

Comparison of Norway spruce (*Picea abies* [L.] Karst.), Scots pine (*Pinus sylvestris* L.) and European larch (*Larix decidua* Mill.) stem shape by means of geometrical methods

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ABSTRACT: In this article the stem shape is compared in three coniferous tree species: Norway spruce, Scots pine and European larch. Stem is investigated by means of geometrical methods. Simplified Bookstein coordinates (stem shape diameters) and Procrustes coordinates were used for variability investigation. The material, originating from the Czech and Slovak territories, involved in total 3,346 spruce stems, 3,082 pine stems and 1,403 larch stems. The accordance of mean stem vectors was assessed by means of Hotelling's T^2 two-sample test. For stem shape diameters and Procrustes tangent coordinates, the variability was examined using the method of principal components analysis. The three most important principal components were diagrammatized and described. The relationship between the stem shape and its size was also investigated, and inflection points of morphological stem curve were described for all three tree species.

Keywords: Norway spruce; Scots pine; European larch; stem shape; stem shape diameters; Procrustes coordinates; principal components analysis

Stem shape is one of the most important subjects of forest mensuration. Traditional forest mensuration describes the stem shape using form quotients, form series, stem profiles, form factors, geometrical bodies (neiloid, paraboloid, cone). In the last 25 years, not only "multivariate morphometrics" but also "geometrical methods" have been developed in biology. These methods use a finite number of points, called *landmarks*, for description of an object's shape. The landmark is a point of correspondence on each object that matches between and within populations (DRYDEN, MARDIA 1998). The shape is intuitively defined as general geometrical information that remains when the location, scale and rotational effects are filtered out of the object. In this investigation we used simplified Bookstein coordinates (stem shape diameters) and Procrustes coordinates for stem shape illustration. Data processing is given in Fig. 1.

Specific location of landmarks on a morphological stem curve enables to simplify the calculation of Bookstein coordinates since the effect of shifting and rotation need not to be removed (KŘEPELA et al. 2005).

Two objects have the same shape if they can be transformed, rescaled and rotated to each other so that they match exactly, i.e. if the objects are similar. In morphometry, definition of average shape and structure of shape variability is often necessary in a dataset. For that purpose, we mostly use multivariate analysis of stem shape diameters, generalized Procrustes analysis (GPA) (GOWER 1975, TEN BERGE 1977 in DRYDEN, MARDIA 1998) and principal components analysis.

Stem in traditional forest mensuration is the term related to relative form factors and relative form series. ŠMELKO (2000) on page 74 presents an example

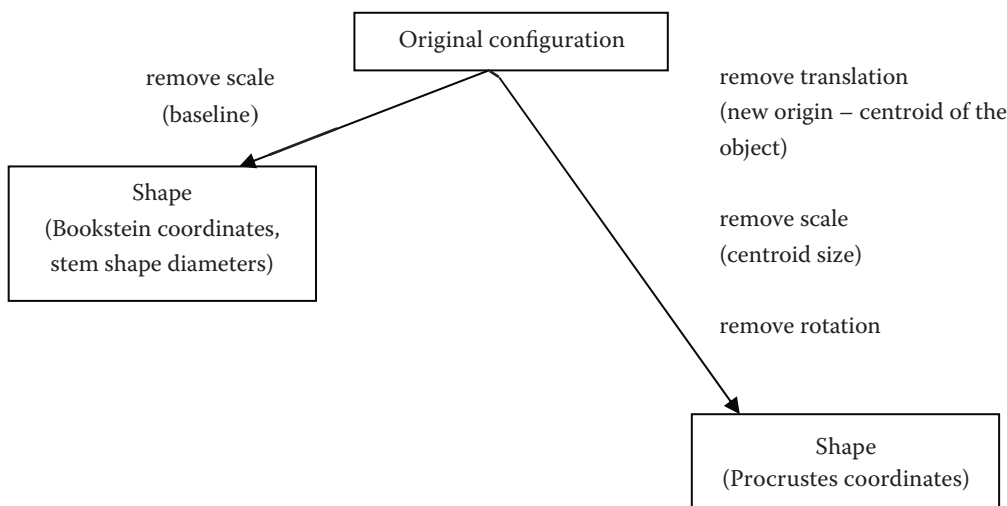


Fig. 1. Scheme of shape evaluation

of two spruce stems. The first stem is 20 m, the other is 10 m high. These stems have the same relative form series and the same relative form factors at 1/10 of the height. The size of relative form series is removed by means of diameter at 1/10 of the height and in the case of relative form factors by means of the volume of an ideal cylinder. Then according to traditional dendrometry the shape of both stems is the same. But according to geometrical methods the situation is different. In Figs. 2 and 3 both stems, represented by means of stem shape diameters and Procrustes coordinates, are not obviously of the same shape.

The aim of this paper is to compare mean stem vectors for three coniferous tree species, to investigate shape variability by means of principal components and to attempt to explain an ecological impact on principal components. The relationship between shape and stem volume will also be investigated as well as inflection points of morphological stem curve.

MATERIAL AND METHODS

The empirical material involved 3,346 spruce stems, 3,082 pine stems and 1,403 larch stems originating from the Czech and Slovak Republic territories. This material is nearly identical with the material used by PETRÁŠ (1989) for the construction of a mathematical model for shape height of stems. In this article stem diameters over bark and stem heights were used for assessments. Diameters were measured in 2m or 1m sections, stump diameter and dbh were also measured. Diameters were measured to the nearest 0.001 m, and stem length and stump height were found out as well. Stem length was measured to the nearest 0.1 m, stump height to the nearest 0.01 m. Characteristics of the experimental material are given in Table 1.

Diameters in improper sections were interpolated. The lower stem part (stump diameter, diameters at

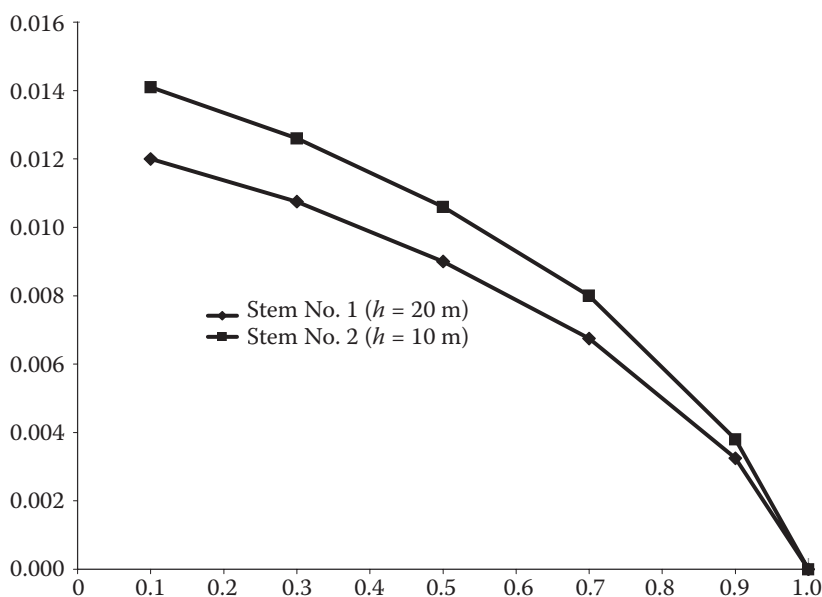


Fig. 2. Shape of both spruce samples (ŠMELKO 2000) illustrated by means of stem shape diameters

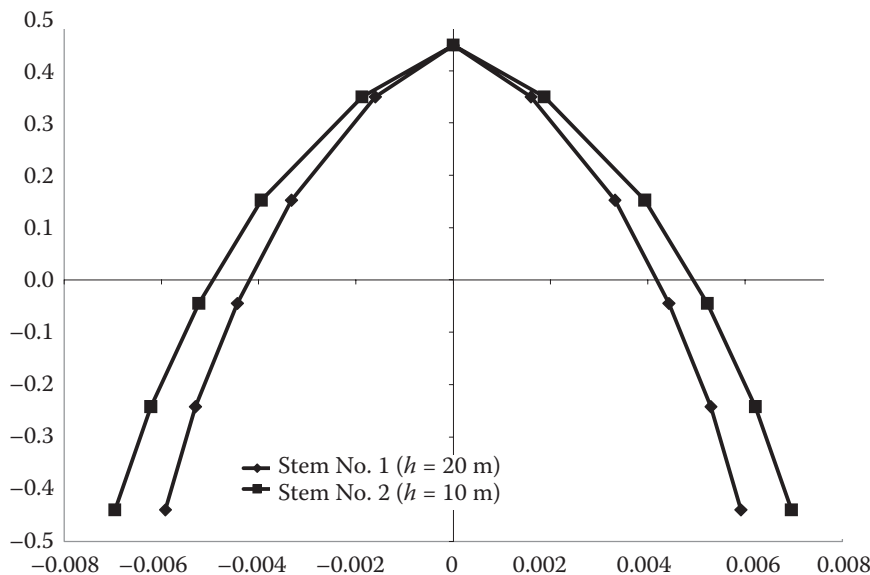


Fig. 3. Shape of both spruce samples (ŠMELKO 2000) illustrated by means of full Procrustes coordinates

1 m, 1.3 m and 2 m, 3 m) was balanced by means of power function $y = ax^k$, parameters of this function were derived for individual stems and then diameters at 1/100 and 1/40 of height were calculated. The remaining diameters (1/20 h , 1/10 h , 2/10 h , ..., 9/10 h) were calculated by linear interpolation.

The stem can be described as a multidimensional object by means of "stem shape diameters". Thus, the stem shape diameters (b_m) are the diameters at the relative sections (in this case $m = 1/100, 1/40, 1/20, 1/10, 2/10, \dots, 9/10$ of the stem height) divided by the stem height (h), therefore $b_m = d_m/h$.

Division by the height is in fact the elimination of the size from the object in the sense of intuitive definition of the shape.

Individual stem is therefore taken as a sample from n objects described by m dimensions (stem shape diameters (b_{ij})). Hence:

$$b_i = (b_{i,1}, \dots, b_{i,m})^T, i = 1, \dots, n$$

For this selection, it is possible to set a sample vector for mean values $\hat{\mu}$ given by the following equation:

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^n b_i \quad (1)$$

Estimation of variance-covariance matrix is ruled by the following equation:

$$S = \frac{1}{n-1} \sum_{i=1}^n (b_i - \hat{\mu})(b_i - \hat{\mu})^T \quad (2)$$

The test of hypothesis that the data are derived from multidimensional normal distribution

In this article, we use a test based on multidimensional skewness ($g_{1,m}$) and kurtosis ($g_{2,m}$), as described in MELOUN and MILITKÝ (1998). We test the simultaneous validity of a hypothesis about symmetry ($H_{01}: g_{1,m} = 0$) and about normality of kurtosis ($H_{02}: g_{2,m} = m(m+2)$) of the examined variable distribution. The estimation of sample skewness is given by the following equation:

$$\hat{g}_{1,m} = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n d_{ij}^3 \quad (3)$$

where: $d_{ij} = (b_i - \hat{\mu})^T S^{-1} (b_j - \hat{\mu})$ is squared Mahalanobis distance.

Considering the H_{01} hypothesis valid, then the test statistics

$$U_1 = \frac{n}{6} \hat{g}_{1,m} \quad (4)$$

Table 1. Characteristics of experimental material: h – total tree height, $d_{1.3}$ – overbark diameter at breast height, SD – standard deviation

	Norway spruce ($n = 3,346$)		Scots pine ($n = 3,082$)		European larch ($n = 1,403$)	
	h (m)	$d_{1.3}$ (cm)	h (m)	$d_{1.3}$ (cm)	h (m)	$d_{1.3}$ (cm)
Mean	22.1	25.9	16.2	19.7	25.4	30.2
Median	24.4	24.5	16.3	18.1	26.4	29
SD	10.8	16.8	7.5	13.2	9.4	16.9
Min.	1.9	1.3	2.2	1.2	4.1	4.1
Max.	49.9	80.5	36.6	63.9	46.8	85

has asymptotically chi-square distribution

$$\chi_{m(m+1)(m+2)/6}^2$$

The estimation of sample kurtosis is given by the following equation:

$$\hat{g}_{2,m} = \frac{1}{n} \sum_{i=1}^n d_{ii}^2 \quad (5)$$

Considering the H_{02} hypothesis valid, then the test statistics:

$$U_2 = (\hat{g}_{2,m} - g_{2,m}) / (8m((m+2)/n))^{0.5} \quad (6)$$

has asymptotically normal distribution $N(0,1)$. This approximation can be used providing the following condition is satisfied:

$$\hat{g}_{2,m} > m(m+2)(n-1)/(n+1) \quad (7)$$

Multivariate test of equality of covariance matrices

For k multivariate populations, the hypothesis of equality of covariance matrices is

$$H_0 : \Sigma_1 = \Sigma_2 = \dots = \Sigma_k$$

The test $\Sigma_1 = \Sigma_2$ for two groups is treated as a special case by setting $k = 2$. We assume independent samples of size n_1, n_2, \dots, n_k from multivariate normal distribution. To make the test, we calculate

$$M = \frac{|\mathbf{S}_1|^{v_1/2} |\mathbf{S}_2|^{v_2/2} \dots |\mathbf{S}_k|^{v_k/2}}{|\mathbf{S}_p|^{\sum v_i/2}} \quad (8)$$

where: $v_i = n_i - 1$,
 \mathbf{S}_i – the covariance matrix of the i -th sample,
 \mathbf{S}_p – the pooled sample covariance matrix.

If $\mathbf{S}_1 = \mathbf{S}_2 = \mathbf{S}_p$, then $M = 1$. As the disparity between \mathbf{S}_1 and \mathbf{S}_2 increases, M approaches zero.

Box (1949, 1950) in RENCHER (2002) gave χ^2 approximation for the distribution of M . This test is referred to as *Box's M-test*. We calculate

$$c_1 = \left[\sum_{i=1}^k \frac{1}{v_i} - \frac{1}{\sum_{i=1}^k v_i} \right] \left[\frac{2m^2 + 3m - 1}{6(m+1)(k-1)} \right] \quad (9)$$

then

$u = -2(1 - c_1) \ln M$ has approximately $\chi_{(k-1)m(m+1)/2}^2$ distribution,

where: M – defined in (8),

and

$$\ln M = \frac{1}{2} \sum_{i=1}^k v_i \ln |\mathbf{S}_i| - \frac{1}{2} \left(\sum_{i=1}^k v_i \right) \ln |\mathbf{S}_p| \quad (10)$$

We reject H_0 , if $u > \chi_{(k-1)m(m+1)/2}^2(\alpha)$.

The test of the hypothesis that the mean vectors are equal

Consider two independent random samples $\mathbf{b}_{x,1}, \dots, \mathbf{b}_{x,n}$ and $\mathbf{b}_{y,1}, \dots, \mathbf{b}_{y,n}$. The vectors $\mathbf{b}_i = (b_{i1}, \dots, b_{im})^T$, $i = 1, \dots, n$ are stem shape diameters. In this case $m = 12$ and $n = 3,346$ for Norway spruce, 3,082 for Scots pine and 1,403 for European larch. The test is provided for three pairs of tree species. We expect that stems from these populations have mean shapes $\boldsymbol{\mu}_x$ and $\boldsymbol{\mu}_y$.

The test of the hypothesis on mean vectors equality ($H_0: \boldsymbol{\mu}_x = \boldsymbol{\mu}_y$ versus $H_1: \boldsymbol{\mu}_x \neq \boldsymbol{\mu}_y$) can be carried out using Hotelling's T^2 two-sample test. Let us use the following test statistics:

$$F_{stat} = \frac{n_1 n_2 (n_1 + n_2 - m - 1)}{(n_1 + n_2)(n_1 + n_2 - 2)m} (\boldsymbol{\mu}_x - \boldsymbol{\mu}_y)^T \mathbf{S}_p^{-1} (\boldsymbol{\mu}_x - \boldsymbol{\mu}_y) \quad (11)$$

where:

$$\mathbf{S}_p = \frac{(n_1 - 1) \mathbf{S}_1 + (n_2 - 1) \mathbf{S}_2}{n_1 + n_2 - 2} \quad (12)$$

is the pooled variance-covariance matrix and \mathbf{S}_1 and \mathbf{S}_2 are variance-covariance matrices for individual samples.

We can express the squared Mahalanobis distance of equation (11) as

$$d_{xy} = (\boldsymbol{\mu}_x - \boldsymbol{\mu}_y)^T \mathbf{S}_p^{-1} (\boldsymbol{\mu}_x - \boldsymbol{\mu}_y) = \sum_{j=1}^m s_j^2 / \lambda_j \quad (13)$$

where: $s_j = (\boldsymbol{\mu}_x - \boldsymbol{\mu}_y)^T \mathbf{y}_j$ – the scores in the direction of the observed group difference,
 \mathbf{y}_j – eigenvectors of matrix \mathbf{S}_p ,
 λ_j – corresponding eigenvalues.

High values of s_j^2 / λ_j indicate which directions of shape variability are associated with the difference between the groups.

Provided the null hypothesis is valid, the test statistics F_{stat} has Fisher's distribution with m and $n_1 + n_2 - m - 1$ degrees of freedom. However, this test can be used only in the case of normality of both sets and homogeneity of variance-covariance matrices.

The assumption of normality and equal covariances turned out to be questionable. Therefore, a Monte Carlo test was carried out with the null hypothesis that the groups had equal mean shapes. The data were randomly split into two groups of the same size as the groups in the data, and the test statistic F_{stat} was evaluated for B random permutations T_1, \dots, T_B . The ranking r of the observed test statistic F_{obs} was then used to give the p -value of the test:

$$p\text{-value} = 1 - \frac{r - 1}{B + 1}$$

Variability

The *principal components analysis* (PCA) was used to analyze the shape variability. In principal components analysis, we seek to maximize the variance of a linear combination of the variables. The first principal component is the linear combination with maximal variance; we are essentially searching for such a dimension that the observation maximally separates or around which the observation data are spread out. The second principal component is the linear combination with maximal variance in a direction orthogonal to the first principal component, and so on.

The orthogonal eigenvectors of variance-covariance matrix, denoted by γ_i , $i = 1, 2, \dots, j$, are the *principal components* of variance-covariance matrix with corresponding eigenvalues

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_j \geq 0$$

where: $j = \min(n-1, m)$.

The principal components are in fact transformed variables and the principal component (PC) score represents transformed objects. PC score for the i -th individual on the j -th principal component is given by

$$s_{ij} = \mathbf{y}_i^T (\mathbf{b}_j - \hat{\boldsymbol{\mu}}) \quad (14)$$

The standardized PC scores are

$$c_{ij} = \frac{s_{ij}}{\sqrt{\lambda_j}} \quad (15)$$

Allometry

Allometry involves the study of relationships between shape and size, and in particular the manner how shape depends on size. Traditional methods in allometry involve the fitting of linear or non-linear regression equations between size and shape measures. Allometry can also be investigated in our geometrical framework, using regression. Principal component score is chosen as a response variable and total tree height is used as the explanatory variable.

Inflection point of morphological stem curve

The landmarks are investigated by geometrical methods. When we want to get the morphological stem curve, we must balance these individual points. Balancing was provided by the same function as used by PETRÁŠ (1989) for the construction of a mathematical model for the stem shape of coniferous tree species:

$$d_i(h_i) = b_1 h_i^{p_1} + b_2 h_i^{p_2} + b_3 \quad (16)$$

where: d_i – the stem diameter at height h_i ,
 b_1, b_2, b_3, p_1, p_2 – parameters of the function.

This function has the only inflection point. In this point the morphological stem curve transfers from the convex to concave course.

If we want to find the height coordinates of inflection point $I(d_i, h_i)$, we define the second function derivation equal to zero:

$$d_i''(h_i) = b_1 p_1 (p_1 - 1) h_i^{p_1 - 2} + b_2 p_2 (p_2 - 1) h_i^{p_2 - 2} = 0 \quad (17)$$

then

$$h_1 = \frac{p_2 - p_1}{b_2 p_2 (p_2 - 1)} \sqrt{\frac{-b_1 p_1 (p_1 - 1)}{b_2 p_2 (p_2 - 1)}} \quad (18)$$

RESULTS AND DISCUSSION

All the following tests are calculated for stem shape diameters. In the case of Norway spruce, Scots pine, European larch, the sample skewness is $\hat{g}_{1,12} = 54.14, 36.13$ and 44.87 , resp. The test statistics U_1 thus equals 30,190, 18,560, 10,490, which is more than the critical value of $\chi_{364}^2(0.05) = 409.49$. Sample kurtosis is $\hat{g}_{2,12} = 330.8, 285.4, 269.2$. Test statistics $U_2 = 256.9, 177.8, 103.4$ and the critical value of standardized normal distribution on the significance level of 0.05 is 1.64. The criterion (7) is satisfied because $\hat{g}_{2,m} > 167.9, 167.9, 167.8$.

In both quantities, skewness and kurtosis, we therefore reject the coincidence with normal distribution.

We also did *Box's M-test* for (a) Norway spruce and Scots pine sets, (b) Norway spruce and European larch sets, (c) Scots pine and European larch sets.

Test statistics:

$$(a) u = -2(1 - c_1) \ln M = 6,692.3 > \chi_{78}^2(0.05) = 99.3, \\ p\text{-value} < 0.001. \text{ We reject } H_0.$$

$$(b) u = -2(1 - c_1) \ln M = 2,147.2 > \chi_{78}^2(0.05) = 99.3, \\ p\text{-value} < 0.001. \text{ We reject } H_0.$$

$$(c) u = -2(1 - c_1) \ln M = 2,913.5 > \chi_{78}^2(0.05) = 99.3, \\ p\text{-value} < 0.001. \text{ We reject } H_0.$$

The Monte Carlo test was used for assessing the coincidence of mean vectors because of problems with normality and doubtfulness of equal variances. The coincidence of mean shapes was tested for (a) Norway spruce and Scots pine sets, (b) Norway spruce and European larch sets, (c) Scots pine and European larch sets. For each pair of samples, 2,000 random permutations were performed. In all three cases $p\text{-value} < 0.01$ and therefore we reject the null hypothesis about coincidence of mean vectors and accept the hypothesis about the difference between mean shape vectors.

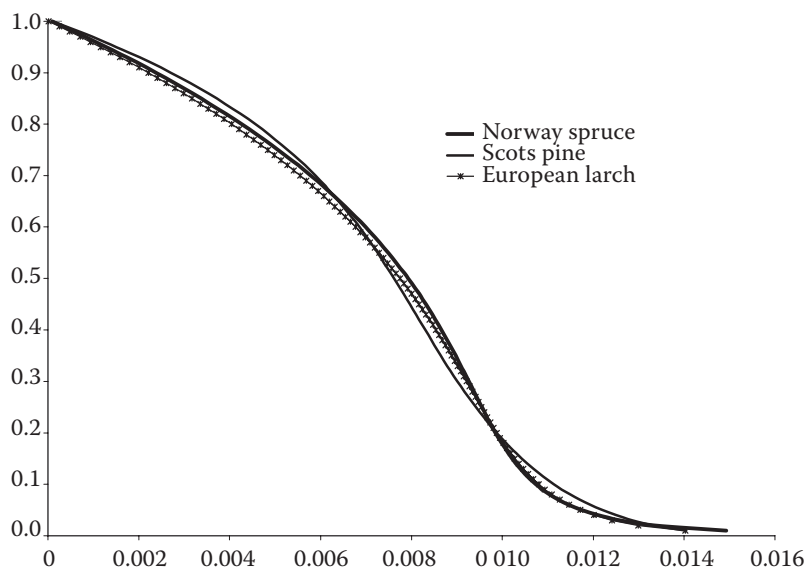


Fig. 4. Average stem shape of all three tree species investigated by means of stem shape diameters and balanced by function No. 16

Mean shapes for stem shape diameters are graphically represented in Fig. 4, and full Procrustes mean shapes for all three species are shown in Fig. 5. Only landmarks express the shape. However, these landmarks do not give a very clear illustration, therefore in the case of stem shape diameters they were balanced by function No. 16 and in the case of full Procrustes coordinates they were connected by means of abscissas. Both figures are very similar to each other. The landmarks are placed at $1/100 h$, $1/40 h$, $1/20 h$, $1/10 h$, $2/10 h$, ..., $9/10 h$. Norway spruce has the largest root swelling. The function of European larch behaviour is closer to Norway spruce than to Scots pine. Scots pine is wider between $1/40 h$ and $2/10 h$, and narrower than Norway spruce and European larch between $2/10 h$ and $6/10 h$ ($7/10 h$). Then Scots pine is markedly wider – the widest at $9/10 h$.

We also found intersection points of all three curves illustrated in Fig. 4 by means of FindRoot function in the programme Mathematica. The intersection point of the curve at about $0.2 h$ is noteworthy. Norway spruce with Scots pine intersects at $0.206 h$, Norway spruce with European larch at $0.2362 h$, Scots pine with European larch at $0.1987 h$.

The height coordinates of inflection points were calculated for all three tree species according to equation No. 18. They are for Norway spruce, Scots pine, and European larch: $0.29 h$, $0.43 h$ and $0.29 h$, resp. DEMAERSCHALK and KOZAK (1977) and PEREZ et al. (1990) found the relative height Z of the inflection point to range between 0.20 and 0.25 and between 0.15 and 0.35, respectively. In another study by ALLEN (1993), who used an average of 0.30, the inflection point was found to vary between 0.29 and 0.32 for small and medium-sized Caribbean pine

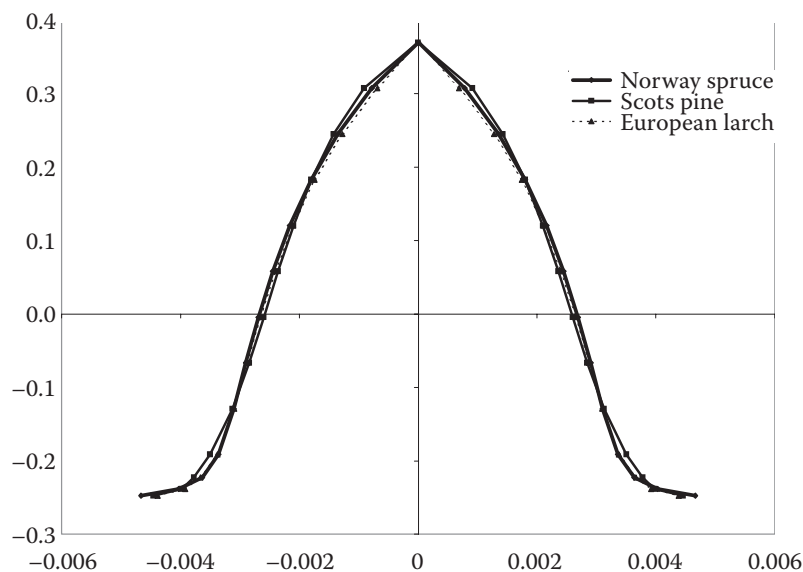


Fig. 5. Stem shapes of all three tree species expressed by full Procrustes mean shapes. The landmarks are connected by abscissas

Table 2. F_{stat} partition of Equation (11) for 12 principal components, for three pairs of tree species, d_{ij} is the squared Mahalano-bis distance

No. of principal components			F_{stat} partition				F_{stat}	d_{ij}
Norway spruce and Scots pine sets								
1–6	0.09	10.51	12.27	329.85	0.45	4.47		
7–12	10.21	0.59	1.78	1.42	15.87	0.02	387.54	2.9
Norway spruce and European larch sets								
1–6	0.67	6.96	31.4	1.02	22.62	7.25		
7–12	4.87	0.42	0.19	17.69	1.7	0.01	94.84	1.15
Scots pine and European larch sets								
1–6	0.21	1.87	12.97	178.68	5.64	2.97		
7–12	8.09	1.90	6.69	0.11	6.96	0.35	226.44	2.83

(*Pinus caribaea* Morelet var. *hondurensis* Barret and Golfari) trees.

The graphic effect of the first three principal components is the same in stem shape diameters as in Procrustes tangent coordinates. For better illustration and with regard to complex programmes prepared by DRYDEN (2000), we carried out an analysis of the first three principal components in Procrustes tangent coordinates as shown in DRYDEN and MARDIA (1998); the definitions were introduced in the same way as in KŘEPELA et al. (2004). Proportional expressions of variability explained by eigenvalues of the variance-covariance matrices of stem shape diameters and Procrustes tangent coordinates are given in Table 3.

Fig. 6 illustrates the graphic effect of the first three principal components for all three tree species. The first three principal components in all three sets show the same graphic effect.

The first principal component (PC 1 see Fig. 6) has a symmetrical graphical effect – the whole stem extension (narrowing) – and explains the great deal of variability (on average 95%). Its cause is not explainable from our data, but based on the preceding

research (KŘEPELA et al. 2001; KŘEPELA 2002), we take it for a competition effect: above-level – the thicker trees, below-level – the narrower ones.

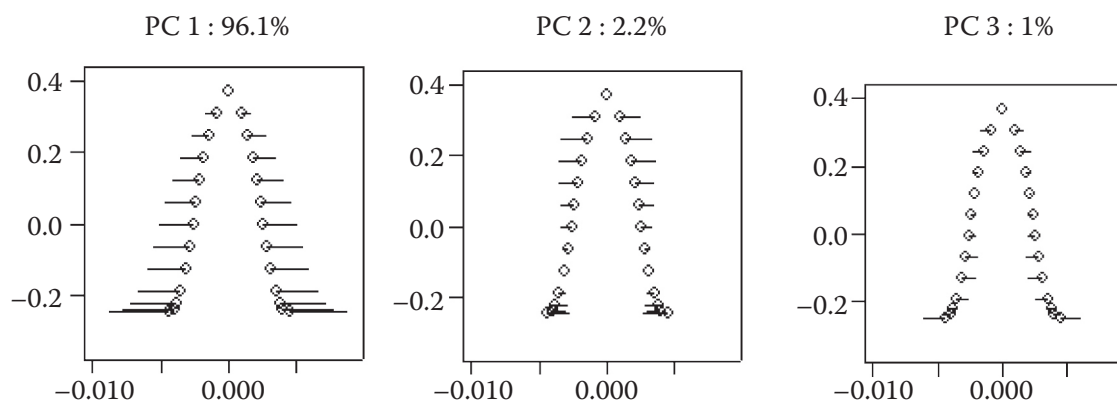
The second principal component (PC 2 see Fig. 6) has an asymmetrical graphical effect: the lower stem part versus the upper part. For Norway spruce up to $1/40 h$ to one side and for $1/20 h$ up to the other side. For European larch the boundary lies between $1/10 h$ and $2/10 h$ – and so higher than for Norway spruce. For Scots pine the boundary is at the highest level – at $2/10 h$. The second principal component for Norway spruce expresses 3.9% of variability – the most for all the tree species. For spruce its effect is the highest at $1/100 h$, and it is the effect of root swellings. For Norway spruce it is the lowest on the stem but is of the greatest importance.

The graphical effect of the third principal component (PC 3 see Fig. 6) changes its direction three times. At $1/100 h$ one direction, at $1/40 h$ null effect, then the reverse direction follows that is up to $4/10 h$ for Norway spruce, up to $5/10 h$ for European larch and up to $5/10 h$ for pine, then the reverse effect follows up to $9/10 h$ for Norway spruce and European larch. For Scots pine also with the exception of $6/10 h$,

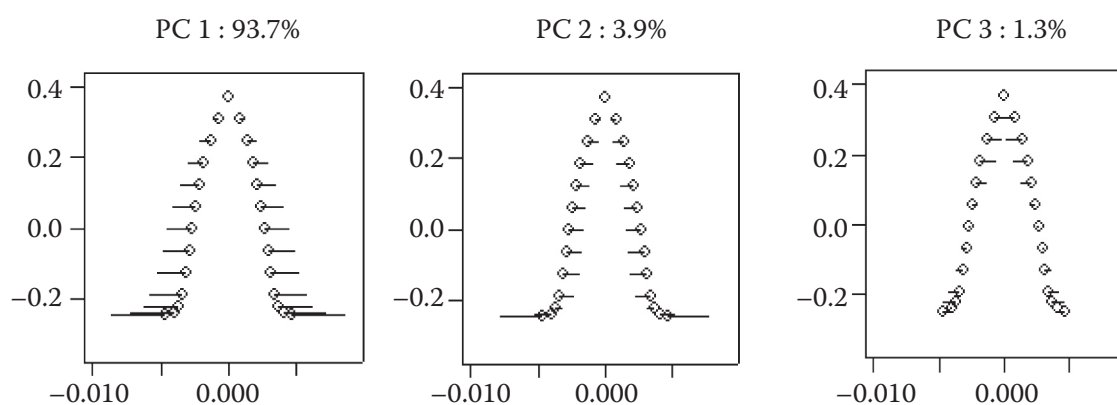
Table 3. Proportional expression of variability explained by eigenvalues of the variance-covariance matrices of stem shape diameters and Procrustes tangent coordinates from the investigated sets

Eigen-value	Norway spruce		Scots pine		European larch	
	Stem shape diameters (%)	Procrustes tangent coordinates (%)	Stem shape diameters (%)	Procrustes tangent coordinates (%)	Stem shape diameters (%)	Procrustes tangent coordinates (%)
	$\lambda_j / \sum_{j=1}^p \lambda_j \times 100$	$\lambda_j / \sum_{j=1}^p \lambda_j \times 100$	$\lambda_j / \sum_{j=1}^p \lambda_j \times 100$	$\lambda_j / \sum_{j=1}^p \lambda_j \times 100$	$\lambda_j / \sum_{j=1}^p \lambda_j \times 100$	$\lambda_j / \sum_{j=1}^p \lambda_j \times 100$
λ_1	93.8	93.7	96.1	96.1	96.1	96.1
λ_2	3.9	3.9	2.2	2.2	2.3	2.3
λ_3	1.3	1.3	1	1	1.1	1.1

Scots pine



Norway spruce



European larch

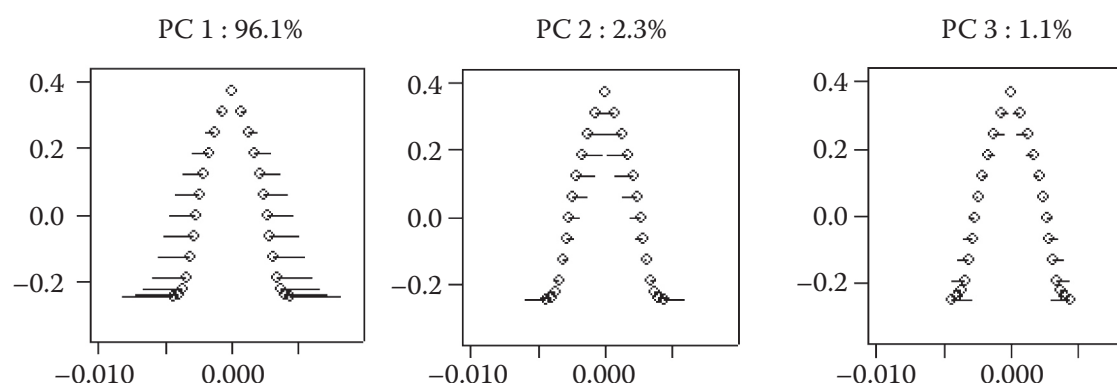


Fig. 6. The first three principal components (PC) with configurations evaluated for 1 (PC 1) and 3 (PC 2, 3) standard deviations along each PC from the full Procrustes mean shape, including proportions of explained variability

where we see the null graphical effect of the third principal component.

Another problem which principal components differ from each other the most was solved. Table 2 contains the components of test statistics F_{stat}^p calculated for individual principal components. Components of Mahalanobis distance s_j^2/λ_j indicate which directions of shape variability are associated with the difference between the groups. Spruce and pine differ the most from each other above all in the 4th principal component, pine and larch in the

4th principal component, spruce and larch in the 3rd principal component.

We also investigated the relationship between principal component score and total tree height (allometry). Principal component score is chosen as a response variable, and total tree height is used as an explanatory variable. The Spearman coefficients of correlation for both variables are calculated in Table 4. Scots pine has the lowest dependence between shape and height. The Norway spruce correlation coefficient between 4 PC score and height is worth

Table 4. The Spearman coefficient of correlation (r_s) for dependence between principal component score and total tree height. Significance is the significance level

PC score	Norway spruce		Scots pine		European larch	
	r_s	significance	r_s	significance	r_s	significance
1.	0.33	5.17E-79	-0.02	1.94E-01	0.40	3.34E-51
2.	-0.23	2.22E-39	0.17	3.92E-20	0.40	2.78E-51
3.	0.19	7.15E-29	-0.05	1.01E-02	0.38	4.17E-47
4.	0.49	1.01E-178	-0.17	2.70E-21	-0.07	1.22E-02
5.	-0.10	6.88E-09	0.02	3.63E-01	0.02	4.96E-01
6.	-0.02	2.74E-01	-0.16	1.03E-18	0.13	5.00E-07
7.	0.01	5.45E-01	-0.05	9.02E-03	0.11	1.86E-05
8.	0.02	1.62E-01	-0.07	4.34E-05	0.11	6.19E-05
9.	0.01	6.44E-01	0.00	9.21E-01	0.01	6.14E-01
10.	0.25	1.25E-45	0.05	7.76E-03	-0.04	1.71E-01
11.	-0.05	4.32E-03	0.05	9.25E-03	0.01	7.73E-01
12.	0.03	7.61E-02	0.10	1.81E-08	0.01	6.95E-01

of attention having the value 0.49, which is a moderate dependence. For larch the first three coefficients document a moderate dependence. Generally it can be said that the height influences the shape of all three tree species only very weakly and therefore the influences on the stem shape must be searched outside the stem.

CONCLUSION

This study showed that the stem shape of three investigated coniferous tree species (Norway spruce, Scots pine, European larch) differed. The term stem is understood in the sense of geometrical methods. In practice the simplified Bookstein coordinates (stem shape diameters) were used. Shape variability was investigated by the method of principal components using stem shape diameters and Procrustes tangent coordinates. The first principal component explains the prevailing part of variability, has a symmetrical graphical effect and we relate it to the competitive pressure onto individual trees. The second principal component has an asymmetrical graphical effect; we connect it with the effect of root swelling. The relationship between stem size (height) and shape showed to be weak, the weakest for Scots pine. Inflection points on the morphological stem curve were situated for Norway spruce, Scots pine, and European larch at: 0.29 h , 0.43 h and 0.29 h , resp.

References

- ALLEN P. J., 1993. Average relative stem profile comparisons for three size classes of Caribbean pine. *Canadian Journal of Forest Research*, 23: 2594–2598.
- DEMAERSCHALK J.P., KOZAK A., 1977. The whole bole system: A conditioned dual-equation system for precise prediction of tree profiles. *Canadian Journal of Forest Research*, 7: 488–497.
- DRYDEN I.L., MARDIA K.V., 1998. *Statistical Shape Analysis*. Chichester, New York, Weinheim, Brisbane, Singapore, Toronto, John Wiley & Sons: 345.
- DRYDEN I.L., 2000. *Statistical shape routines*. University of Nottingham. <http://www.maths.nottingham.ac.uk/personal/ild/Shape-R/Shap>.
- KŘEPELA M., SEQUENS J., ZAHRADNÍK D., 2001. Dendrometric evaluation of stand structure and stem forms on Norway spruce (*Picea abies* [L.] Karst.) sample plots Doubravčice 1, 2, 3. *Journal of Forest Science*, 47: 419–427.
- KŘEPELA M., 2002. Point distribution form model for spruce stems (*Picea abies* [L.] Karst.). *Journal of Forest Science*, 48: 150–155.
- KŘEPELA M., SEQUENS J., ZAHRADNÍK D., 2004. A contribution to the knowledge of Scots pine (*Pinus sylvestris* [L.] stem shape. *Journal of Forest Science*, 50: 211–218.
- KŘEPELA M., ZAHRADNÍK D., SEQUENS J., 2005. Possible methods of Norway spruce (*Picea abies* [L.] Karst.) stem shape description. *Journal of Forest Science*, 51: 244–255.

MELOUN M., MILITKÝ J., 1998. Statistické zpracování experimentálních dat. Praha, East Publishing: 839.
PEREZ D.N., BURKHART H.E., STIFF C.T., 1990. Avariable-form taper function for *Pinus oocarpa* Schiede in Central Honduras. *Forestry Science*, 36: 186–191.
PETRÁŠ R., 1989. Matematický model tvaru kmeňa ihličnatých drevín. *Lesnictví*, 35: 867–878.

RENCHER A.C., 2002. *Methods of Multivariate Analysis*. Chichester, New York, Weinheim, Brisbane, Singapore, Toronto, John Wiley & Sons: 708.
ŠMELKO Š., 2000. *Dendrometria*. Zvolen, TU: 399.

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Porovnání tvaru kmene smrku ztepilého (*Picea abies* [L.] Karst.), borovice lesní (*Pinus sylvestris* L.) a modřínu opadavého (*Larix decidua* Mill.) pomocí geometrických metod

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ABSTRAKT: Článek se zabývá porovnáním tvaru kmene tří jehličnatých dřevin: smrku ztepilého, borovice lesní a modřínu opadavého. Tvar je zkoumán pomocí geometrických metod. Byly použity zjednodušené Booksteinovy souřadnice (kmenové tvarové průměry) a pro zkoumání variability také Prokrustovy souřadnice. Materiál tvořilo celkem 3 346 kmenů smrku, 3 082 kmenů borovice a 1 403 kmenů modřínu. Tento materiál pochází z území České a Slovenské republiky. Shoda středních tvarových vektorů byla zkoumána pomocí Hotellingova T^2 testu vždy pro dva výběry. Variabilita byla zkoumána pro kmenové tvarové průměry a pro Prokrustovy tangentské souřadnice pomocí metody hlavních komponent. Byly znázorněny a popsány tři nejdůležitější hlavní komponenty. Dále byl zkoumán vztah mezi tvarem a velikostí a byly popsány inflexní body morfologické křivky kmene pro všechny tři dřeviny.

Klíčová slova: smrk ztepilý; borovice lesní; modřín evropský; tvar kmene; kmenové tvarové průměry; Prokrustovy souřadnice; analýza hlavních komponent

Pro popis tvaru kmene jsou v článku použity geometrické metody (DRYDEN, MARDIA 1998), které vycházejí z přesné definice tvaru. Tvar je definován jako geometrická informace o konfigurační matici po odstranění efektu posunutí, otočení a přeškálování. Tato konfigurační matice je složena ze souřadnic hraničních bodů, umístěných na morfologické křivce kmene. Klasická lesnická dendrometrie používá k vyjádření tvaru kmene rotační geometrická tělesa, pravé tvarové řady nebo pravé výtvarnice. V článku je na příkladu dvou smrkových kmenů, který uvádí ŠMELKO (2000), demonstrováno, že kmeny, které mají stejnou pravou tvarovou řadu a stejné pravé výtvarnice, nemusejí mít na základě geometrických metod stejný tvar (obr. 2 a 3).

Ke zkoumání tvaru kmene byly z geometrických metod použity zjednodušené Booksteinovy souřad-

nice (kmenové tvarové průměry) a dále Prokrustovy souřadnice. Všechny statistické testy byly provedeny s kmenovými tvarovými průměry. Prokrustovy souřadnice byly použity zároveň s kmenovými tvarovými průměry pro výzkum variability a znázornění efektu jednotlivých hlavních komponent, a to pro jejich lepší grafickou vypovídací schopnost a také pro srovnání obou geometrických metod.

Pokusný materiál tvořilo 3 346 kmenů smrku, 3 082 kmenů borovice a 1 403 kmenů modřínu. Tento materiál pochází z území České a Slovenské republiky. Je téměř totožný s materiálem, který použil PETRÁŠ (1989) ke konstrukci matematického modelu tvaru kmene jehličnatých dřevin.

Hraniční body byly umístěny na morfologickou křivku kmene do 1/100, 1/40, 1/20, 1/10, 2/10, ..., 9/10 výšky kmene. Podle schématu, znázorněného na

obr. 1, byly vypočteny tvarové kmenové průměry a Prokrustovy souřadnice pro všechny kmene.

Statistické testy směřovaly k porovnání středních tvarových vektorů tvarových kmenových průměrů všech tří zkoumaných dřevin pomocí Hotellingova T^2 testu. Před vlastním provedením tohoto testu jsme zkoumali vícerozměrnou normalitu kmenových tvarových průměrů jednotlivých dřevin a shodu variančně-kovariančních matic pro tři dvojice dřevin. V obou případech byly nulové hypotézy zamítnuty. Nezbyvalo tedy než provést neparametrický Monte Carlo test, kterým jsme zamítli nulovou hypotézu o rovnosti středních tvarových vektorů pro tři dvojice dřevin – tedy zkoumané dřeviny nemají ve smyslu geometrických metod stejný tvar.

Tvarová variabilita v rámci kmenových tvarových průměrů a Prokrustových souřadnic byla zkoumána pomocí metody hlavních komponent. Přehled výsledků podává obr. 6 a tab. 3. Procenta vysvětlené variability pomocí hlavních komponent jsou pro kmenové tvarové průměry i pro Prokrustovy souřadnice téměř shodná. Také geometrické efekty prvních tří hlavních komponent, které vysvětlují přes 99 % variability, jsou pro všechny tři dřeviny stejné. Ekologický vliv vyjádřený první hlavní komponentou způsobuje rozšiřování (zužování) kmene po celé jeho délce. Stejný efekt první hlavní komponenty popsal KŘEPELA et al. (2001) a KŘEPELA (2002). V těchto pracích byl zkoumán tvar kmene smrku podle Konšelových stromových tříd. Usuzujeme, že i v tomto případě se jedná o efekt konkurence. Nadúroveň – silnější stromy, podúroveň – užší. Přímý důkaz však nemůže být podán, protože u empirického materiálu nebyly zkoumány stromové třídy ani konkurenční indexy.

Druhá hlavní komponenta má asymetrický grafický efekt: spodní část kmene kontra zbývající horní část. Tato komponenta u smrku vyjadřuje 3,9 % variability

– tedy nejvíce ze všech dřevin. U smrku je její efekt v $1/100 h$ největší. Jde tedy o efekt kořenových náběhů. U smrku zasahuje na kmeni nejnižší, má ale největší váhu, což odpovídá prostému okulárnímu pozorování v přírodě. Grafický efekt třetí hlavní komponenty mění směr třikrát. Jaký ekologický vliv jej způsobuje, nebylo možné z našich dat zjistit.

Zajímavý problém představuje zkoumání závislosti mezi velikostí kmene a jeho tvarem. V případě kmenových tvarových průměrů představuje velikost výška kmene, neboť její pomocí je z objektu (kmene) odstraňována jeho velikost. V případě geometrických metod se vztah mezi velikostí a tvarem zkoumá pomocí korelační závislosti mezi velikostí a jednotlivými skóre hlavních komponent, kterých je 12, protože máme na kmeni 12 kmenových tvarových průměrů. Spearmanovy korelační koeficienty jsou demonstrovány v tab. 4. Tato závislost se ukázala nejnižší u borovice, kde ji můžeme s určitým nadhledem charakterizovat jako téměř žádnou. U modřínu se slabá závislost projevuje mezi prvními třemi skóre a výškou, u smrku je zajímavější 4. skóre. Celkově můžeme říci, že tvar kmene zkoumaných jehličnanů je závislý na jiných ekologických vlivech daleko více než na velikosti.

V dendrometrických pracích se často spodní část kmene vyjadřuje pomocí geometrického tělesa neiloidu a část nad ní pomocí paraboloidu. Právě inflexní bod, tedy bod, kde jedno těleso přechází v druhé, byl také předmětem našeho zkoumání. Geometrické metody pracují s hraničními body. Tyto hraniční body bylo nejdříve nutno vyrovnat pomocí funkce, která by zachytila morfologickou křivku kmene. Byla zvolena funkce č. 16. Ta má jediný inflexní bod, a jí byly vyrovnány střední tvarové průměry. Pro smrk, borovici a modřín vycházejí inflexní body do 0,29, 0,43 a 0,29 výšky kmene.

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