

## A contribution to the knowledge of Scots pine (*Pinus sylvestris* L.) stem shape

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**ABSTRACT:** This is a study on the evaluation of stem shape in Scots pine. The experiments were carried out on 430 stems without bark, taken from 30 fully analysed sample trees that grew in 5 different locations in the Czech Republic. Generalized Procrustes analysis was used as the method of study. The stems were described by the use of landmarks. Full Procrustes coordinates were calculated for all stems, and the full Procrustes mean shape was set for individual sample trees, locations and all stems. For Procrustes tangent coordinates, variability was examined using the method of principal components. The two most important principal components were diagrammatized and described. Furthermore, statistical tests of mean shape vectors for individual locations were carried out.

**Keywords:** Scots pine (*Pinus sylvestris* L.); stem shape; Procrustes analysis; principal components analysis

Stem shape study is one of the most important subjects of forest mensuration. Traditional forest mensuration describes the stem shape by the use of form quotients, form series, stem profiles, form factors. In the last 25 years, not only “multivariate morphometrics” but also “geometrical methods” have been developed in biology. These methods use a finite number of points, called *landmarks*, for description of the object’s shape. A landmark is a point of correspondence on each object that matches between and within populations (DRYDEN, MARDIA 1998). A shape is intuitively defined as all the geometrical information that remains when the location, scale and rotational effects are filtered out of the object. Two objects have the same shape if they can be translated, rescaled and rotated to each other so that they match exactly, i.e. if the objects are similar. In morphometry, it is often necessary to define an average shape and the structure of shape variability in a dataset. For that purpose, we mostly use generalized Procrustes analysis (GPA) [GOWER (1975), TEN BERGE (1977) in DRYDEN, MARDIA (1998)] and principal components analysis.

### MATERIAL AND METHODS

The objective of the study was to evaluate full Procrustes mean shape for all 430 stems of the thirty sample trees examined by the method of full analysis. Furthermore, we examined shape variability by the method of principal components for all stems, individual locations and sample trees. Examined stems come from five different locations. The locations Doubravčice, Brník and Oleška are situated

within the area of the University Forest Enterprise in Kostelec nad Černými lesy, the location Třeboň is situated in the area of the former forest enterprise Třeboň, and the location Manětín is in the former forest enterprise Plasy. For the detailed localization of the sources of the material, including mensurational characteristics of the source locations see SEQUENS (1985, 1994).

Selected dominant, exceptionally co-dominant sample trees were cut down, and their total stem length and timber to the top of 7 cm o.b. were measured. Consequently, width and length of green crown was measured together with one-year height increments. After debranching, the sample tree was divided into sections from the lower part of the stem up to the top section. The first section is 0.6 m long, the second 1.4 m, and all the following sections are 2 m long. These sections were cut in the middle to obtain the discs, i.e. each sample tree was cut at 0.3 m, 1.3 m, 3 m and then at every odd metre up to the middle of the top section of the stem and of the timber to the top of 7 cm o.b. The discs were examined by laboratory methods. Their diameter increments were measured using the Eklund mensuration instrument: yearly increments of the 1.3 section, five-year increments of the remaining sections. Consequently, height analysis was carried out. Table 1 shows the summary of characteristics of the examined material.

### Full ordinary Procrustes analysis

The scheme of shape evaluation, the procedure of the calculation of full Procrustes mean shape including the

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determination of full Procrustes coordinates using Kent's approach (1994) in DRYDEN and MARDIA (1998) was described in the Czech forestry literature in earlier articles – KŘEPELA et al. (2001) and KŘEPELA (2002). In this study, some basic terms are explained and shown on a simplified example:

Consider sample tree No. 26 (see Table 1), from which we choose two stems at the age of 102 and 57 years. We place landmarks at the 1/10 of their height and at the top of the stem. The origin of the system of coordinates is in the middle of the stem base.

Thereby, we obtain two  $k \times m$  configuration matrices  $X_1$  and  $X_2$  (in our example both matrices consist of coordinates from 3 points in 2 dimensions, thus  $k = 3, m = 2$ ):

$$X_1^T = \begin{bmatrix} -0.102 & 0.102 & 0 \\ 0 & 0 & 15.13 \end{bmatrix}, X_2^T = \begin{bmatrix} -0.043 & 0.043 & 0 \\ 0 & 0 & 7.94 \end{bmatrix}$$

A basic technique for comparing shapes is to superimpose them and then to note the differences in the positions of the landmark points.

We define a new origin, which is the *centroid* of the object – we remove translation. Configuration matrices have been centred by

$$X_C = CX$$

where  $C$  is centering matrix

$$C = I_k - \frac{1}{k} \mathbf{1}_k \mathbf{1}_k^T$$

$I_k$  is the  $k \times k$  identity matrix, and  $\mathbf{1}_k$  is the  $k \times 1$  vector of ones.

$$X_{1C}^T = \begin{bmatrix} -0.102 & 0.102 & 0 \\ -5.04333 & -5.04333 & 10.08667 \end{bmatrix}$$

$$X_{2C}^T = \begin{bmatrix} -0.043 & 0.043 & 0 \\ -2.64667 & -2.64667 & 5.29333 \end{bmatrix}$$

The *centroid size* is given by

$$S(X) = \sqrt{\sum_{i=1}^k \sum_{j=1}^m (X_{ij} - \bar{X}_j)^2}$$

where  $X_{ij}$  is the  $(i, j)$ -th entry of  $X$  and the arithmetic mean of the  $j$ -th dimension is

$$\bar{X}_j = \frac{1}{k} \sum_{i=1}^k X_{ij}$$

The centroid size is the square root of the sum of squared Euclidean distances from each landmark to the centroid. In our example:  $S(X_1) = 12.354, S(X_2) = 6.483$ .

Now, we compare both centroid configuration matrices using the method of *full ordinary Procrustes analysis* (full OPA).

The method of full OPA (DRYDEN, MARDIA 1998) involves the least squares matching of two configurations using the similarity transformations. Estimation of the similarity parameters  $\chi, \Gamma$  and  $\beta$  is carried out by minimizing the squared Euclidean distance:

$$D_{OPA}^2(X_{1C}, X_{2C}) = \|X_{2C} - \beta X_{1C} \Gamma - \mathbf{1}_k \chi^T\|^2$$

where  $\|X\| = (\text{Tr}(X^T X))^{1/2}$  is the Euclidean norm,  $\Gamma$  is an  $(m \times m)$  rotation matrix,  $\beta > 0$  is a scale parameter and  $\chi$  is an  $(m \times 1)$  location vector (it is not considered, as the configuration matrices have been centred).

$$\Gamma = UV^T$$

Matrices  $U$  and  $V$  can be obtained from *singular value decomposition* of matrix

$$\frac{X_{2C}^T X_{1C}}{\|X_{1C}\| \|X_{2C}\|}$$

$$\beta = \frac{\text{Tr}(X_{2C}^T X_{1C} \Gamma)}{\text{Tr}(X_{1C}^T X_{1C})}$$

Finally, full Procrustes coordinates of  $X_{1C}$  onto  $X_{2C}$  are

$$X_1^p = \beta X_{1C} \Gamma + \mathbf{1}_k \chi^T$$

In our example we have  $\Gamma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \beta = 0.5247711,$

$$D_{OPA}^2(X_{1C}, X_{2C}) = 0.0002216511,$$

$$(X_1^p)^T = \begin{bmatrix} -0.053527 & 0.053527 & 0 \\ -2.646594 & -2.646594 & 5.293191 \end{bmatrix}$$

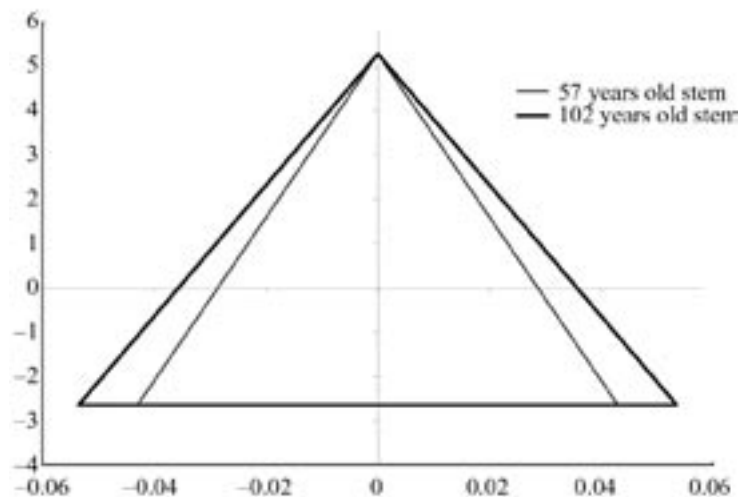


Fig. 1. Diagrammatized stem shape of a 102 years old Scots pine and a 57 years old Scots pine; basis in 1/10 of height. Triangles compared by the OPA method

Graphic matching of both triangles, which show diagrammatized stem shapes, is presented in Fig. 1. The 102 years old stem has a wider stem base than the 57 years old stem.

The term ordinary Procrustes refers to Procrustes matching of one observation onto another. Where at least two observations are to be matched, the term generalized Procrustes analysis is used.

Total amount of 21 landmarks was placed onto the stem curve of all the 430 stems. The bottom pair was placed at the height of 0.30 m, the remaining pairs by 1/10s of the stem height. Consequently, full Procrustes mean shapes and full Procrustes coordinates were calculated for each sample tree, location and, eventually, for all of the 430 examined stems. All statistical calculations were performed by the use of S-plus software using Statistical shape analysis routines written by DRYDEN (2000).

### Test of equality of the mean shape vectors for each location

Consider two independent random samples  $X_1, \dots, X_{n_1}$  and  $Y_1, \dots, Y_{n_2}$  from independent populations with mean shapes  $[\mu_1]$  and  $[\mu_2]$ . To test between

$$H_0 : [\mu_1] = [\mu_2] \text{ versus } H_1 : [\mu_1] \neq [\mu_2]$$

we could carry out *Hotelling's T<sup>2</sup> two sample test* in the Procrustes tangent space (DRYDEN, MARDIA 1998), where the pole corresponds to the overall pooled full Procrustes mean shape  $\bar{\mu}$  (i.e. the full Procrustes mean shape calculated by GPA on all  $n_1 + n_2$  individuals). Let  $v_1, \dots, v_{n_1}$  and  $w_1, \dots, w_{n_2}$  be the partial Procrustes tangent coordinates (with pole  $\bar{\mu}$ ). The multivariate normal model is proposed in the tangent space, where

$$v_i \sim N(\xi_1, \Sigma), \quad w_j \sim N(\xi_2, \Sigma), \quad i = 1, \dots, n_1; \quad j = 1, \dots, n_2.$$

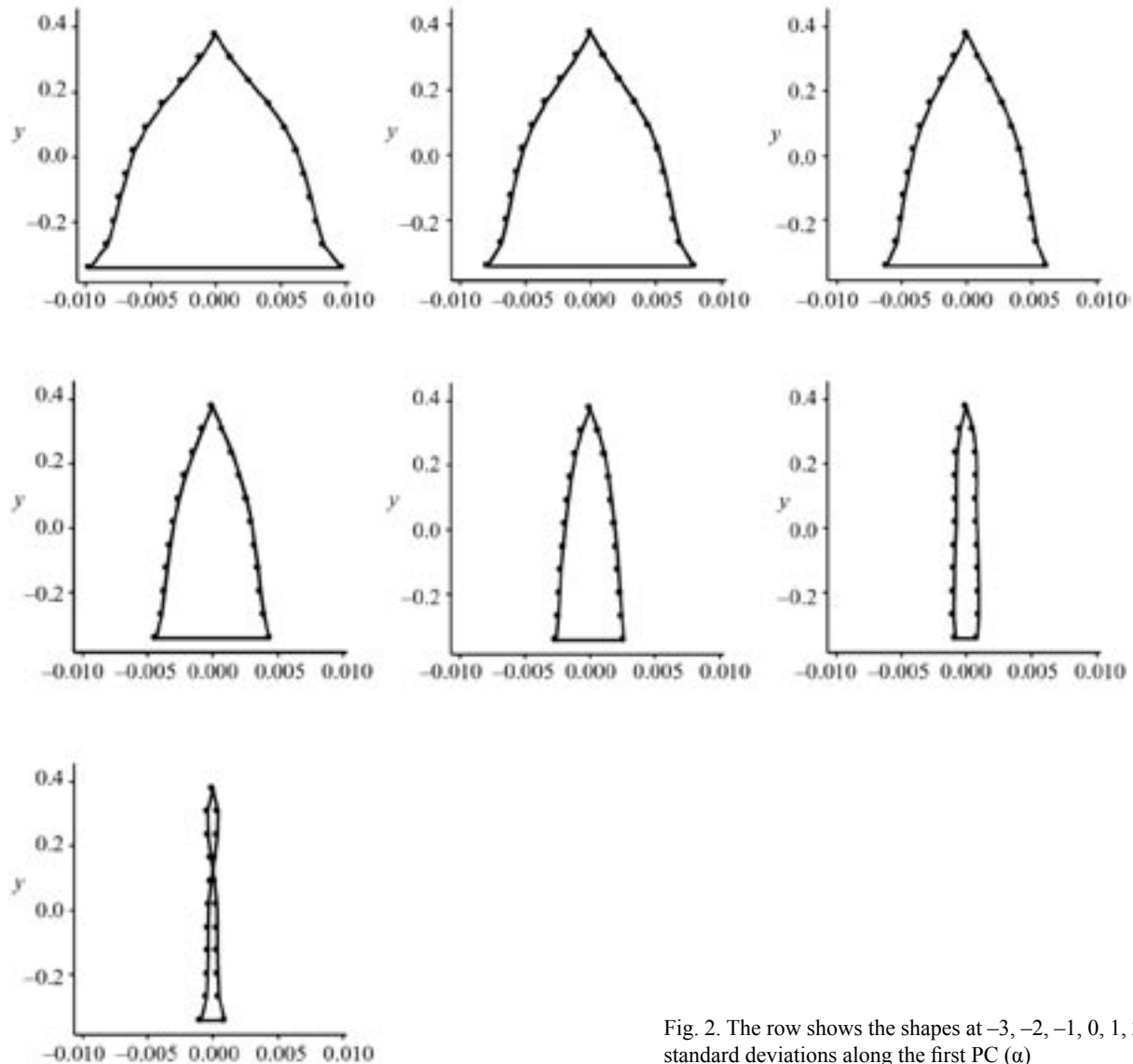


Fig. 2. The row shows the shapes at -3, -2, -1, 0, 1, 2, 3, standard deviations along the first PC ( $\alpha$ )

Table 1. Characteristics of the examined material

Location Group of forest types	Number of the sample tree	Number of stems	Age (years)	Height (m)	Volume (m <sup>3</sup> )	Centroid size (m)	PCs $\alpha$ ; $\beta$ (%)
Doubravčice 2 K	1	16	28–103	10.3–28.9	0.07–1.21	14.4–40.3	59; 25
	2	18	18–103	7.0–28.9	0.02–1.22	9.7–40.2	75; 12
	3	17	24–104	8.1–27.6	0.05–1.24	11.3–38.4	74; 2
	4	16	28–103	9.6–27.1	0.04–1.32	13.3–37.8	86; 6
	5	17	25–105	7.5–29.5	0.03–1.35	10.4–41.1	72; 23
Brník 2 M	6	15	33–103	7.6–23.0	0.03–0.74	10.5–32.0	81; 13
	7	14	35–100	7.9–23.8	0.03–0.58	11.0–33.1	60; 30
	8	14	38–103	7.7–25.5	0.02–1.01	10.7–35.5	80; 14
	9	15	30–100	8.1–25.0	0.02–0.67	11.3–34.8	77; 16
	10	15	30–99	7.0–24.6	0.01–0.60	9.7–34.2	77; 14
	11	8	29–64	7.0–19.6	0.02–0.36	9.7–27.3	74; 11
	12	9	25–65	8.5–22.7	0.03–0.61	11.8–31.6	74; 6
	13	10	23–68	7.2–21.8	0.01–0.48	10.1–30.4	87; 7
Oleška 0 M	14	10	49–94	7.8–22.0	0.01–0.40	10.9–30.6	92; 6
	15	11	41–91	6.7–21.6	0.02–0.38	9.3–30.1	90; 7
	16	11	45–95	7.3–20.6	0.03–0.46	10.2–28.7	88; 10
	17	11	40–90	6.7–19.6	0.02–0.23	9.3–27.2	98; 1
	18	10	45–90	8.0–21.4	0.03–0.52	11.1–29.8	79; 11
	19	11	42–92	7.0–22.0	0.01–0.36	9.7–30.6	18; 72
Třeboň 0 O	20	20	23–118	7.0–28.8	0.02–0.72	9.7–40.1	96; 2
	21	19	27–117	8.0–28.0	0.03–1.43	11.1–39.0	70; 16
	22	19	32–122	7.5–25.7	0.04–0.80	10.5–35.8	95; 3
Manětín 0 Q, 0 M	23	19	27–117	8.0–27.4	0.02–0.87	11.1–38.1	71; 22
	24	14	45–110	7.0–19.4	0.02–0.38	9.7–27.0	85; 8
	25	11	58–108	7.4–16.1	0.02–0.17	10.3–22.3	88; 7
	26	13	47–107	7.0–18.2	0.02–0.39	9.7–25.4	97; 3
	27	14	58–123	7.2–17.7	0.02–0.32	10.1–24.6	50; 34
	28	15	60–130	6.8–18.7	0.01–0.41	9.5–26.0	75; 21
	29	17	54–134	8.1–14.8	0.01–0.15	8.1–20.6	89; 8
	30	21	32–132	8.1–18.7	0.03–0.38	11.3–26.0	76; 17

Parameters  $\xi_1, \xi_2$ , and  $\Sigma$  are estimated by sample means  $\bar{v}, \bar{w}$  and sample variance-covariance matrices  $S_1, S_2$  (with divisors  $n_1 - 1$  and  $n_2 - 1$  in each group). The Mahalanobis distance squared between  $\bar{v}$  and  $\bar{w}$  is

$$D^2 = (\bar{v} - \bar{w})^T S^{-1} (\bar{v} - \bar{w}), \text{ where}$$

$$S = \frac{(n_1 - 1) S_1 + (n_2 - 1) S_2}{n_1 + n_2 - 2}$$

and  $S^{-1}$  is the Moore-Penrose generalized inversion of  $S$ . Under  $H_0$  we have  $\xi_1 = \xi_2$ , and we use the test statistic

$$F_{stat} = \frac{n_1 n_2 (n_1 + n_2 - M - 1)}{(n_1 + n_2) (n_1 + n_2 - 2) M} D^2$$

$M$  is the dimension of shape space. The null hypothesis is rejected if  $F_{stat} \geq F_{M, n_1 + n_2 - M - 1}(\alpha)$ .

In our case,  $\alpha = 0.05$ .

Subsequently, a test of homogeneity of covariances was carried out. A formal test such as Box's M test [Box (1949) in MARDIA et al. (1979) and in HAVRÁNEK (1993)]:

$$H_0 : [S_1] = [S_2] \text{ versus } H_1 : [S_1] \neq [S_2]$$

we use the test statistic

$$M_B = \frac{(n_1 + n_2 - 2) \ln(\det S) - (n_1 - 1) \ln(\det S_1) + (n_2 - 1) \ln(\det S_2)}{1 - \frac{2M^2 + 3M - 1}{6(M + 1)} \left[ \frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} - \frac{1}{n_1 + n_2 - 2} \right]}$$

The null hypothesis is rejected if  $M_B \geq \chi^2_{M(M+1)/2}(\alpha)$ .

The assumption of equal variances turned out to be doubtful. Therefore, a Monte Carlo test was carried out with the null hypothesis that the groups have equal mean shapes. The data are randomly split into two groups of the same size as the groups in the data, and the test statistic

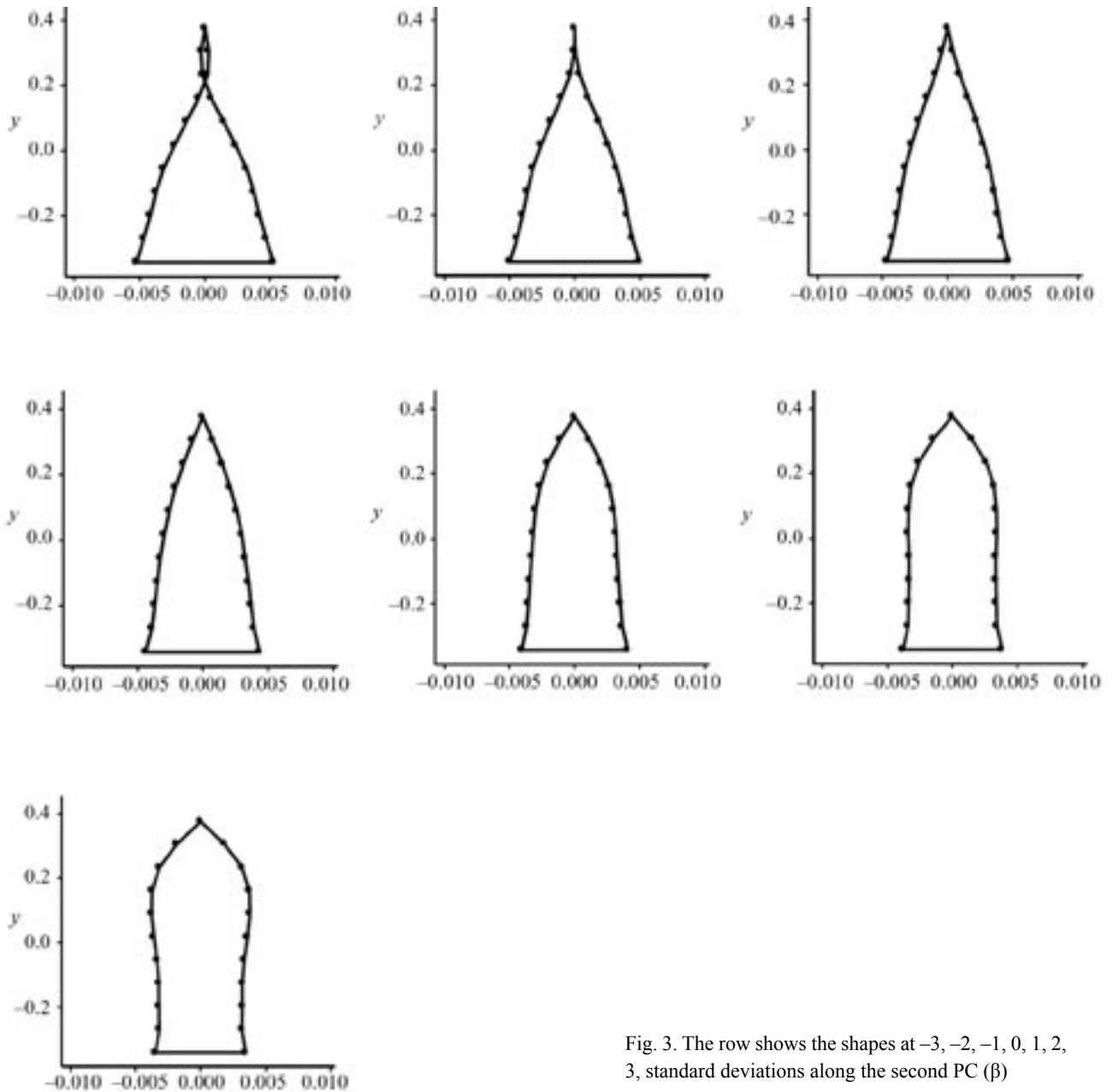


Fig. 3. The row shows the shapes at -3, -2, -1, 0, 1, 2, 3, standard deviations along the second PC ( $\beta$ )

$F_{stat}$  is evaluated for  $B$  random permutations  $T_1, \dots, T_B$ . The ranking  $r$  of the observed test statistic  $T_{abs}$  is then used to give the  $p$ -value of the test:

$$p\text{-value} = 1 - \frac{r-1}{B+1}$$

Table 2. Eigenvalues of the variance-covariance matrix from the set of Scots pine stems and proportional expression of variability explained by them

Eigenvalue	$\lambda_j \cdot 10^{-3}$	$\lambda_j / \sum_{j=1}^p \lambda_j \cdot 100$ (%)
$\lambda_1$	1.691	87
$\lambda_2$	0.496	8
$\lambda_3$	0.290	3

For each pair of locations, 1,000 random permutations were performed.

### Variability

The *principal component analysis* was used to analyse the shape variability. The sample variance-covariance matrix of tangent coordinates was calculated for the tree set:

$$S = \frac{1}{n} (\mathbf{v}_i - \bar{\mathbf{v}}) (\mathbf{v}_i - \bar{\mathbf{v}})^T$$

where  $\bar{\mathbf{v}} = \frac{1}{n} \sum \mathbf{v}_i$

The orthogonal eigenvectors of  $S$ , denoted by  $\chi_j$  are the principal components of  $S$  with corresponding eigenvalues  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_j \geq 0$ , where  $j = \min(n-1, M)$ .

## RESULTS AND DISCUSSION

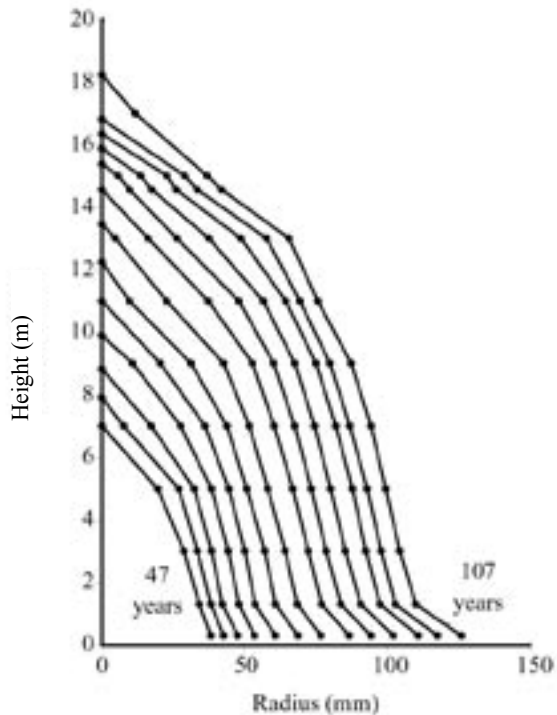


Fig. 4. Full stem analysis of the sample tree No. 26

The principal component (PC) score for the  $i$ -th individual on the  $j$ -th principal component is given by

$$s_{ij} = \mathbf{x}_i^T (\mathbf{v}_j - \bar{\mathbf{v}}_j)$$

The standardized PC scores are

$$c_{ij} = \frac{s_{ij}}{\sqrt{\lambda_{ij}}}$$

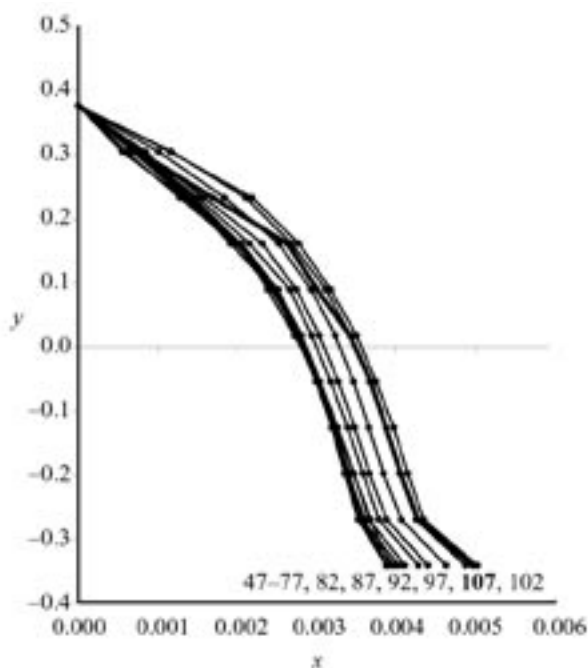


Fig. 5. Stem shapes of the sample tree No. 26 expressed by full Procrustes coordinates

As can be seen in Table 2, the first three principal components (PC) include 98% of all variability. Out of it, the first two cover 95% of the variability. Figs. 2 and 3 show a series of stem shapes evaluated in accordance with the first two PCs. In each row, the middle plot is the mean shape. First PC is labelled ( $\alpha$ ). Similarly like in spruce, see KŘEPELA et al. (2001) and KŘEPELA (2002), the first PC is symmetric, it points across the vertical stem axis. The second PC is asymmetric. It has an opposite direction in the bottom section (up to 4/10 of the stem height) and top section of the stem. This PC is labelled ( $\beta$ ) and it has not occurred at spruce amongst the first three principal components in the cited works of the authors, nor has the third PC shown in Table 2 occurred at spruce. It is not caused by the effect of buttresses or hypertrophy of the lower part of the stem. Representation of the first two PCs for each sample tree is shown in Table 1. Full stem analysis of sample tree No. 26 between the age of 47 and 107 years is presented in Fig. 4. Here, considerable increment in the last five-year period is noticeable. Figs. 5 and 6 show shape changes in sample tree No. 26. In the age group of 47–77 years, we see only slight changes of PC ( $\alpha$ ). In the age group of 77–107 years, the stem steadily widens in its full length. See also diagrammatized Fig. 1, where the triangle basis in 1/10 of the stem has widened. In the age group of 102–107 years, the examined stem had a considerable height increment. The increment was not accompanied by a corresponding diameter increment. It means that PC ( $\beta$ ) changed considerably.

Mean shape of Scots pine within the Třeboň location differed from those in all other locations, except the Oleška location. The Manětín location differed from the Brník and Doubravčice locations. In the other location pairs, the test did not show any statistically significant difference.

Fig. 7 shows the full Procrustes mean shapes of Scots pine in the Doubravčice, Oleška and Třeboň locations. We see a noticeable difference of the Třeboň location. It is explained by the second PC. Scots pine stems in the Třeboň location are narrow in the lower part of the stem and wide

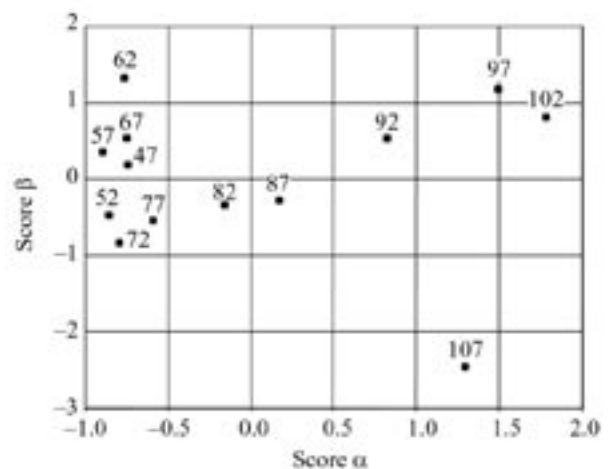


Fig. 6. The standardized PC scores of individual stems of the sample tree No. 26

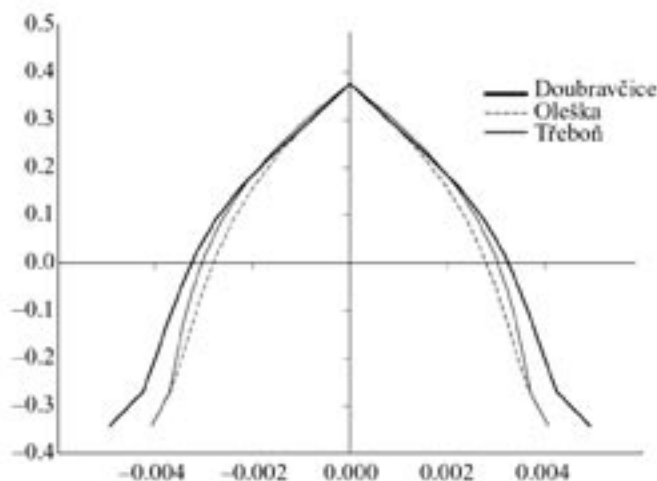


Fig. 7. Full Procrustes mean shapes of Scots pine in Doubravčice, Oleška and Třeboň locations

in the top part of the stem. This difference is also evident in comparison with the Brník and Manětín locations, whose mean shapes were not included in the figure.

### CONCLUSIONS

Shape variability analysis performed by the method of principal components for all 430 stems of Scots pine in five locations showed that 95% of variability can be explained by the first two PCs. The first PC explains 87% of variability and has the identical graphic effect as the most prominent PC at spruce – see earlier articles and studies of the authors. Within 98% of shape variability, the effect of buttresses was not recorded, in comparison with spruce. The second PC explains 8% variability. It is noticeable at Scots pine in Třeboň location. Together with the Manětín location, the Třeboň location shows the most marked difference from the other locations. The difference cannot be caused by the social position of the tree within the stand, as most of the trees were dominant, or, exceptionally, co-dominant, nor is it influenced by the age. It can point, though, to another ecotype of the tree species.

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## Příspěvek k poznání tvaru kmene borovice lesní (*Pinus sylvestris* L.)

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**ABSTRAKT:** Článek se zabývá studiem tvaru kmene borovice lesní. Pokusný materiál tvořilo celkem 430 kmenů bez kůry získaných z třiceti plně analyzovaných vzorníků, které rostly v pěti lokalitách ČR. Jako metoda zkoumání byla použita generalizovaná Procrustova analýza. Kmeny byly popsány pomocí hraničních bodů. Pro všechny kmeny byly vypočteny plné Procrustovy souřadnice, byl nalezen plný Procrustův průměr pro jednotlivé vzorníky, lokality a všechny kmeny. Dále byla pro Procrus-

tovy tangentové souřadnice zkoumána variabilita metodou hlavních komponent. Dvě nejdůležitější hlavní komponenty byly graficky znázorněny a popsány. Dále byly provedeny statistické testy mezi středními tvarovými vektory pro jednotlivé lokality.

**Klíčová slova:** borovice lesní (*Pinus sylvestris* L.); tvar kmene; Procrustova analýza; analýza hlavních komponent

Tvar kmene borovice lesní byl zkoumán na třiceti vzornících z pěti lokalit na území ČR. Z těchto 30 vzorníků bylo metodou plné analýzy zrekonstruováno celkem 430 kmenů (tab. 1). Na tyto kmeny bylo umístěno celkem 21 hraničních bodů – první dvojice do 0,3 m a dále výšky stoupaly po 1/10 výšky kmene až po vrchol. Jednotlivé výšky a tloušťky vytvářejí konfigurační matice a z nich byly metodou zobecněné Procrustovy analýzy (GPA) odhadnuty plné Procrustovy tvarové souřadnice a plný Procrustův tvarový průměr pro všechny kmeny, jednotlivé lokality a vzorníky. Metoda GPA byla zjednodušeně vysvětlena na metodě běžné Procrustovy analýzy (OPA) pro dva trojúhelníky, které zjednodušeně reprezentují tvar dvou kmenů starých 102 a 57 let v rámci vzorníku č. 26. Na obr. 1 je patrná širší základna kmene o stáří 102 let. Základny byly umístěny do 1/10 výšky kmene.

Pro střední tvarové vektory jednotlivých lokalit, odhadnuté z tangentových souřadnic [ty jsou získány z plných Procrustových souřadnic jejich převedením do lineární tangentové sféry podle postupu DRYDENA a MARDII (1998)], byly provedeny Hotellingovy  $T^2$  testy rovnosti středních tvarových vektorů. Předtím byl proveden test homogenity variančně-kovariančních matic. Test se nazývá Boxův M test [Box (1949) in MARDIA et al. (1979) a in HAVRÁNEK (1993)]. Shoda variančně-kovariančních matic nebyla prokázána, proto byl proveden neparametrický Monte Carlo test. Pro každou dvojici lokalit bylo provedeno celkem 1 000 náhodných permutací. Tvarový průměr borovic z lokality Třeboň se odlišoval od všech ostatních lokalit s výjimkou lokality Oleška. Lokalita Manětín se odlišovala od lokality Brník a Doubravčice. V ostatních dvojicích lokalit neprokázal použitý test na hladině významnosti  $\alpha = 0,05$  statisticky významný rozdíl. Na obr. 7 jsou zachyceny plné Procrustovy průměrné tvary borovic pro lokality Doubravčice,

Oleška a Třeboň. Vidíme zde nápadnou odlišnost lokality Třeboň. Tato odlišnost je vysvětlena hlavní komponentou ( $\beta$ ). Kmeny třeboňské borovice jsou úzké ve spodní části kmene a široké v horní části kmene. Tato odlišnost je patrná i vzhledem k lokalitám Brník a Manětín, jejichž tvarové průměry nebyly pro přehlednost do obrázku zařazeny.

Variabilita byla zkoumána metodou hlavních komponent. V souboru 430 kmenů první tři hlavní komponenty vysvětlují 98 % variability. Obr. 2 a 3 znázorňují grafický efekt prvních dvou hlavních komponent. První hlavní komponenta byla označena ( $\alpha$ ). Obdobně jako u smrku – viz KŘEPELA et al. (2001) a KŘEPELA (2002) – je její grafický efekt symetrický podél svislé osy kmene. Druhá hlavní komponenta je asymetrická. Její grafický efekt má opačný směr ve spodní (do 4/10 výšky kmene) a horní části kmene. Tato hlavní komponenta byla označena ( $\beta$ ). Tato hlavní komponenta se v citovaných pracích autorů tohoto článku u smrku nevyskytovala mezi prvními třemi hlavními komponentami. Ani v tab. 2 uvedená třetí hlavní komponenta se u smrku nevyskytovala. Nejde o efekt kořenových náběhů či zbytnění spodní části kmene. Zastoupení prvních dvou hlavních komponent pro jednotlivé vzorníky zachycuje tab. 1. Plná kmenová analýza vzorníku č. 26 mezi stářími 47 až 107 let je zachycena na obr. 4. Zde je nápadný velký výškový přírůst v poslední pětileté periodě. Na obr. 5 a 6 jsou zachyceny tvarové změny vzorníku č. 26. V periodě 47–77 let dochází u PC ( $\alpha$ ) jen k malým změnám. V periodě 77 až 107 let dochází k rovnoměrnému rozšiřování kmene po celé jeho délce. Mezi stářími 102–107 let došlo u sledovaného kmene k velkému výškovému přírůstu. Tento přírůst nebyl doplněn odpovídajícím přírůstem tloušťkovým. Došlo tedy k velké změně u hlavní komponenty ( $\beta$ ).

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