A DEA approach for merging dairy farms

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Abstract: This paper proposes a model for determining the most advantageous merger within a set of dairy farms. It uses data envelopment analysis (DEA) to estimate the total technical efficiency improvement that the merger would produce and for decomposing it into a learning effect and a pure merger effect. A design of experiments has also been carried to test the effects of various factors (the total number of farms, the standard deviation of herd size, the percentage of farms exhibiting increasing returns to scale, the standard deviation of the current technical efficiency of the farms) on different response variables (the percentage of farms involved in the merger, the reduction of herd size and the efficiency improvement obtained by the merger). The results show that the disparity in the herd size of the farms in a region and the percentage of farms that exhibit increasing returns to scale increase the number of farms that enter into the most advantageous merger. The disparity of herd size also increases the number of cows that are not needed after the merger. Finally, the expected efficiency improvement increases with the total number of farms.

Keywords: dairy farming; land consolidation; learning effect; pure merger gains; technical efficiency

According to FAO (2014), the vast majority of farms around the world (more than 80%) are family-run and operate at a small scale (less than 2 ha). Agriculture at a small scale has many advantages, including providing employment and food security to millions of people and improving environmental sustainability. However, the small scale implies many sources of inefficiency, which makes this type of agriculture less competitive in the market. Innovation, technology and resource access are challenges difficult to overcome when operating at small scale, making it more difficult to compete in price long-term (Raup 1972).

Given the importance of fixing the rural population and guaranteeing a sustainable agriculture system, some governments have passed various land consolidation policies to revert that situation since mid-last century. Promoting group farming and farmers’ co-ops has led individual farmers to pool their resources, sharing costs and profits, and gaining a larger scale and greater competitiveness. Agarwal and Dorin (2019) present many cases of group farming in countries such as Norway and Ireland for the case of milk, Japan with rice crops, different former socialist countries or India. Piet et al. (2012) report that the number of French farms decreased one third in the period 1970–2007, increasing the farms average size from 18.84 ha to 53.96 ha during that period. Monke et al. (1992) remarked on the viability and economic benefit obtained by farm size enlargement in the case of northern Portugal. In the case of cow-calf farms, in Sweden Holmström et al. (2018) analysed how creating larger pastures from scattered plots yields positive effects, thanks to lower operating costs resulting from economies of scale. For the case of milk production, similar results were observed by Corral et al. (2011) in Spain, with higher profitability for dairy farms (DFs) with lower land fragmenta-

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tion. Different studies conclude that in industrialised countries, there has been a clear tendency over the last decades to decrease the number of farms and increase their size, although the growth is not uniform across all countries (Piet et al. 2012).

This paper deals with a specific consolidation case, i.e. merging DFs in a limited geographic area. The goal is efficiency improvement. Gloy et al. (2002) reported that larger farm size and milking parlours in the dairy sector significantly increase profitability. In fact, the number of small DFs has decreased at a high rate in the USA in recent decades (Tauer and Mishra 2006). One of the first studies on the consolidation of DFs was Corral et al. (2011). According to their analysis, reducing the average number of plots per farm by 80% increases profits by 12%, with greater effect when the milk price is low.

The goal of the paper is to develop an approach to identify the most promising mergers among a set of DFs, assessing and decomposing the efficiency improvements that these mergers would bring about. This research topic has not been dealt with in the literature. In order to assess the relative efficiency of the existing DFs as well as of the unit that would result from merging them, data envelopment analysis (DEA) is proposed. DEA is a non-parametric methodology that, from the observed data about input consumption and output production and making some simple assumptions, infers the production possibility set (PPS; also known as DEA technology). PPS contains all the operating points that are deemed feasible. The non-dominated region of the PPS corresponds to the efficient frontier, and the DFs whose operating points lie on that efficient frontier are said to exhibit the best practices (Le et al. 2019). Conversely, those DFs that can reduce their inputs without reducing their outputs or increase their outputs without increasing their inputs are deemed inefficient, and an inefficiency score that measures the margin for improvement can be computed. Apart from the technical efficiency improvement that each DF can individually achieve by adopting the best practices of their peers (i.e. the so-called learning effect), often the DF can achieve additional gains by merging. This is the case when their relative sizes are small, and they operate in a region of the PPS in which increasing returns to scale (IRS) prevail. This is the problem addressed in this paper, in which a new DEA model to identify the most advantageous DF merger, quantifying the potential efficiency gains, is proposed.

A large number of studies have applied DEA to DF efficiency assessment (Iribarren et al. 2011; Mugera 2013; Mu et al. 2018; Siafakas et al. 2019). Some of these studies have used second-stage regression to test the influence of farm size on technical efficiency. In general, it seems that technical efficiency in DFs is positively affected by farm size measured by herd size or by dairy production (Barnes 2006; Demircan et al. 2010). Hanson (2008), however, found a non-linear relationship so that this positive effect applies only for larger farms and linked this with the capacity for adopting technological innovations.

The DEA methodology can also be used to study the desirability of mergers from a technical efficiency point of view. A distinction can be made between those approaches that aim to pre-evaluate the potential gains of a given merger from those approaches that aim to identify the units to be merged. In the first category of studies (Bogetof and Wang 2005), the units to be merged are known beforehand. It is common that all possible combinations (often pairwise) are considered and pre-evaluated (Li et al. 2018). Sometimes only certain mergers fulfilling certain geographical restrictions are meaningful (Zschille 2015), and sometimes such constraints are considered to reduce the number of possible combinations (Mydland 2020). Depending on the application, specific types of mergers may be interesting to study (e.g. merging hospitals in the same city or merging rural hospitals in the same region). In some cases, specific mergers that had taken place (Mattson and Tidanå 2019) or that had been proposed in official reports (Flokou et al. 2017) are studied. There are other approaches to estimate efficiency gains of mergers that are not based on Bogetof and Wang (2005), such as Lozano and Villa (2010) or Blancard et al. (2016).

All the above DEA merger approaches have one thing in common: the units to be merged are given. However, there are a few approaches that aim to identify the best partner for a merger and set the corresponding input and output targets (Lozano 2013; Zhu et al. 2017). The mixed-integer linear programming (MILP) model used in Blancard et al. (2016) to identify the firms to enter into a merger with a given unit is also of this type. Those authors also impose an upper bound on the number of units to merge. This is an indirect way of taking into account the unobserved transaction costs derived from a reorganisation involving a significant number of units.

An important aspect that has to be taken into account when studying mergers is the returns to scale (RTS). Scenarios, consistent with the additivity assumption [constant returns to scale (CRS) and non-decreasing returns to scale (NDRS)], guarantee that the operating point that results from just adding the
inputs and outputs of any subset of units is always feasible. Often, however, variable returns to scale (VRS) are considered more realistic (Mydland 2020), but in that case, not all mergers generate feasible operating points. In any case, the estimated efficiency gains from mergers are smaller, and hence, fewer mergers appear advantageous in the VRS case than in the CRS case (Wu et al. 2011).

**MATERIAL AND METHODS**

In this paper, a new DEA model for identifying the most advantageous dairy farm (DF) merging is presented. The proposed approach belongs to the second category of DEA merger approaches, i.e. the units to merge are not given but are determined by the model based on the observed data. A difference with other approaches is that no constraint is imposed on the number of units that should enter into the merger, i.e. the optimal number is left to the model to decide. The VRS technology considered precludes the merger of a large number of units unless the sample contains units of widely diverging sizes. Also, different to the radial or cost efficiency approaches generally considered in the literature, the proposed approach uses a slacks-based inefficiency (SBI) measure (Fukuyama and Weber 2009).

**DEA merger model.** The proposed SBI DEA merger model is [Model (1)]:

\[
SBI_R = \text{Max} \frac{1}{m+s} \left( \sum_{i=1}^{m} s_{ir}^x + \sum_{k=1}^{s} s_{kr}^y \right)
\]

\[s.t.
\]

\[
\hat{x}_{ir} = \sum_{j=1}^{n} \lambda_j x_{ij} = \sum_{r=1}^{n} \delta_r x_{ir} - s_{ir}^x \quad \forall i
\]

\[
\hat{y}_{kr} = \sum_{j=1}^{n} \lambda_j y_{kj} = \sum_{k=1}^{n} \delta_r y_{kr} - s_{kr}^y \quad \forall k
\]

\[
\sum_{j=1}^{n} \lambda_j = 1
\]

\[
s_{ir}^x \geq 0 \quad \forall i \quad s_{kr}^y \geq 0 \quad \forall k \quad \lambda_j \geq 0 \quad \forall j \quad \delta_r \in \{0,1\} \quad \forall r
\]

where: \(j, r = 1, 2, ..., n\) – indexes on decision making units (DMUs); \(i = 1, 2, ..., m\) – index on inputs; \(k = 1, 2, ..., s\) – index on outputs; \(x_{ij}\) – amount of input \(i\) consumed by DF \(j\); \(y_{kj}\) – amount of output \(k\) produced by DF \(j\); SBI\(_R\) – estimated merger efficiency gains; \(g = (g^x, g^y) \in \mathbb{R}_+^{m+s}\) – SBI directional vector (i.e. slacks-normalisation coefficients); \((\lambda_1, \lambda_2, ..., \lambda_n)\) – intensity variables used to compute the merger operating point as a linear combination of the observed DMUs; \(\delta_r\) – binary variable indicating if DF \(r\) will enter into the merger or not; \(R = \{ r : \delta_r = 1 \}\) – set of DMUs that will enter into the merger; \(s_{ir}^x\) – slack (i.e. potential improvement) of input \(i\) after the merger; \(\hat{x}_{ir}\) – target value of input \(i\) after the merger; \(\hat{y}_{kr}\) – target value of output \(k\) after the merger.

The above MILP model always has a feasible solution. If we add the constraint that at least two units should be selected for merging:

\[
\sum_{r=1}^{n} \delta_r \geq 2
\]

Then feasibility is no longer guaranteed as it may occur, in some cases, that none of the possible mergers lies within the VRS PPS. This does not occur if CRS or NDRS are assumed.

If so desired, we can impose an upper bound on the maximum number of DF entering into the merger or on the maximum size (in terms of one or more inputs or outputs) of the operating point after the merger. It is also possible to include incompatibility constraints preventing, for example, two specific DFs \(r'\) and \(r''\) from entering both into the merger, i.e. \(\delta_{r'} + \delta_{r''} \leq 2\).

It is also possible to explore the merger options available to a specific DF \(r'\) by forcing the model to include it in the merger compulsorily, i.e. \(\delta_{r'} = 1\).

Another possible scenario is the case that only DFs that belong to a certain subset \(A\) are allowed to merge. For example, \(A\) may contain the indexes of the DFs that belong to a certain region. Making the model consider the potential mergers of only those units can be imposed with \(\delta_r = 0\) \(\forall r \notin A\).
These are just examples of the multiple modelling possibilities that the proposed approach allows, which can be customised to each specific application. Finally, note that all DF DEA applications consider an important input: the herd size, which is an integer variable. For that variable (and any other integer variable considered), integrality constraints should be imposed (Lozano and Villa 2006).

**Merger efficiency gains decomposition.** The optimal value of the objective function of Model (1) $SBI_r$, represents the average input and output improvement that can be obtained from the merger. Thus, if:

$$
\left( \sum_{r=1}^{n} \delta_{r} x_{ir}, \sum_{r=1}^{n} \delta_{r} y_{ir} \right) = \left( \sum_{r \in R} x_{ir}, \sum_{r \in R} y_{ir} \right)
$$

is the operating point that results from aggregating the inputs and outputs of the DMUs prior to the merger, then the input and output slacks computed by the model correspond to the input and output improvements that can be made from that aggregated operating point. That is what the first two constraints of Model (1) mean. In other words, the estimated merger efficiency gains $SBI_r$ correspond to the technical inefficiency of that aggregated operating point measured with respect to the target $(\hat{x}_{ir}, \hat{y}_{ir})$ computed by the Model (1). Moreover, since Model (1) projects onto the efficient frontier, the target $(\hat{x}_{ir}, \hat{y}_{ir})$ is technically efficient. However, it may occur (it is quite likely) that the DFs that will enter into the merger were not technically efficient. That means that part of the estimated efficiency gains can be due to removing the technical inefficiency present in the observed DFs prior to the merger. The technical inefficiency of any observed DF $r$ can be computed using the following conventional SBI DEA model [Model (4)]:

$$
SBI_r = \text{Max} \left\{ \frac{1}{m + s} \left( \sum_{i=1}^{m} \frac{S_{ir}^x}{\delta_i^x} + \sum_{k=1}^{n} \frac{S_{kr}^y}{\delta_k^y} \right) \right\}
$$

s.t.

$$
\begin{align*}
\hat{x}_{ir} &= \sum_{j=1}^{n} \lambda_j x_{ij} = x_{ir} - s_{ir}^x \quad \forall i \\
\hat{y}_{kr} &= \sum_{j=1}^{n} \lambda_j y_{kj} = y_{kr} - s_{kr}^y \quad \forall k \\
\sum_{j=1}^{n} \lambda_j &= 1 \\
s_{ir}^x &\geq 0 \quad \forall i \quad s_{kr}^y &\geq 0 \quad \forall k \quad \lambda_j &\geq 0 \quad \forall j
\end{align*}
$$

where: $SBI_r$ – slacks-based inefficiency of DF $r$.

Following the above reasoning, the estimated merger efficiency gains $SBI_r$ can be decomposed into a technical efficiency (or learning effect) component $SBI_{rLE}$ that measures the technical efficiency improvements that can be achieved without carrying out the merger, only by making the inefficient DFs adopt the best practices available, plus the pure merger efficiency gains $SBI_{rP}$, i.e.:

$$
SBI_r^* = SBI_r - SBI_{rLE}
$$

$$
SBI_{rLE} = \sum_{r \in R} SBI_r
$$

where: $SBI_{rLE}$ – technical efficiency (or learning effect) component; $SBI_{rP}$ – pure merger efficiency gains.

A merger $R$ is said to be advantageous if $SBI_{rP} > 0$, i.e. if there are pure merger efficiency gains, and not all the merger efficiency gains are due to learning effects that are independent of the merger.

**APPLICATION OF THE PROPOSED APPROACH**

Let us consider a set of 20 dairy farms (DFs) that consume two inputs [herd size (HS), and operating costs (OC)] to produce a single output [milk production (MP)]. The corresponding data are shown in Table 1 and have been generated randomly using a similar procedure to the one described in the next section for the Monte Carlo experiments. Table 1 also shows the results of projecting each DMU onto the efficient frontier using Model (4) and using Equation (2) as slacks-normalisation coefficients. It can be seen that eight of the DFs (namely, DF1, DF4, DF7, DF8, DF10, DF11, DF16, and DF20) are efficient. The rest have technical inefficiencies measured by their $SBI_r$ inefficiency score. Table 1 shows the corresponding input and output slacks. The sum of the reductions of the input herd size is 147, which represents 6.77% of the total herd size of the sample. Similarly, the sum of the reduction in operating costs is EUR 246 288.5 (9.1% of the total OC of the sample). The sum of the output increases is much more modest, just 12 366 L of milk (0.12% of the total milk production of the sample).

Solving Model (1) with the slacks-normalisation coefficients [Equation (2)] and imposing [Equation (3)] to identify mergers of at least two DFs, the merger shown as Iteration 1 in Table 2 is obtained. This represents the
most advantageous merger and involves four relatively small DFs, namely DF3, DF11, DF15, and DF20. Note that of these four DFs, two are technically inefficient. Adding the corresponding \( \text{SBI}_r \) inefficiency scores of the learning effect component can be obtained and subtracting it from the overall merger efficiency gains \( \text{SBI}_r = 0.563 \) the pure merger efficiency gains can be obtained, i.e.:

\[
\text{SBI}_r = \text{SBI}_{DF3} + \text{SBI}_{DF11} + \text{SBI}_{DF15} + \text{SBI}_{DF20} =
\]

\[
= 0.086 + 0.000 + 0.063 + 0.000 = 0.149
\]

\[
\text{SBI}_{r,LE} = \text{SBI}_r - \text{SBI}_{r,LE} = 0.563 - 0.149 = 0.414
\]

Assuming that this most advantageous DF merger takes place, we can identify the second most advantageous merger in the sample. This can be done using the proposed Model (1) imposing the constraints:

\[
\delta_r = 0 \quad \forall r \in \{DF3, DF11, DF15, DF20\}
\]

The results obtained are shown in Table 2 on the Iteration 2 row. In this case, the merger involves just two DFs (DF2 and DF13). The estimated overall efficiency gains of this merger and its corresponding learning effect and pure merger efficiency gains are, respectively, \( \text{SBI}_r = 0.292 \), \( \text{SBI}_{r,LE} = 0.199 \) and \( \text{SBI}_{r,LE} = 0.093 \). In this case, most of the merger efficiency gains are due to learning effect. This is due to the fact that both DFs entering into the merger are rather inefficient. Note that although the objective function of Model (1) looks for the most advantageous merger from the point of view of the overall merger efficiency gains, the criterion chosen may be, alternatively, the merger that leads to the maximum pure merger efficiency gains.
Table 2. Illustrative dataset: Five most advantageous DF mergers

<table>
<thead>
<tr>
<th>Merger iteration</th>
<th>R</th>
<th>( \sum_{x_{ir}} \sum_{y_{kr}} )</th>
<th>((\hat{x}<em>{ir}, \hat{y}</em>{kr}))</th>
<th>((\hat{s}^x_{ir}, \hat{s}^y_{kr}))</th>
<th>(SBI^*_g)</th>
<th>(SBI^{LE}_{kr})</th>
<th>(SBI^{*}_{kr})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[DF3, DF11, DF15, DF20]</td>
<td>310</td>
<td>163</td>
<td>147</td>
<td>0.563</td>
<td>0.149</td>
<td>0.414</td>
</tr>
<tr>
<td></td>
<td>OC (EUR)</td>
<td>384 781.3</td>
<td>207 678.2</td>
<td>177 103.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MP (L)</td>
<td>990 704.0</td>
<td>990 704.0</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>[DF2, DF13]</td>
<td>245</td>
<td>163</td>
<td>82</td>
<td>0.292</td>
<td>0.199</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td>OC (EUR)</td>
<td>293 566.3</td>
<td>209 218.5</td>
<td>84 357.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MP (L)</td>
<td>994 516.0</td>
<td>994 516.0</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>[DF9, DF17]</td>
<td>211</td>
<td>153</td>
<td>58</td>
<td>0.219</td>
<td>0.107</td>
<td>0.112</td>
</tr>
<tr>
<td></td>
<td>OC (EUR)</td>
<td>263 566.0</td>
<td>195 572.3</td>
<td>67 993.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MP (L)</td>
<td>921 925.0</td>
<td>921 925.0</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>[DF6, DF12]</td>
<td>197</td>
<td>150</td>
<td>47</td>
<td>0.183</td>
<td>0.084</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>OC (EUR)</td>
<td>249 941.9</td>
<td>191 497.6</td>
<td>58 444.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MP (L)</td>
<td>899 992.0</td>
<td>899 992.0</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>[DF14, DF18]</td>
<td>216</td>
<td>166</td>
<td>50</td>
<td>0.125</td>
<td>0.033</td>
<td>0.092</td>
</tr>
<tr>
<td></td>
<td>OC (EUR)</td>
<td>228 862.7</td>
<td>211 635.7</td>
<td>17 227.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MP (L)</td>
<td>1 011 343.0</td>
<td>1 012 293.1</td>
<td>950.1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

DF – dairy farm; HS – herd size; MP – milk production; n – number; OC – operating cost; R – merging DFs; \(SBI^*_g\) – slacks-based inefficiency; \(SBI^{LE}_{kr}\) – technical efficiency (or learning effect) component; \(SBI^{*}_{kr}\) – pure merger efficiency gains; \(\hat{s}^x_{ir}\) – slack (i.e. potential improvement) of input \(i\) after the merger; \(\hat{s}^y_{kr}\) – slack (i.e. potential improvement) of output \(k\) after the merger; \(x_{ir}\) – amount of input \(i\) consumed by DF \(r\); \(\hat{x}_{ir}\) – target value of input \(i\) after the merger; \(y_{kr}\) – amount of output \(k\) produced by DF \(r\); \(\hat{y}_{kr}\) – target value of output \(k\) after the merger.

Successive optimal mergers identified using Model (1); in each case, the set \(R\) corresponds to the merging DFs; the results show the total inputs and outputs of those DF as well as the corresponding targets for the merged unit \((\hat{x}_{ir}, \hat{y}_{kr})\); the resulting improvements (i.e. input and output slacks), the total merger efficiency gain and the proposed decomposition of the merger efficiency gains (i.e. learning effect and pure merger efficiency gain) are also shown.

Source: Authors’ own calculations

That would just require substituting objective function of Model (1) by:

\[
SBI^*_r = \text{Max} \left( \frac{1}{m + s} \left( \sum_{i=1}^{m} \frac{\hat{s}^x_{ir}}{g_i} + \sum_{k=1}^{s} \frac{\hat{s}^y_{kr}}{g_k} \right) - \sum_{r=1}^{n} \delta_i SBI^*_r \right) \tag{5}
\]

Of course, this requires having previously solved Model (4) for each \(DF_r\).

Proceeding as before, we can compute the third, fourth, ..., most advantageous mergers, evaluating each of them in terms of their overall and pure merger efficiency gains. For the dataset considered in this illustration, the 3\textsuperscript{rd}, 4\textsuperscript{th}, and 5\textsuperscript{th} rounds of mergers involve each of the two DFs, technically inefficient in all cases. After those five mergers take place, of the original 20 DFs, only eight remain unmerged, and of those eight, all are efficient except two (DF5 and DF19). These unmerged, inefficient DFs can still benefit from the efficiency improvements computed by Model (4).

**EXPERIMENTAL RESEARCH**

**Experimental design.** To further explore the influence of different factors on some characteristics of dairy farm (DF) mergers, a Monte Carlo experiment has been designed. This section presents the factors and response variables considered and the results of the experiments carried out. For the sake of results consistency, in all the random instances generated, the total number of cows in the whole set of DFs was constant \((N = 10\ 000\ cows)\). The DFs were assumed to belong to a certain geographical region, and this was the total number of cows in that region. For each instance, the characteristics of the various farms in the sample were determined by four experimental factors which were considered to have a possible influence. As shown in Table 3, two levels were considered for each experimental factor. The first
two factors refer to the number of DFs and the herd size distribution, which correspond to the first input of the DEA model. Thus, experimental factor F1 is the number of farms to be considered (with 100 or 500 farms as low and high levels, respectively), while experimental factor F2 controls the similarity of the farms regarding the herd size. The herd size of each farm is generated using a normal distribution with mean $\mu = N/F1$, and a standard deviation of $\sigma = \mu/5$ or $\sigma = \mu/3$ (low and high levels of F2, respectively). This was done so that the sum of the herd sizes of all the farms equalled $N$.

The second input of the DEA model [operating costs (OC)] results from multiplying the DF herd size by an OC per capita ratio randomly generated using a uniform distribution $c_\mu \pm 0.2c_\mu$, where $c_\mu$ is the mean operating cost per animal in the area. According to the data gathered in DFs in the north of Spain, a value $c_\mu$ of EUR 1 250 per year was used.

The third and fourth experimental factors, F3 and F4, are related to the output of the DEA model (milk production). To generate this output, a single-output Cobb-Douglas production function was considered (Chen and Delmas 2012):

$$Y = K X_1^{\alpha_1} X_2^{\alpha_2} e / F$$  \hspace{1cm} (6)

where: $K$ – constant (total factor productivity); $X_1, X_2$ – the two inputs; $e$ – noise random variable (mean value = 1); $F$ ($z1$) – random inefficiency variable that reduces the actual output obtained with respect to the maximum technical production achievable.

Taking logarithms,

$$\ln Y = \ln K + \alpha_1 \ln X_1 + \alpha_2 \ln X_2 + e - f$$  \hspace{1cm} (7)

It has been assumed that $e \sim N(0; 0.03)$ and $f \sim |N(0; \sigma_f)|$ is a non-negative half-normal. Note that the RTS depend on the value of $\alpha_1 + \alpha_2$ so that CRS apply if $\alpha_1 + \alpha_2 = 1$ while if $\alpha_1 + \alpha_2 > 1$, the production function exhibits IRS (Coelli et al. 2005). Therefore, the last two factors of the experiments characterise the efficiency of the farms involved. Experimental factor F3 takes into account the percentage of farms exhibiting IRS (with their output using $\alpha_1 + \alpha_2 > 1$); for the remaining DFs, half are generated using CRS and the other half using DRS). Finally, factor F4 controls the values of the standard deviation of the inefficiency variable $\sigma_f$ (Table 3).

Regarding the complexity of the experiment, for each of the $2^4$ factor level treatments in this full factorial design, 10 random instances were generated, which makes a total of 160 problem instances. For each instance, the single most advantageous merger was calculated according to the proposed DEA model, and three measures of the solution found were recorded as response variables for the statistical analysis. The response variable considered were: i) num_cows: the reduction of the herd size due to the merger, ii) perc_farms: the percentage of farms that are chosen to enter into the merger, and iii) var_eff: the expected efficiency improvement due the merger, using the aggregated inputs and outputs

$$\left( \sum_{r \in R} X_{ir}, \sum_{r \in R} Y_{kr} \right)$$

as slacks-normalisation coefficients.

### RESULTS AND DISCUSSION

The proposed DEA merger model was solved for each of the 160 instances using Lingo 10.0, each run requiring a negligible amount of computation time (less than 1 second). The effects of the four experimental factors on the three response variables mentioned above were statistically analysed. Figure 1 shows some boxplots corresponding to the two more significant measures. A bigger influence of the factors can be seen in the percentage of farms that merge (perc_farms), mainly by the influence of factors F2 (disparity in size among the farms) and F1 (total number of farms in the region). Regarding the efficiency improvement response (var_eff), there is a clear influence of F1 in the results, with significant higher values for instances with a higher number of farms involved.
Focusing on these two measures, Figure 2 shows their average value for each level of the four factors. The percentage of farms involved in the merger increases up to 1.71 points when there is a higher disparity in the herd size of the farms (F2). Also, as there are 5 times more farms in the higher level of F1 with respect to the lower level, the percentage of farms involved in the merger decreases by 0.73 points on average. Regarding the num_cows measure, its mean value increases slightly (by an amount of 0.21) when the number of farms (F1) is higher, not showing any relevant effect for the other three factors. A half-normal plot confirms these results for F2 in the case of measure perc_farms and F1 for var_eff (Figure 3).

A Wilcoxon rank-sum test was performed with the results shown in Table 4. As it can be seen, the factors considered have an influence, mainly affecting the percentage of farms involved in the merger (perc_farms). Not only F2, as seen before, but also F1 and F3 give results that differ depending on the level of those factors. For the number of cows merged (num_cows), only the disparity in herd size seems to be significant. Regarding the var_eff measure, only the number of farms (F1) has an influence.

F4 does not seem to significantly influence the results for any of the three response variables considered. However, an analysis of the interactions among the four factors does not show relevant interactions, except for F1 and F4. Figure 4 shows the presence of interaction between those two factors for num_cows and perc_farms. In the latter case, which seemed to be statistically significant, the reduction in perc_farms for higher values of F1 is smaller when factor F4 (the technical inefficiency of the farms) is higher.

Summarising, the results indicate that: i) the greater the disparity in herd sizes, the more farms enter into the merger and the larger the reduction in the number of cows that the merger can obtain; ii) the number of farms that enter into the merger, expressed in relative terms as a percentage of the total number of farms, de-
increases as the total number of farms increases; iii) the number of farms that enter into the merger increases when more farms exhibit IRS; and iv) the expected efficiency improvement due to the merger increases as the number of farms in the region increases.

CONCLUSION

This paper has proposed a new DEA-based approach for identifying the most advantageous dairy farm (DF) merger. The model is flexible in the sense that it allows including additional constraints to reflect the preferences and circumstances of each specific application. It can also be used iteratively to identify multiple merger possibilities. For each merger identified, the proposed approach provides the estimated merger efficiency gains as well as its decomposition into a learning effect and a pure merger efficiency gain.

The proposed approach has been illustrated using a small dataset, showing the simplicity of the methodology and its usefulness. The dataset was generated randomly using actual unit costs and production rates.
The proposed model has also been used to carry out experimental research to gain insight into the factors that can influence the most advantageous DF merger characteristics in a region. It has been found that the disparity in the herd size of the farms in a region and the percentage of farms that exhibit IRS increase the number of farms that enter into the merger. The disparity of herd sizes also increases the number of cows that are not needed after the merger. Finally, as regards the expected efficiency improvement, it increases with the total number of farms considered.

These results can provide some hints regarding how to proceed with the required consolidation of herds to improve their profitability, especially in a sector so regulated as DFs, which are experiencing a push to increase their size in many parts of the world. Clearly, some other factors could also be taken into account for a full understanding of the impact of merging DFs on efficiency gains. Thus, for instance, Špička and Mackev (2015) discuss aspects such as the regional investment culture and macroeconomic performance as having an influence on milk production efficiency. Also, as suggested by Luik-Lindsaar et al. (2019), farm hygiene (somatic cell count) and managerial decisions (age of first calving, culling rate) can also have an influence.

REFERENCES


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