

# An estimation of subsurface total drainage quantity in non-steady state drainage flow, and its verification in loamy soils

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## ABSTRACT

The subsurface total drainage quantity is a very important hydrological indicator to solve the drainage problems in a field of water management in the landscape, especially in a situation after massive floods. Described in this paper is an estimation of the subsurface total drainage quantity, which was developed by the operation of a subsurface pipe drainage system in saturated, middle permeable soil under unsteady state drainage flow with the application of the Dupuit's assumptions and Darcy's law, by analytical approximation. The correctness and applicability of this estimation of the subsurface total drainage quantity was verified by field measurements on the loamy soils of an experimental watershed area of the Research Institute for Soil and Water Conservation (RISWC) Prague-Zbraslav, Czech Republic. The parameters and the shape of this subsurface total drainage quantity equation were also proved with the help of nonlinear regression analysis, with application of the method of Marquardt. This analytical approximation should serve as an elementary tool of water engineering practice for an immediate estimation of the values of subsurface total drainage quantities from field pipe drainage systems in saturated loamy soils. It requires only a minimum amount of information (fundamental soil hydrology data and drainage system basic design parameters) and its use is often viewed, it is simple, user-friendly and is possible for a wide range of drainage policies.

**Keywords:** analytical approximation; drain discharge; subsurface pipe drainage system; subsurface total drainage quantity; unsteady state drainage conditions

It is obvious, that subsurface total drainage quantity is one of the most important factors of drainage hydrology. The consequence of the subsurface total drainage quantity in the Czech Republic drainage hydrology conditions, especially in connection with massive floods of August 2002, is evident.

The detailed description of the way of derivation of the simple analytical approximation of the subsurface total drainage quantity mentioned above, was presented by the author on the Department of Land Use and Improvement, Environmental and Forestry Faculty, Czech University of Agricultural Prague for the evaluation of the landscape retention capacity (Kovář et al. 2001).

The correctness and applicability of this estimation of the subsurface total drainage quantity was verified by the field measurement on the loamy soils of an experimental watershed area of the Research Institute for Soil and Water Conservation (RISWC) Prague-Zbraslav, Czech Republic.

An availability of the simple analytical approximation of the subsurface total drainage quantity in non-steady state drainage flow on loamy soils was also proven by nonlinear regressions analysis, with the application of the method of Marquardt.

Tens of square kilometers in the flat area of the Central Czech Upland, situated at the Elbe river

basin, in the central part of the Czech Republic, were totally flooded by massive floods of August 2002. The problems with the high position of the ground water table level so far linger there.

To regulate ground water regime in this area, the design of a controllable subsurface drainage system with an adequate reservoir, and sponsored by EU funds, is under consideration.

An estimation of subsurface total drainage quantity should be the basis for the calculation of the reservoir capacity and for the design of the subsurface drainage system parameters.

At the present time, the strong cooperation between CAU Prague, Department of Land Use and Improvement and the Department of Water Management at the town office of Terezín, placed just in the middle of the flooded area, to manage the local landscape water regime, is supposed.

## MATERIAL AND METHODS

### Study area

The field area is part of the experimental watershed of RISWC Prague-Zbraslav with loamy soils, which was selected for the purpose of the

research (CUA Prague) to increase ecological stability, retention and accumulation of water in the landscape and comes under the watershed area of Cerhovice brook, which is situated in Central Czech Upland with an altitude of 350–500 meters above sea level. Total watershed surface area is in the neighbourhood of 7.3 square kilometers.

From the soil hydrology measurements, soil investigations and other drainage hydrology observations, the level of the approximately horizontal impervious layer, can be considered 3.5 m below the ground surface. Long-term annual average of precipitation amounts to 560 mm.

This experimental field area is drained by the subsurface horizontal parallel systematic drain system with the drain spacing  $L$  (m) = 30, average of the drain depth  $h_d$  (m) = 0.75, and diameter of the lateral drain  $r_0$  (m) = 0.06. This case represents a typical example of the low down subsurface drainage system of homogenous porous soils, which is characterized by the one value of saturated hydraulic conductivity and one value of drainable pore space. Those values were set up assuming high space, time, and experimental variability of soil hydraulic properties (Gribb et al. 2004).

The drainage discharge data used for verification were measured from June 2000 to July 2001. Measured values of the subsurface drainage rate were chosen between May 4 and May 17, 2001 after relatively more intensive precipitation (30 mm of recharge during of the May 4–May 6, 2001).

The initial position of the water table level, at the beginning of the tested period, was ap-

proximated from the records of the water levels in piezometers. During the water table recession by the subsurface pipe drainage system was not recorded any recharge (e.g. following the rain-falls, irrigations, heavy rains or floods) to the water table level.

### Theory of analytical approximation

The principle of derivation of analytical approximation of the subsurface total drainage quantity in non-steady state drainage flow is based on the analytical solution of Boussinesq's Equation (Boussinesq 1904), which describes the unsteady-state groundwater flow. Its linearized form can be expressed as:

$$HK \frac{\partial^2 h}{\partial x^2} = P \frac{\partial h}{\partial t} \quad (1)$$

where:  $K$  – saturated hydraulic conductivity (M/T),  $h$  – height of the water table level (M),  $P$  – drainable pore space or effective porosity (–),  $H$  – average depth of the aquifer (M),  $t$  – time (T),  $x$  – horizontal  $x$ -direction ( $x$ -coordinate) (M),  $M$  – unit of length,  $T$  – time unit

Equation (1) has the same shape as Fourier's equation for one-dimensional heat flow in orthogonal systems and can describe the unsteady-state saturated drainage flow, without any recharge to the water table. The analytical solution of the equation (1) is based, besides others, on application of the Fourier's half-range sine series.

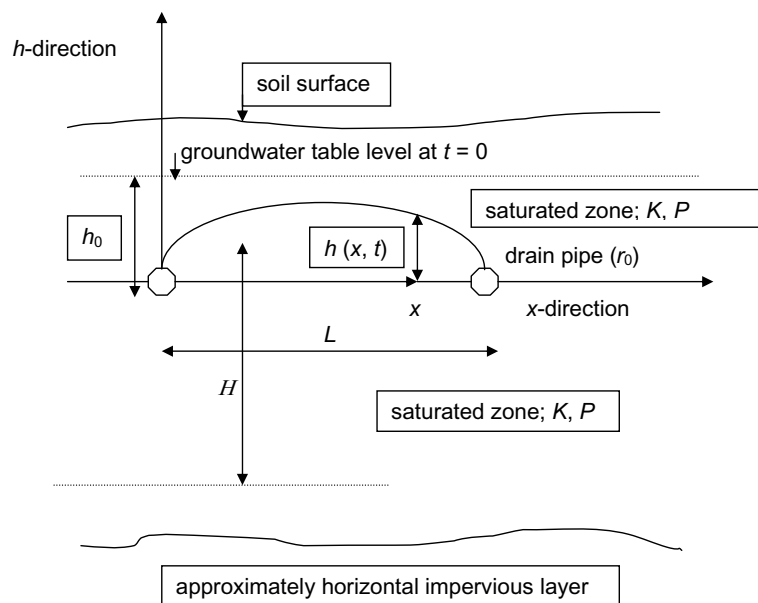


Figure 1. Height of the groundwater table  $h(x, t)$  at distance  $x > 0$  at time  $t > 0$  on saturated unsteady flow (long-depth of the impervious layer with loamy soils); all symbols of Figure 1 are precisely explained in the text on page 3 under the equation (1) and on page 4 under the equation (3)

Dumm and Glover (Wesseling 1969, Ritzema 1994) for the lowering of an initially horizontal water table level  $h_0$  (M) shaped:

$$h(x, t) = \frac{4h_0}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n} e^{-n^2 at} \sin\left(\frac{n\pi x}{L}\right) \quad (2)$$

For the next process will be used approximation with  $n = 1$ , with use of the first term of the equation (2) only, expressed as:

$$h(x, t) = \frac{4h_0}{\pi} e^{-at} \sin\left(\frac{\pi x}{L}\right) \quad (3)$$

where:  $h(x, t)$  – height of the water table (M) above the level of the drain at distance  $x$  (M) (from the drain pipe) at time  $t$  (T) (see Figure 1),  $h_0$  – initial water table level (M) at time  $t = 0$ ,  $L$  – drain spacing (M),  $t$  – time (T) after rise of the water table (after period of recharge),  $a$  – drainage intensity factor ( $T^{-1}$ ),  $a = (\pi^2 KH/L^2 P)$

Dumm (1954), Kraijenhoff van de Leur (1958), Štibinger (1985) showed the similar process of the analytical approximation of a linearized equation (1). The equation (3), which shows Figure 1, served as an initial step to get a final analytical form of the approximation of subsurface total drainage quantity, which can be written as:

$$Q(t) = h_0 P \left( 1 - \frac{8}{\pi^2} e^{-at} \right) \quad (4)$$

where:  $Q(t)$  (M) can be defined as the total drainage quantity in a certain time  $t > 0$ . It should be noted, that  $Q(t)$  (M) of an approximation (4), represents, from the point of view of physics, the scalar. It means volume, quantity, amount or mass (in this case expressed in units of length).

With physical laws, the ground water quantity scalar can be transformed to the drainage rate vector (drainage intensity, drain discharge) by the differentiation in time.

If the intensity of the drain discharge in any time  $t > 0$  is denoted by  $q(t)$ , it can be then described as:

$$q(t) = \delta[\text{water quantity}]/\delta t = \delta[Q(t)]/\delta t = (8h_0 Pa/\pi^2) e^{-at} \quad (5)$$

where: the parameter  $(8h_0 Pa/\pi^2)$  represents  $q(0)$  (M/T) and  $q(t)$  (M/T) represents drainage rate, discharge intensity, which is a variable quantity in time, expressed per unit surface area.

Equation (5), or its approximation, is used for the estimation of the drainage rate and is presented in the corresponding literature in a similar, or in the same, form. Ritzema (Ritzema 1994a) derived from Darcy's law drainage rate as:

$$q(t) = (8Kl'/L^2) h_0 e^{-at} \quad (6)$$

where:  $l'$  (M) is Hooghoudt's equivalent depth (M) of the soil layer below the level of the drain. By substituting  $a = (\pi^2 KH/L^2 P)$  into the first term of equation (5), which was derived from approximation (4), can be defined:

$$q(t) = (8KH/L^2) h_0 e^{-at} \quad (7)$$

It is obvious, that the Ritzema's derivation and expression (7) are identical and their physical laws are equal. The explorations, theory and analysis mentioned above affirm the correctness of the shape of the final equation (4), for the analytical approximation of the subsurface total drainage quantity in non-steady state drainage flow.

### Analytical approximation and nonlinear regression

The correctness of the shape of the final equation (4), for the analytical approximation of the subsurface total drainage quantity in non-steady state drainage flow can be validated by the nonlinear regression analysis, by the following method.

The data of the subsurface total drainage quantities  $Q(1), Q(2), \dots, Q(i), \dots, Q(n)$  at the corresponding time  $t_1, t_2, \dots, t_i, \dots, t_n$  were get from the experimental field area under the non-steady state drainage flow.

Soil conditions are described by hydraulic saturated conductivity  $K$ , by drainable pore space  $P$ , by the thickness of the permeable soil profile and by the initial position of the water table level  $h_0$ . The basic design parameters of the subsurface pipe drainage system are drain spacing  $L$ , drain depth  $h_d$ , and the diameter of the lateral drain  $r_0$ .

From the known values of the drainage system parameters and soil characteristics can be evaluated the drainage intensity factor  $a$ .

Equation (4) can be written as an integral curve by vector  $Y$ , vector  $X$ , parameter  $P1$ , parameter  $P2$  and constant  $h_0$  as:

$$Y = h_0 P1 \left( 1 - \frac{8}{\pi^2} e^{-P2 X} \right) \quad (8)$$

where:  $Y$  – the subsurface total drainage quantity  $Q(t)$  (M),  $X$  – time  $t$  (T),  $P1$  – drainable pore space  $P$  (–),  $P2$  – drainage intensity factor  $a$  ( $T^{-1}$ ),  $a = \pi^2 KH/L^2 P$ ,  $h_0$  – an initial position of the water table level (M) at a time  $t = 0$

Equation (8) mikes up in reality time series and can be shaped as

$$Y[Q(1), Q(2), \dots, Q(i), \dots, Q(n)] = h_0 P1 \left( 1 - \frac{8}{\pi^2} e^{-P2 X(t_1, t_2, t_3, \dots, t_i, \dots, t_n)} \right) \quad (9)$$

where the vector  $Y[Q(1), Q(2), \dots, Q(i), \dots, Q(n)]$  represents dependant variables and vector  $X(t_1, t_2, \dots, t_i, \dots, t_n)$  represents independent variables.

By nonlinear regression analysis of the equation (9) can be calculated parameter  $P1$ , which represents drainable pore space  $P$  (–) and parameter  $P2$ , which represents drainage intensity factor  $a$  ( $T^{-1}$ ) and it is possible to verify hydraulic properties of soils and drainage system investigation.

The vector  $Y[Q(1), Q(2), \dots, Q(i), \dots, Q(n)]$ , vector  $X(t_1, t_2, \dots, t_i, \dots, t_n)$ , constant  $h_0$  and the shape of the equation (9) are known values (information), unknown values of  $P1$  and  $P2$  will be calculated by nonlinear regression, with use the method of Marquardt (1963), known also as the method of Marquardt and Levenberg.

The estimation of the values of  $P1$  and  $P2$ , that fit the data best, is the goal of the nonlinear regression processing. An infinite number of curves by varying  $P1$  and  $P2$  will be generated. The sum-of-squares should be computed for each of the generated curves to estimate how well the corresponding curve fits the data (GraphPad Software, Inc. 2001).

It is supposed the  $x$ -axis represents  $P1$  variable and  $y$ -axis represents  $P2$  variable that should be fit by non-linear regression. The value of the sum-of-squares represents the  $z$ -axis. By this way will be in the orthogonal three-dimensional system ( $X, Y, Z$ ) molded the special spatial formation with its surface area. Each point of this surface area will correspond to one possible curve.

The non-linear regression goal is to locate the values of  $P1$  and  $P2$  that create the minimum of the sum-of-square. It means to find the bottom of this modeled surface area.

The key role of the method of linear descent is at the beginning of the iteration process, when works very well and is fast enough. But at the closing of the best-fit values (tested surface area is almost flat) works very slowly.

The Gauss-Newton method is suitable tool in the latter iterations, but at the start works very badly.

These two methods were blended in with the method of Marquardt to optimize an iterative algorithm.

In this manner it is possible to evaluate the values of  $P1$  and  $P2$ . The vector  $Y[Q(1), Q(2), \dots, Q(i), \dots, Q(n)]$  and corresponding vector  $X(t_1, t_2, \dots, t_i, \dots, t_n)$  represents known values, unknown values of  $P1$  and  $P2$  will be calculated by nonlinear regression, with the use of the method of Marquardt, described above.

The value of the parameter  $P1$  and  $P2$ , can be compared with the real values of the drainable pore space  $P$  (–) and with the real value of the drainage intensity factor  $a$  ( $T^{-1}$ ), which are known from the

soil conditions and drainage investigations of the experimental fields mentioned above.

Equation (9), which comprises (besides parameters  $P1$  and  $P2$ ) drainage characteristics and parameters from the experimental field, is identical with equation (4).

By evaluation of the parameters  $P1$  and  $P2$  and by its comparison with real soil conditions and drainage parameters, can be checked not only the time series of the subsurface total drainage quantities in non-steady state drainage flow, but also the correctness of the shape of the equation (9), in reality (4).

## RESULTS AND DISCUSSION

### Starting and calculated values

Equation (4) and equation (8) were verified by the results of field's tests measurements from the part of experimental watershed area of the Research Institute for Soil and Water Conservation (RISWC) Prague-Zbraslav, Czech Republic (Soukup et al. 2000).

The complete experimental field of 40.5 hectare with subsurface pipe drainage system was established in 1976/1977 by the RISWC Prague-Zbraslav, with an aim to evaluate and to describe the impact of the subsurface drainage policy on water regime in landscape.

By means of the measured data and with help of the selected hydraulic methods, the role of the subsurface drainage hydrology in land use can be explained and predicted.

Soil conditions of experimental field, the thickness of the soil profile, position of the groundwater table level and some other characteristics of soil hydrology were analysed in a part of the complex soil survey of the locality.

Soil conditions can be described by saturated hydraulic conductivity  $K$  (m/day) = 0.9, by drainable pore space  $P$  (–) = 0.07, by the approximative thickness of the middle permeable soil profile 3.5 (m) and by the initial water table level  $h_0$  (m) = 0.50.

Subsurface pipe drainage system comprises the drain spacing  $L$  (m) = 30, average of the drain depth  $h_d$  (m) = 0.75, and diameter of the lateral drain  $r_0$  (m) = 0.06.

A simple scheme of the drainage system parameters and soil conditions views Figure 2.

The results of the initial hydraulic calculations indicate that the value of  $l'$  (m) = 1.9, the value of  $H$  (m) =  $l' + (h_0/4)$  = 2.03 and indicate a value of the drainage intensity factor  $a$  ( $day^{-1}$ ) =  $(\pi^2 KH/L^2 P)$  = 0.285.

The measurement of the drainage of the subsurface flow to drains under the unsteady-state drainage conditions started at May 7, 2001 ( $t = 1$ ).

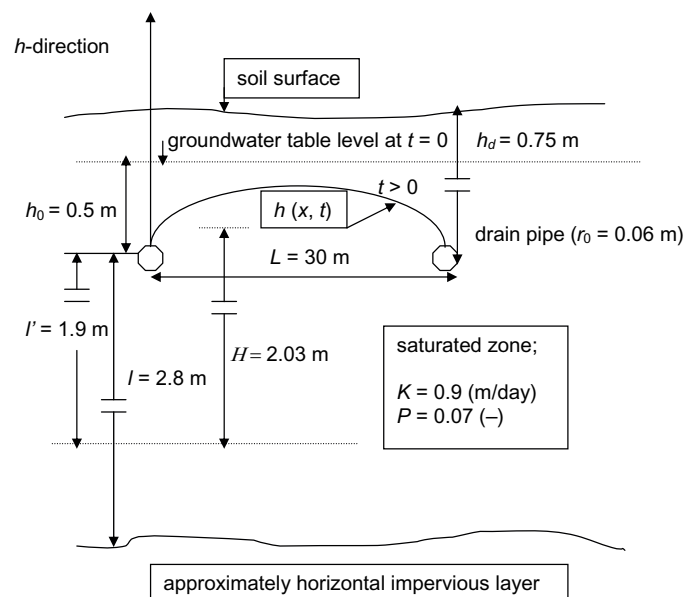


Figure 2. Drainage system parameters on non-steady state conditions of the RISWC Prague-Zbraslav experimental field; all symbols of Figure 2 are precisely explained and described in the text just at the end of page 565

The unsteady state drainage process was terminated at May 29, 2001, when the drainage rate fell under 0.1 mm per day. The measured value of the subsurface total drainage quantity from an experimental area at the end of the process (May 29, 2001) was approximately about 10 mm.

After May 16, 2001,  $t = 10$ , the daily value of the drainage rate were not measured continuously every day.

During the water table recession by the subsurface pipe drainage system no recharge was recorded (e.g., irrigation, heavy rains, following rainfall or floods) to the water table level.

It is supposed, that the validity of equation (4) [equation (8), respectively equation (9)] according Dieleman and Trafford (1976) will be defined from the point of time  $t$  (days) =  $0.4 (-)/a$  (days<sup>-1</sup>) =  $0.4/0.285 = 1.4$  days.

The daily values of the drainage rate (mm/day) were measured in the period from May 7 to May 16, 2001 and are shown on the second column in Table 1.

By the mass-drainage rate curve the daily values of the subsurface total drainage quantity (mm) were generated and are presented in the third column in Table 1.

Table 1. Measured and calculated daily values of the subsurface total drainage quantity from the RISWC Prague-Zbraslav experimental field

Days	Drainage rate (mm/day) measured values	Subsurface total drainage quantity (mm) measured values	Subsurface total drainage quantity (mm) calculated values	Subsurface total drainage quantity (mm), nonlinear regression values
1	14.5	14.4	13 .65	14.69
2	5.9	20.3	18.94	20.27
3	4.4	24.6	22.93	24.61
4	3.4	28.0	25.92	27.99
5	2.6	30.6	28.17	30.61
6	2.0	32.6	29.86	32.65
7	1.6	34.4	31.13	34.24
8	1.2	35.6	32.09	35.47
9	0.8	36.4	32.81	36.43
10	0.6	37.0	33.35	37.17



Calculations of the daily values of the subsurface total drainage quantity  $Q(t)$  (mm) according to equation (4), where  $t$  (day) is gradually 1, 2, 3, ...10 and drainage intensity factor  $a$  ( $\text{day}^{-1}$ ) = constant = 0.285, are shown in the fourth column of Table 1.

With the use of the nonlinear regression, actually with help of the Marquardt search algorithm, was determined the value of the parameter  $P1$  (–) and parameter  $P2$  ( $\text{day}^{-1}$ ) from the equation (8).

In equation (8), cumulative daily values of the subsurface total drainage quantity (mm), which are in the third column in Table 1, represent a Y-vector as a dependent variable (known values). The values of  $t$  (day) = 1, 2, 3, ...10 represent X-vector as an independent variable (known values).

The results of nonlinear regression model fitting show that  $P1$  (–) = 0.079,  $P2$  ( $\text{day}^{-1}$ ) = 0.251 and determination coefficient  $R$ -squared = 0.999.

This means, that according to nonlinear regression analysis drainable pore space will be  $P$  (–) = parameter  $P1$  = 0.079 and the value of the drainage intensity factor  $a$  ( $\text{day}^{-1}$ ) = parameter  $P2$  = 0.251.

The results of model fitting and the analysis of variance for the full regression are presented in Table 2. With the help of the known value of  $P1$  (–) and  $P2$  ( $\text{day}^{-1}$ ) and by X-vector [ $t$  (day) = 1, 2, 3, ...10], according equation (8), the new daily values of the subsurface total drainage quantity (mm) were recalculated and are shown in the last column in Table 1.

Cumulative data and calculated data by equation (4), inclusive recalculation by nonlinear regression, are presented in Figure 3.

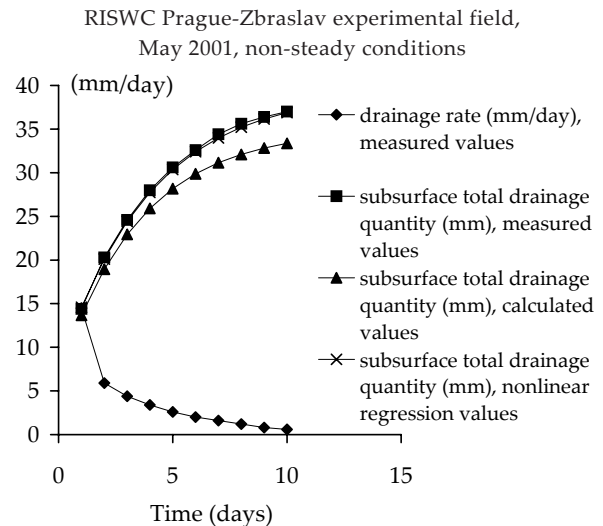


Figure 3. Measured and calculated daily values of the subsurface total drainage quantity from the RISWC Prague-Zbraslav experimental field

### Discussion of the analytical approximation results

On computing the experiment by equation (4) and its verification including nonlinear regression analysis, the data from May 7 to May 16, 2001 (for from  $t = 1, 2, 3, \dots t = 10$ ) were calculated.

From the comparison of the daily values of the subsurface total drainage quantity  $Q(t)$  (mm), calculated according to equation (4) for the RISWC Prague experimental field conditions and the cumulative daily values of the subsurface total drainage quantity from the same field (Figure 3) it

Table 2. Results of nonlinear regression analysis from the RISWC Prague-Zbraslav experimental field

Parameter	Estimation	Standard error	Ratio	
Model fitting results				
$P1$	0.07953	0.0003849	206.593	
$P2$	0.25161	0.0038267	65.751	
Total iterations = 6		Total function evaluations = 21		
Analysis of variance for the full regression				
Source	Sum of squares	Degrees of freedom	Mean square	Ratio
Model	0.009	2	0.05	99999
Error	0.0000002	8	0.000000	
Total	0.0091524			
Total (corr.)	0.0005147			
$R^2 = 0.999681$				

is obvious, that the shape of the curve of equation (4) and the shape of the curve of the cumulative daily values is identical, even the certain difference between the curves is apparent.

The course of the time series of the absolute magnitude of the differences [absolute magnitude from the daily cumulative values of the subsurface total drainage quantity minus  $Q(t)$  calculated by equation (4)] is monotone and very slightly increasing in the whole course.

This case shows that differences will be directly proportional with the values of the subsurface total drainage quantity. The higher values of the subsurface total drainage quantity, the greater the differences (errors).

The course of the differences can be viewed in Figure 4 and are presented in Table 3.

The biggest daily difference (approximately 3.64 mm, meaning 9.84% of the subsurface total drainage quantity in the time of the ending of the drainage process) in this tested time series, will be at the ending of the validity of equation (4) [approximately at time  $t$  (day) = 10] on the 10<sup>th</sup> day.

Other differences will be smaller. The minimal error is at the beginning of the unsteady state drainage process, respectively at  $t$  (day) = 2, because of validity equation 4 is from  $t$  (day) = 1.4.

This minimal error, the smallest (corresponding) difference of time  $t$  (day) = 2, will be less than 1.35 mm, meaning less than 3.65%.

The linearization, introduced by equation (1), is relevant for relatively deep barriers. This approximation (even as  $H$  was substituted by  $l' + h_0/4$ ), introduces errors in the estimations of the drain flow rates.

RISWC Prague-Zbraslav experimental field,  
May 2001, non-steady conditions,  
curve of differences

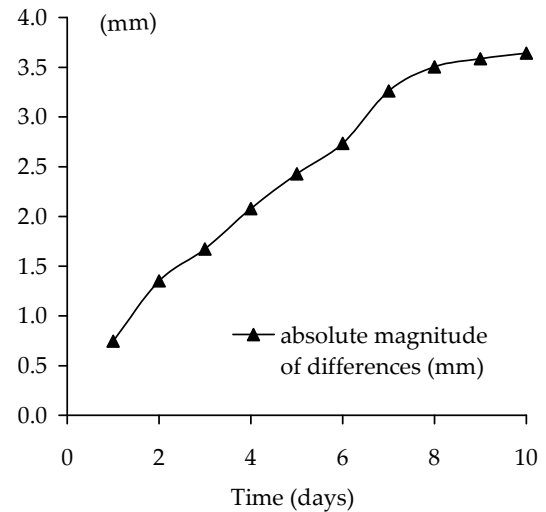


Figure 4. Course of differences between measured and calculated daily values of the subsurface total drainage quantity from the RISWC Prague-Zbraslav experimental field with application of equation (4)

The initially flat shape of the water table [ $h(x, 0) = h_0$  at  $t = 0$  for  $0 \leq x \leq L$ ] may explain the smaller calculated values than the measured and fitted values at the end of the tested period.

At the end of the tested unsteady state drainage process the difference will be 3.64 mm, meaning that 9.84%.

It seems that the equation (4) is useful for subsurface total drainage quantity approximation and pos-

Table 3. Differences between measured and calculated (by equation 4) daily values of the subsurface total drainage quantity from the RISWC Prague-Zbraslav experimental field

Days	Subsurface total drainage quantity (mm) measured values	Subsurface total drainage quantity (mm) calculated values	Differences (mm) absolute magnitude	Differences (%)
1	14.4	13.6	0.8	2.01
2	20.3	18.9	1.4	3.65
3	24.6	22.9	1.7	4.51
4	28.0	25.9	2.1	5.61
5	30.6	28.1	2.5	6.55
6	32.6	29.8	2.8	7.38
7	34.4	31.1	3.3	8.81
8	35.6	32.0	3.6	9.46
9	36.4	32.8	3.6	9.68
10	37.0	33.3	3.7	9.84

sibly approximates the real values of the subsurface total drainage quantity in these conditions.

### Discussion of the nonlinear regression model results

According to the nonlinear regression analysis effective will be  $P(-) = 0.079$ , measured value of  $P(-)$  is 0.07. The value of the drainage intensity factor  $a$  ( $\text{day}^{-1}$ ) estimated according nonlinear regression is 0.251 and the measured and calculated value of the drainage intensity factor  $a$  ( $\text{day}^{-1}$ ) is 0.285. It is obvious, that the differences between those indicators are negligible.

The high value of the determination coefficient  $R$ -squared = 0.999 reflects the particularly good predictive ability of the tested model, in this case represented by equation (8), respectively by equation (9).

It means that equations (8) and (9) explain very well the relation between  $t$  (as independent variable) and daily cumulative values of the subsurface total drainage quantity (as dependant variable).

The absolute magnitude of the differences between the daily cumulative values of the subsurface total drainage quantity and the values of  $Q(t)$  calculated by equation (9), but with nonlinear regression parameters, varies between 0.2 mm and 0.5 mm, which means between 0.47% and 1.15% (see Table 4).

The trend of the differences is increasing and directly proportional to the values of the subsurface total drainage quantity until the time  $t$  (day) = 7.

RISWC Prague-Zbraslav experimental field,  
May 2001, non-steady conditions,  
curve of differences on nonlinear regression applications

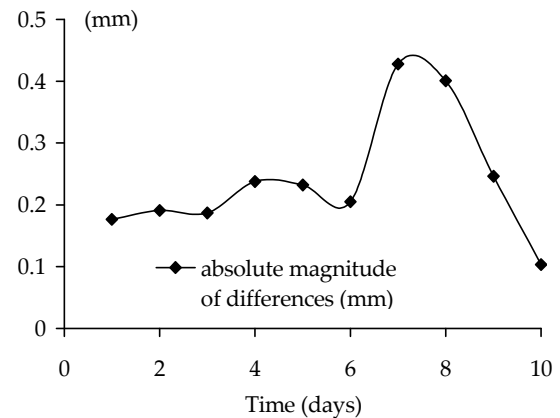


Figure 5. Course of differences between measured and calculated daily values of the subsurface total drainage quantity from the RISWC Prague-Zbraslav experimental field with nonlinear regression applications; the comment and the description of this course is presented on the page 569

From the time  $t$  (day) = 7 to the end of the drainage process the set-up will be reversed, the course of the differences continues to the monotone, but clearly decreasing (see Figure 5).

It seems that also the nonlinear regression analysis fully confirmed the correctness of the application of equation (4), as a good tool for subsurface total drainage quantity estimation during non-steady state drainage flow in porous medium.

Table 4. Differences between measured and calculated (by nonlinear regression) daily values of the subsurface total drainage quantity from the RISWC Prague-Zbraslav experimental field

Days	Subsurface total drainage quantity (mm) measured values	Subsurface total drainage quantity (mm) nonlinear regression values	Differences (mm) absolute magnitude	Differences (%)
1	14.4	14.5	0.1	0.47
2	20.3	20.1	0.2	0.51
3	24.6	24.4	0.2	0.51
4	28.0	27.7	0.3	0.64
5	30.6	30.3	0.3	0.64
6	32.6	32.3	0.3	0.64
7	34.4	33.9	0.5	1.15
8	35.6	35.1	0.4	1.08
9	36.4	36.1	0.3	0.64
10	37.0	36.8	0.2	0.51



## CONCLUSIONS

The correct estimation of subsurface total drainage quantity plays a key role in drainage policy and is necessary for the impact evaluation of existing subsurface drainage system or for the calculation of parameters of new ones.

Verification of analytical approximation of subsurface total drainage quantity, expressed by equation (4), showed a good conformity between calculations and measured data under unsteady state drainage flow in middle permeable soils. Verification in heavy soils were presented by the author at the end of his residency in July 2002 on Wageningen University, Department of Environmental Sciences, Sub-department Water Resources, Wageningen, The Netherlands.

The correctness of this analytical approximation of subsurface total drainage quantity was also validated by nonlinear regression analysis.

At the Department of Land Use and Improvement, Faculty of Forestry and Environment, Czech University of Agriculture in Prague runs research of the drainage testing in sandy soils and in gravel sandy soils. The partial results from this investigation allow the application of equation (4) also in very high permeable soil conditions.

The single final form of this analytical approximation of subsurface total drainage quantity allows its use also in an inversion situation.

From time series of the subsurface total drainage quantities and with the help of nonlinear regression analysis it is possible very easily and very quickly to determine and verify not only basic soil hydraulic properties (hydraulic saturated conductivity, effective drainable pore space) but also parameters of the subsurface drainage system.

Of course, in complex cases of drainage problems (unsaturated zone, cracked soils, transient drainage processes) the applications of the models as DRAINMOD (Skaggs 1999), SWAP (Dam 2000), MODFLOW or the other available models of similar type, may be necessary.

This analytical approximation should be used as a simple tool for immediate estimation of the value of subsurface total drainage quantity, before being further corrected and specified.

It should serve as a tool that requires only a minimum amount of information (the basic soil hydrology data and drainage system basic design parameters).

The verification of the field test results and measurements reflects that the possibilities of application and their user benefits, mentioned above, can be fulfilled.

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## ABSTRAKT

### **Hydraulický výpočet podzemního drenážního odtoku v podmínkách neustáleného drenážního proudění a jeho ověření v prostředí středně propustných půd**

Hodnota celkového podzemního drenážního odtoku je jedna z nejdůležitějších hydrologických charakteristik při řešení problémů souvisejících s odtokem vody v krajině, zejména pak v „po-povodňových“ situacích. Článek popisuje analytickou aproximaci hodnoty celkového podzemního drenážního odtoku, který byl vyvolán přítomností systematické trubkové drenáže v nasyceném, středně propustném prostředí hlinitých půd v podmínkách neustáleného drenážního proudění a za předpokladu platnosti Dupuitových postulátů a Darcyho zákona. Správnost a použitelnost hydraulického vyhodnocení celkového podzemního drenážního odtoku byla ověřena pomocí skutečných naměřených hodnot drenážních odtoků z experimentálního povodí VÚMOP Praha-Zbraslav v podmínkách neustáleného drenážního proudění na středně propustném prostředí hlinitých půd. Parametry a tvar rovnice pro odhad hodnot celkového podzemního drenážního odtoku byly též ověřeny modelem nelineární regrese s použitím Marquardtova algoritmu. Výše uvedená analytická aproximace by měla sloužit jako jednoduchý inženýrský nástroj vodohospodářské praxe pro okamžitý odhad hodnot celkových podzemních drenážních odtoků z drenážních systémů v nasyceném, středně propustném půdním prostředí. Tento prostředek vyžaduje pouze minimum informací (základní hydrologická a hydropedologická data, údaje o parametrech drenážního systému), jeho použití je názorné, jednoduché, uživatelsky příjemné a nalezne uplatnění v širokém rozsahu drenážní problematiky.

**Klíčová slova:** analytická aproximace; intenzita drenážního odtoku; podzemní trubkový drenážní systém; celkový podzemní drenážní odtok; neustálené drenážní proudění

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