio, according to He et al. (2020), and this is the reason why we also calculate \( \text{HEI} \) for \( \text{VaR} \) and \( \text{CVaR} \). \( \text{VaR} \) measure is explained as a maximum possible loss that some portfolio might endure during particular time horizon and under certain probability \( (P) \). \( \text{VaR} \) is calculated as in Equation (10):

\[ \text{VaR}_t = \mu + Z_\omega \sigma \quad (10) \]

where: \( Z_\omega \) – left quantile at \( \omega \% \) of the distribution; \( \mu, \sigma \) – estimated mean and standard deviation of a particular portfolio, respectively.

However, \( \text{VaR} \) does not consider the expected size of a loss in the event that this loss exceeds \( \text{VaR} \) of a portfolio. In order to overcome this \( \text{VaR} \) setback, we also consider conditional value-at-risk that measures the mean loss, conditional upon the fact that the \( \text{VaR} \) has been exceeded. \( \text{CVaR} \) is calculated as in Equation (11):

\[ \text{CVaR}_t = \frac{1}{\omega} \int_0^\omega \text{VaR}(x) \, dx \quad (11) \]

where: \( \text{VaR}(x) \) – value at risk of a particular two-asset portfolio; \( \omega \) – left quantile of the normal distribution, and we apply a confidence level of \( \omega = 95\% \).

Portfolio hedging effectiveness indices of particular risk measure \( \text{(HEI}_{RM}) \) are calculated in the following way:

\[ \text{HEI}_{RM} = \frac{RM_{\text{unhedged}} - RM_{\text{hedged}}}{RM_{\text{unhedged}}} \quad (12) \]

where: \( RM \) – particular risk measure of a portfolio, i.e. \( \text{Var}, \text{VaR}, \) and \( \text{CVaR}; \text{unhedged} \) – investment only in corn; \( \text{hedged} \) – investment in a two-asset portfolio. The closer the \( \text{HEI} \) index is to 1, the higher the hedging effectiveness is, and vice versa.

**DATASET AND SELECTION OF AN OPTIMAL DYNAMIC CORRELATION MODEL**

This study uses daily prices of corn spot commodity and four futures prices of precious metals – gold, silver, platinum, and palladium. All time-series are retrieved from Stooq.com website. The data span for corn, platinum, and palladium ranges from January 1, 2005 to September 15, 2020, while for gold and silver futures, the samples start from June 1, 2005. We selected a rather long time-sample because we want to cover both tranquil and turbulent periods, such as huge corn price upswings of 2006 and 2011 (Figure 1). In this way, our \( \text{VaR} \) and \( \text{CVaR} \) risk measures are more realistic because we also consider crisis periods during which...
Table 2. Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>JB</th>
<th>LB(Q)</th>
<th>LB(Q^2)</th>
<th>DF-GLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn</td>
<td>0.014</td>
<td>1.888</td>
<td>–0.280</td>
<td>6.991</td>
<td>2576.7</td>
<td>0.000</td>
<td>0.000</td>
<td>–12.612</td>
</tr>
<tr>
<td>Gold</td>
<td>0.037</td>
<td>1.159</td>
<td>–0.282</td>
<td>9.657</td>
<td>7080.8</td>
<td>0.755</td>
<td>0.000</td>
<td>–59.635</td>
</tr>
<tr>
<td>Silver</td>
<td>0.031</td>
<td>2.036</td>
<td>–0.836</td>
<td>9.416</td>
<td>6975.8</td>
<td>0.021</td>
<td>0.000</td>
<td>–60.758</td>
</tr>
<tr>
<td>Platinum</td>
<td>0.001</td>
<td>1.493</td>
<td>–0.494</td>
<td>9.447</td>
<td>6920.2</td>
<td>0.000</td>
<td>0.000</td>
<td>–6.778</td>
</tr>
<tr>
<td>Palladium</td>
<td>0.055</td>
<td>1.979</td>
<td>–0.561</td>
<td>7.346</td>
<td>3278.9</td>
<td>0.516</td>
<td>0.000</td>
<td>–15.581</td>
</tr>
</tbody>
</table>

DF-GLS – Dickey-Fuller generalized least square; JB – value of Jarque-Bera coefficients of normality; LB(Q), LB(Q^2) – P-values of Ljung-Box Q-statistics of level and squared residuals for 20 lags; 1% and 5% critical values for DF-GLS test with 10 lags, assuming only constant, are –2.566 and –1.941, respectively

Source: Authors’ own calculations based on data from Stooq (2020)

investors recorded significant losses. All pairs of corn–precious metal time-series are synchronized according to the existing observations. The concise summary statistics that contain first four moments, Jarque-Bera (JB) and Ljung-Box (LB) tests as well as Dickey-Fuller generalized least square (DF-GLS) unit root tests are presented in Table 2.

According to Table 2, all time-series are left-skewed with heavy tails, and none of the commodities has a normal distribution. Also, corn and platinum time-series have autocorrelation, while all commodities have time-varying variance feature, justifying the usage of ARMA-GARCH models in DCC framework. None of the time-series has a unit root, which is a necessary requirement for GARCH modelling.

As have been said earlier, we want to recognize the empirical time-series in the best possible way. Therefore we test four different univariate GARCH specifications in DCC framework. The decision about the best DCC model is based upon the lowest AIC. Table 3 shows that in tree out of four cases the APARCH model has an advantage over other GARCH models, indicating the presence of an asymmetry in the empirical time-series.

**EMPIRICAL RESULTS**

Results of DCC model. This subsection presents the results of the estimated DCC models, taking into account the best-fitting univariate GARCH model. According to Table 4, we find an asymmetric effect in models for corn, because parameter γ is statistically significant, while for precious metals, this is not the case. In addition, all models report the absence of autocorrelation and heteroscedasticity, which means that all univariate GARCH specifications handle these issues very well. Table 4 also shows estimated parameters in the multivariate DCC model. In particular, multivariate DCC model estimates (a and b) are statistically significant and nonnegative in all cases, also satisfying the condition a + b < 1. This means that estimated dynamic correlations are reliable. Figure 2 presents graphical illustration of the estimated DCCs.

According to both Table 4 and Figure 2, it can be concluded that dynamic correlations between corn and precious metals are relatively low. In particular, all average DCCs are below 15%, whereas DCC for gold is the lowest one and is around 11%. This preliminary finding indicates that precious metals could successfully serve as a diversification tool in a portfolio with corn, which is in line with some recent papers (Mirović et al. 2017; Kang and Yoon 2019). In addition, all DCC plots have the same distinctive feature, and that is the rise of DCCs around the period of global financial crisis (GFC). This characteristic could be attributed to the phenomenon known as commodity financialization, which happens when commodity prices are driven away from rational levels, determined by their supply and demand. In other words, the relative rise of DCCs...
during GFC may be caused by the continuous investments from institutional investors into commodities. This happened because all financial markets fell sharply during GFC, and as a consequence, commodities emerged as a suitable alternative. These activities kept high co-movement between commodities during GFC, and that is why we find increased DCCs during GFC in all plots.

Portfolio construction and risk measurement in the full sample. This section presents the results of four constructed portfolios in terms of risk measures and hedge effectiveness indices. All portfolios are designed according to Equation (9), which minimizes the variance of the portfolio by default. Referring to Živkov et al. (2020), we use this portfolio to calculate three different risk measures – Var, VaR, and CVaR. Figure 3 presents dynamic weights of four precious metals, whereas Table 5 presents their average values. Table 5 shows that the highest weight in the portfolio with corn is that of gold with 73%, while platinum follows with 64%. These findings coincide perfectly with previously calculated dynamic correlations in the sense that precious metals with the lowest correlation with corn have the highest weight in the portfolio. However, these results do not tell anything about actual risk metrics, nor guarantee that gold is the best auxiliary asset in a combination with corn. Table 6 serves this purpose.

In particular, it can be seen in Table 6 that the corn-gold portfolio outperforms all other portfolios in all three risk measures. Var gauges the average risk of the portfolio, giving equal weight to positive and negative

Table 4. Dynamic correlation model estimation results

<table>
<thead>
<tr>
<th>Panel A: Variance estimates of corn</th>
<th>Corn vs. gold (APARCH)</th>
<th>Corn vs. silver (EGARCH)</th>
<th>Corn vs. platinum (APARCH)</th>
<th>Corn vs. palladium (APARCH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>0.078***</td>
<td>–0.493***</td>
<td>0.076***</td>
<td>0.076***</td>
</tr>
<tr>
<td>β</td>
<td>0.921***</td>
<td>0.984***</td>
<td>0.923***</td>
<td>0.923***</td>
</tr>
<tr>
<td>γ</td>
<td>0.125**</td>
<td>–0.057*</td>
<td>0.185**</td>
<td>0.189**</td>
</tr>
<tr>
<td>δ</td>
<td>1.141***</td>
<td>–</td>
<td>1.097***</td>
<td>1.096***</td>
</tr>
<tr>
<td>Diagnostic tests</td>
<td>LB(Q)_10</td>
<td>13.44</td>
<td>12.95</td>
<td>14.49</td>
</tr>
<tr>
<td></td>
<td>LB(Q^2)_10</td>
<td>15.58</td>
<td>15.60</td>
<td>15.39</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Variance estimates of precious metals</th>
<th>Corn vs. gold (APARCH)</th>
<th>Corn vs. silver (EGARCH)</th>
<th>Corn vs. platinum (APARCH)</th>
<th>Corn vs. palladium (APARCH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>0.057***</td>
<td>–0.543***</td>
<td>0.068***</td>
<td>0.081***</td>
</tr>
<tr>
<td>β</td>
<td>0.940***</td>
<td>0.989***</td>
<td>0.934***</td>
<td>0.920***</td>
</tr>
<tr>
<td>γ</td>
<td>–0.055</td>
<td>–0.004</td>
<td>0.055</td>
<td>0.013</td>
</tr>
<tr>
<td>δ</td>
<td>1.651***</td>
<td>–</td>
<td>1.484***</td>
<td>1.464***</td>
</tr>
<tr>
<td>Diagnostic tests</td>
<td>LB(Q)_10</td>
<td>5.15</td>
<td>8.95</td>
<td>12.73</td>
</tr>
<tr>
<td></td>
<td>LB(Q^2)_10</td>
<td>11.08</td>
<td>15.50</td>
<td>14.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: DCC parameters</th>
<th>Average ρ (%)</th>
<th>10.73</th>
<th>14.50</th>
<th>13.67</th>
<th>14.47</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>0.004***</td>
<td>0.004**</td>
<td>0.011*</td>
<td>0.004***</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>0.995***</td>
<td>0.995***</td>
<td>0.980***</td>
<td>0.993***</td>
</tr>
</tbody>
</table>

***, **, *Statistical significance at the 1, 5 and 10% level, respectively; APARCH – asymmetric power autoregressive conditional heteroscedasticity; GARCH – generalized autoregressive conditional heteroscedasticity; EGARCH – exponential GARCH; GJRGARCH – Glosten, Jagannathan and Runkle GARCH; LB(Q), LB(Q^2) tests – P-values of Ljung-Box Q-statistics for level and squared residuals for 10 lags

Source: Authors’ own calculations based on data from Stooq (2020)

Table 5. Average weight of precious metals in a portfolio with corn

<table>
<thead>
<tr>
<th>Metal</th>
<th>Weight (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>0.73</td>
</tr>
<tr>
<td>Silver</td>
<td>0.49</td>
</tr>
<tr>
<td>Platinum</td>
<td>0.64</td>
</tr>
<tr>
<td>Palladium</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Source: Authors’ own calculations based on data from Stooq (2020)
returns, whereby the \( \text{Var} \) value for corn-gold portfolio stands as the best of all portfolios considered. In terms of \( \text{Var} \), corn-platinum is the second-best option, while the corn-silver combination is the worst of all. On the other hand, \( \text{VaR} \) measures down-side risk, and for corn-gold, it amounts to \(-1.56\%\) under a probability level of 95%. This means that there is a 5% chance that investors will lose 1.56% or more in value of the portfolio in a single day. Once again, we find that the corn-platinum portfolio follows a corn-gold combination with the value of \(-1.89\%\), whereas palladium and silver perform the worst, and they are relatively equal in terms of \( \text{VaR} \). With respect to the \( \text{CVaR} \) value, the portfolio with gold has the best results, the portfolio with platinum follows, while corn-silver is the worst performing combination. \( \text{CVaR} \) for corn-gold is \(-2.5\%\) under 95% probability, which means that in the worst 5% of returns, the average loss will be 2.5%. We calculate \( \text{CVaR} \) because this risk-measure gives a better risk approximation than \( \text{VaR} \). This is because \( \text{CVaR} \) gauges an average expected loss rather than a range of potential losses that \( \text{VaR} \) provides. In other words, \( \text{VaR} \) may lead to an underapproximation of potential losses because it ignores all returns worse than the given \( \text{VaR} \) level. Once again, the portfolio with platinum is the second-best, while palladium is the third one. The portfolio with silver has the worst results. Although, we cannot directly compare our research with other studies, because this paper is the first one that put together corn and precious metals, we can say that our findings are well in line with other papers which constructed portfolio with these metals.

Table 6. Results for three risk metrics and \( \text{HEI} \) indices in the full sample

<table>
<thead>
<tr>
<th></th>
<th>Corn vs. gold</th>
<th>Corn vs. silver</th>
<th>Corn vs. platinum</th>
<th>Corn vs. palladium</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Var} )</td>
<td>1.078</td>
<td>2.162</td>
<td>1.201</td>
<td>1.696</td>
</tr>
<tr>
<td>( \text{VaR} )</td>
<td>(-1.56%)</td>
<td>(-2.22%)</td>
<td>(-1.89%)</td>
<td>(-2.24%)</td>
</tr>
<tr>
<td>( \text{CVaR} )</td>
<td>(-2.50%)</td>
<td>(-3.57%)</td>
<td>(-2.80%)</td>
<td>(-3.28%)</td>
</tr>
</tbody>
</table>

\( \text{CVaR} \) – conditional value-at-risk; \( \text{HEI} \) – hedge effectiveness index; \( \text{Var} \) – variance; \( \text{VaR} \) – value-at-risk; values for \( \text{VaR} \) and \( \text{CVaR} \) are given under probability level of 95%; bolded values are the best ones

Source: Authors’ own calculations based on data from Stooq (2020)

Figure 2. Estimated dynamic correlations for four pairs of commodities

Dynamic correlation could range between \(-1\) and 1

Source: Authors’ calculation based on data from Stooq (2020)
and globally traded financial (commodity) assets. For instance, the papers of Hammoudeh et al. (2013) and Jiang et al. (2019) found that gold in combination with financial and commodity assets gives the best risk-minimizing results of all metals from this group.

As for HEI values, it can be seen that values of risk measures perfectly translate to hedge indices in terms of their performance. In other words, the portfolio with gold has the highest HEI values for all three risk measures, while platinum follows. In order to depict different risk aversions of investors, we present VaR and CVaR values in Figure 4 calculated under five different levels of probabilities.

If we want to provide a logical explanation why gold is the best asset in combination with corn, considering all three risk measures, we have to consider two important categories – dynamic correlation and the level of risk of all selected metal commodities. Namely, according to Table 4, gold has the lowest dynamic correlation with corn, and that is an excellent characteristic of gold in terms of diversification. Besides, what is even more important is the fact that gold has the lowest

Figure 3. Time-varying weight of precious metals in a portfolio with corn
Y-axis denotes a weight measure of auxiliary asset in two-asset portfolio, and it can range between 0 and 1
Source: Authors’ own calculations based on data from Stooq (2020)

Figure 4. Measures of VaR and CVaR for full sample under different probability levels
CVaR – conditional value-at-risk; VaR – value-at-risk
Source: Authors’ own calculations based on data from Stooq (2020)
risk of all assets (Table 2), and according to Khalfaoui et al. (2015), the instrument with the lowest risk makes its presence in a two-asset portfolio very favourable. Therefore, due to its lowest DCC with corn and its lowest risk vis-à-vis corn, gold is the most suitable hedging tool of all selected precious metals.

SUMMARY AND CONCLUSION

This paper researches which precious metal futures provide the best risk-minimizing result in combination with corn spot commodity in terms of Var, VaR, and CVaR. For computation purposes, we use several DCC-GARCH models.

Based on the results, we can strongly advise investors who have a long position in corn spot commodity to combine corn with gold futures, in an approximate ratio of 73% gold futures and 27% corn spot, because gold lowers the risk of such portfolio the most effectively with respect to all three risk measures. In other words, the variance of such portfolio is the lowest one and amounts to 1.078. For investors who pursue minimum VaR, the results indicate that there is a 5% chance that investor will lose 1.56% or more in a value of the portfolio with gold in a single day. As for investors who target CVaR, our findings suggest that in the worst 5% of returns, the average loss will be 2.5%.

The reason why gold is the best hedging tool for corn lies in the fact that gold has the lowest dynamic correlation with gold and, additionally, gold has the lowest risk of all precious metals, which makes gold an ideal instrument to combine with corn. The second-best combination is corn-platinum, while the worst pair is corn-silver, with respect to all three risk measures.

We believe that corn producers, corn traders, and portfolio managers who invest in corn spot market can find these results useful because they could benefit based on the knowledge with which precious metal to couple their long position in corn and in which percentage in order to lower the portfolio risk. This paper considers precious metals for hedging with corn, but future studies can take into account different assets in combination with corn, such as ferrous metals, different agricultural products, different energy commodities, stock indices, ETFs, bonds.

REFERENCES


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