Utilisation of mathematical models and growth simulators for creating forest management plans and planning the tending felling

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ABSTRACT: The paper describes modelling of forest development and its utilisation for forest management planning, specifically modelling of growth mensurational variables of the two main commercial species – spruce and beech. Further, utilisation of growth simulator SILVA 2.2 for modelling the tending felling is described, focusing on data collection and main outputs of the modelling. Researched methodology was applied to a sample forest near the Racková municipality, property of Kroměříž Municipal Forests.

Keywords: mathematical modelling of development of mensurational variables; spruce; growth simulators; SILVA 2.2; tending felling

Growth simulators (GS) or mathematical models focusing on forest stand development can be used as an important instrument of decision making when collecting data for forest management. Currently, the main source of information for the forest owner or the forest manager is Forest Management Plans (FMP). The problem is that FMPs are static works, created for 10 years, which do not directly deal with the modelling of mensurational variables. Therefore the informative value of FMP for the owner is rather limited. FMP offer very exact information in the first year of their validity, but they do not enable to determine the development and behaviour of mensurational variables during the whole period while the plan is still valid, and thus give a more accurate information base for successful forest management. This problem could be solved by mathematical modelling of development of mensurational variables, or rather complicated growth simulators, which would supply the above-mentioned information. These problems are not new, Kneifl (2000) and Simon (2001) mentioned them in their works.

Pretzsch et al. (2002) approached the term model as a system of equations and relations describing tree growth and competitive relationships, while simulator is an implementation of the model into a computer program, which enables us to interactively influence the process of simulation. The main objectives of the growth process modelling are prognosis of forest stand development and determination of optimum management strategy.

Based on the known and given behaviour regularities of the individual growth processes, simulation models enable to foresee the condition in which the given forest stand will be with certain probability under the conditions of quantitatively chosen factors. Many Czech authors were dealing with mathematical modelling and its possible utilisation in their works. To name the most important: Polák (1972), Simon et al. (1996) and Zach (2001). On the international level, the following authors focused on forming the growth simulators – Pretzsch and Kahn (1998), Nagel (1975), and Sterba and Monsrud (1995). As Pretzsch et al. (2002) presents, it is difficult to use foreign models because of different environment and that is why such models do not give realistic results. The main problem of creating a GS that would be possible to use for simulation of greater territory units, is the need for enormous database and validation, which demands extensive measuring. This is, of course, very demanding and becomes an obstacle when constructing the GS. However, it is possible to use quite simple mathematical models that can be created while processing the FMP. On the one hand, their utilisation restricted is to the given property, but they give valuable information for the management and also for the processing of FMP.

For the mathematical modelling of development of mensurational variables it is necessary to create mock time series from non-mixed forest stands of the same age on the given estate. According to Zach (2001) the mock time series are defined as an entity of stands:

- in the same or related habitat,
- of various age classes,
- planted in the same way,
- of the same origin,
- and of the same tending.

One of the hybrid models, SILVA 2.2, product of a team of researchers headed by Prof. H. Pretzsch from the Munich Technical University (for more detailed information...
see PRETZSCH 2002), makes it possible to forecast the growth of pure and mixed stands of all age classes. The identification parameters of forest stands, stand variables and forest stand treatment basis are used as the starting and controlling quantities. The starting quantities are information about the tree proportions and tree heights – that is variables \( d_{13}, h \), crown setting height and variables delta \( x \) and \( y \) which describe the position of the trees. These quantities are gathered by measuring. The controlling quantities are the individual stand parameters. These include nutrient supply, Nox and Cox content in the air, length of vegetation period, annual amplitude, average temperature, aridity index, total precipitation of vegetation period and soil moisture. The forest stand treatment basis is the assessment of tending felling.

Before the prognosis, the program SILVA 2.2 must calculate capacity model of the stand, which is the controlling model for the development of individual tree proportions through the factors of the stands. In this model, most of the growth functions for the given stand conditions provided by the GS SILVA 2.2 will be set.

The prognosis itself consists of four steps, repetitious in five-year cycles.

1. The first criterion is the time sequence of the sampling according to the tree classes, sampling according to the average classes. For the research, two groups could be regarded as suitable: sampling according to the tree classes and sampling according to the target diameter. When sampling according to the tree classes, GS SILVA 2.2 takes into account individual proportions of the trees and the competition indices. When sampling according to the target diameter, it is the set value of the diameter.

2. As criterion No. 2 is regarded the thinning, enabling combinations of more styles during the stand evolution. SILVA 2.2 distinguishes between four basic groups: sampling according to the tree classes, sampling ac-

cording to the target diameter, sampling according to the selected or prospective trees and sampling according to the average classes. For the research, two groups could be regarded as suitable: sampling according to the tree classes and sampling according to the target diameter. When sampling according to the tree classes, GS SILVA 2.2 takes into account individual proportions of the trees and the competition indices. When sampling according to the target diameter, it is the set value of the diameter.

3. The third criterion is intensity. At the level of the forest stand, the intensity is based on the default curves of the number of trees or of the fundamental area. At the level of the individual trees, it is based on the set degree of the release of selected or prospective trees. It is also possible to combine these with setting the maximum height of felling at one particular event.

At the moment the calibration to growth conditions in the Czech Republic is carried out. One of the institutions dealing with growth simulators is the Faculty of Forest Management at Mendel University of Agriculture and Forestry in Brno, where the simulations for this work were elaborated. Even though the calibration to the stands conditions is more or less finished (SIMON 2003), the results can be regarded as only theoretical for the time being.

**Timber and methodology**

Research was conducted at the estate of Kroměříž Municipal Forests, cadastral area Racková. The estate is situated in Natural Forest Area 41 Hostýnsko-Vsetínské vrchy and Javorníky, between the towns of Zlín and Holešov. The area of the forests is 642 ha. The estate was selected because of its typology and stand homogeneity. More than 95% of the whole surveyed estate falls within target forest management No. 45 – beech/spruce, fertile site of medium location.

In mock time series for the given forest stand, the following static units of quantities were measured for the individual age classes: diameter (\( d_{13} \)), height (\( h \)) and crown parameters (height of the crown setting, its length and breadth). Measurements were conducted on the two most frequent species, the spruce (\( Picea abies \)) and the beech (\( Fagus sylvatica \)). Because of the paper’s limitations, only the results about spruce will be discussed. At each age class, 100 specimens were measured and the values of about 900 trees in total were used for subsequent calculations. For determining the number of trunks per ha (\( N \)), sample plots of 20 × 20 cm (or 10 × 10 cm for the immature forest stands) were used.

Statistical data were processed by the Adstat and Excel programs, basic statistical characteristics were calculated and tested for the congruence of the theoretical normal distribution with the distribution of the tested unit, specifically the selective skewness and acuteness. Autocorrelation test was conducted according to ‘von Neumann Ratio’ (MELOUN, MILITKY 1994). At all the age classes, normality and independence were proved for the quantities \( d_{13} \) as
well as h of the spruce. Next steps of the calculation were dealing with the modelling of the given mensurational characteristics (diameter, height and stock).

The growth and increment models of $d_{1.3}$, $h$ and stock were calculated according the Korf function (ZACH 2001). The equation is:

$$y = A \cdot e^{(1-n) \cdot t}$$

The real stock was calculated for a given age class by the method of volume tables and recalculated for the area of 1 ha. For the production analysis it was necessary to convert the real stock into potential stock with full density. Growth and Mensurational Tables of the main species in the CR were regarded as the standards (ČERNÝ et al. 1996). Applicability of the growth tables was tested by one-selection T-test for the congruence between the measured and table quantity values of the $d_{1.3}$ mean (DRÁPELA, ZACH 2002).

Input data for GS SILVA 2.2 are the stand conditions, obtained from the climatic station Holešov, and proportions and distribution of the trees, obtained by special measurements. Suitable forest stands were selected from the given pseudo-time series that best meet the requirements of minimal species distribution. For the given forest stand, an area of 0.5 ha was marked out and all the specimens were located there by the Delta x- and y-co-ordinates, thus determining the exact positions of all the individual specimens in the forest stand. For each tree, $d_{1.3}$, $h$ and the crown setting are measured and these data are then processed by the GS SILVA 2.2, including the simulation of tending fellings.

**RESULTS**

The Korf function obtained by non-linear optimisation by the Gauss-Newton method proved to be a suitable development model for the monitored mensurational variables in the given estate. The calculated coefficients are:

- $d_{1.3}$: $A = 50.572$, $k = 7.898$, $n = 1.707$, correlation index = 0.965
- $h$: $A = 51.351$, $k = 2.977$, $n = 1.507$, correlation index = 0.966.

**Table 1. Results of measurements and Korf function models**

<table>
<thead>
<tr>
<th>Age</th>
<th>$d_{1.3}$ (cm)</th>
<th>$h$ (m)</th>
<th>$d_{1.3}$ of Korf function</th>
<th>$h$ of Korf function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>GF (cm)</td>
<td>AI (cm/year)</td>
</tr>
<tr>
<td>5</td>
<td>5.91</td>
<td></td>
<td>3.64</td>
<td>0.73</td>
</tr>
<tr>
<td>15</td>
<td>11.19</td>
<td>9.75</td>
<td>13.57</td>
<td>0.55</td>
</tr>
<tr>
<td>25</td>
<td>15.38</td>
<td>13.87</td>
<td>22.31</td>
<td>0.64</td>
</tr>
<tr>
<td>35</td>
<td>19.40</td>
<td>19.54</td>
<td>27.11</td>
<td>0.61</td>
</tr>
<tr>
<td>45</td>
<td>22.16</td>
<td>22.21</td>
<td>30.02</td>
<td>0.55</td>
</tr>
<tr>
<td>55</td>
<td>25.01</td>
<td>22.41</td>
<td>31.93</td>
<td>0.49</td>
</tr>
<tr>
<td>65</td>
<td>28.49</td>
<td>26.87</td>
<td>33.27</td>
<td>0.44</td>
</tr>
<tr>
<td>75</td>
<td>31.48</td>
<td>27.15</td>
<td>34.26</td>
<td>0.40</td>
</tr>
<tr>
<td>85</td>
<td>32.79</td>
<td>28.63</td>
<td>35.01</td>
<td>0.37</td>
</tr>
</tbody>
</table>

GR – growth function, AI – average increment, CI – current increment
Eventual data obtained by measuring and those calculated by the model are described in Table 1. Growth function (GF), average increment (AI) and current increment (CI) curves are represented in Figs. 1 and 2. Correlation indices were tested by the tests of significance with the reliability level of alpha at 0.05; and the significance was proved to be valid. High values of these indices define the relation as close.

The results of one-selection T-tests for the congruence of the mean when modelling the stock prove that divergences between the measured quantities and values from the growth tables (Table 2a) are minimal. Growth tables were therefore advantageously used for assessing the potential stock. Another suitable stock development model was also the Korf function, whose coefficients are as follows:

<table>
<thead>
<tr>
<th>Age class</th>
<th>( d_{1.3} ) (cm) measurement</th>
<th>( d_{1.3} ) (cm) GT</th>
<th>Ho</th>
<th>( h ) (m) measurement</th>
<th>( h ) (m) GT</th>
<th>Ho</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>10</td>
<td>not accepted</td>
<td>10</td>
<td>10</td>
<td>accepted</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>14</td>
<td>not accepted</td>
<td>14</td>
<td>15</td>
<td>not accepted</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
<td>19</td>
<td>accepted</td>
<td>20</td>
<td>20</td>
<td>accepted</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
<td>22</td>
<td>accepted</td>
<td>22</td>
<td>22</td>
<td>accepted</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>22</td>
<td>not accepted</td>
<td>22</td>
<td>22</td>
<td>accepted</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
<td>28</td>
<td>accepted</td>
<td>27</td>
<td>28</td>
<td>not accepted</td>
</tr>
<tr>
<td>8</td>
<td>31</td>
<td>30</td>
<td>not accepted</td>
<td>27</td>
<td>28</td>
<td>not accepted</td>
</tr>
<tr>
<td>9</td>
<td>33</td>
<td>32</td>
<td>accepted</td>
<td>29</td>
<td>29</td>
<td>accepted</td>
</tr>
</tbody>
</table>

GT – growth and mensurational tables, Ho – zero hypothesis about equality of values (measured results are equal to table values)
For models of the tending felling, growth simulator as well as three basic types of tending felling are used, which corresponds to the works of BOGACHEV (1990), COURBAUD (2000) and NIEMISTO (1989). When choosing the thinning methods, the real age of forest stand, its condition and recommendations of the management instructions for the given estate were taken into account. The following methods of tending fellings were selected for the estate, by virtue of the management instructions within SILVA 2.2:

a) crown thinning is a negative selection placed at the main level, where the decisive criterion is non-figurability of the stem. Two tending fellings in 20-year spans with the maximum value of 100 m$^3$/ha per felling were set;

b) selection thinning (Schädelin’s) is a positive selection designed for all the five tree classes. When selecting trees for felling, it is necessary to bear in mind the quality of the tree, materiality, social position, distribution of the trees on the plot and stability. Two extensive tending fellings with the maximum value of 100 m$^3$/ha per felling were set in this case as well;

c) target diameter method is adjusted for the trees selected for felling at the $d_{1.3}$ value of 45 cm and four fellings in 10-year spans with the maximum of 100 m$^3$/ha.

Table 2b. Model of reserves development

<table>
<thead>
<tr>
<th>Age class</th>
<th>Reserves measurement</th>
<th>GF</th>
<th>AI</th>
<th>CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>93</td>
<td>92</td>
<td>6.14</td>
<td>14.3</td>
</tr>
<tr>
<td>2</td>
<td>220</td>
<td>234</td>
<td>9.35</td>
<td>13.1</td>
</tr>
<tr>
<td>3</td>
<td>380</td>
<td>349</td>
<td>9.96</td>
<td>9.1</td>
</tr>
<tr>
<td>4</td>
<td>451</td>
<td>435</td>
<td>9.68</td>
<td>7.5</td>
</tr>
<tr>
<td>5</td>
<td>433</td>
<td>501</td>
<td>9.12</td>
<td>5.8</td>
</tr>
<tr>
<td>6</td>
<td>603</td>
<td>553</td>
<td>8.51</td>
<td>4.6</td>
</tr>
<tr>
<td>7</td>
<td>588</td>
<td>594</td>
<td>7.92</td>
<td>3.7</td>
</tr>
<tr>
<td>8</td>
<td>628</td>
<td>628</td>
<td>7.38</td>
<td>3.0</td>
</tr>
</tbody>
</table>

GF – growth function m$^3$/ha, AI – average increment m$^3$/ha/year, CI – current increment m$^3$/ha/year

$V$ m$^3$/ha: $A = 536.755$, $k = 204.888$, $n = 2.717$, correlation index = 0.986.

Stock at the level of 630 m$^3$/ha (Table 2b) classifies this stand as a stand with high production, which proves the hypothesis that the management set of stands No. 45 is one of the optimal sets for the spruce (PRŮŠA 2001).

For models of the tending felling, growth simulator as well as three basic types of tending felling are used, which corresponds to the works of BOGACHEV (1990), COURBAUD (2000) and NIEMISTO (1989). When choosing the thinning methods, the real age of forest stand, its condition and recommendations of the management instructions for the given estate were taken into account. The following methods of tending fellings were selected for the estate, by virtue of the management instructions within SILVA 2.2:

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b) selection thinning (Schädelin’s) is a positive selection designed for all the five tree classes. When selecting trees for felling, it is necessary to bear in mind the quality of the tree, materiality, social position, distribution of the trees on the plot and stability. Two extensive tending fellings with the maximum value of 100 m$^3$/ha per felling were set in this case as well;

c) target diameter method is adjusted for the trees selected for felling at the $d_{1.3}$ value of 45 cm and four fellings in 10-year spans with the maximum of 100 m$^3$/ha.
The owner's intention to assess individual types of thinning was chosen in agreement with the management instructions. As WOLF stated (1998), production criterion (stock at the exploitable age and total volume increment) and production safeguarding criterion (defined by the slenderness coefficient $h/d_{1.3}$) were monitored.

The simulation results – as described in Table 3 and Figs. 6 to 5 – make it clear that the method of target diameters is better from the production aspect. The disadvantage of this method is the lowering of slenderness coefficient and irregular tree distribution in the area (Figs. 6–9). The best alternative for tending the estate is selection thinning, which ensures the slenderness coefficient value at 80 while losing only 8% of the total volume increment. Crown thinning results prove the hypothesis that this type of thinning can lower the stability and balance and thus become the least suitable alternative in subsequent years.

It is important for the owner that the examined spruce stands are the ones with very high timber production for which the management instructions recommend to switch the tending from stability to quality. It is obvious from the simulation results that the best tending alternative for the given estate is the selection thinning which renders the timber at 660 m$^3$/ha while achieving the best value of slenderness coefficient.

DISCUSSION AND CONCLUSIONS

The use of growth simulators and mathematical modelling for development of mensurational variables should be integrated into the forest management planning. The use of growth simulators and mathematical modelling for development of mensurational variables should be integrated into the forest management planning.
of models is a particularly valuable source of information for creating the FMP (ZACH 2001; SIMON 2003; ŃURSKY 2003). The problem that remains is the need of extensive amount of measured data (PRETZSCH et al. 2002; ZACH 2001), which is very time-consuming and with the present methodology of FMP processing impossible. Another limiting criterion for creating the models of development of mensurational variables is certainly also the size of the estate (KNEIFL 2000) – not only the area, but also the representation of the age classes in particular. This is important when creating the quasi-time series and with not as large estates: there might be a danger that some age classes could be missing and would have to be substituted.

The great contribution for forest management planning is the substitution of growth simulators for growth tables in mixed forest stands with various age classes, where the growth tables currently use methods not suitable for the given forest stands (ĎURSKY 2003) and might not be the optimal criterion for the given forest stand. When compared with the growth tables for spruce monocultures, no significant divergences were discovered. With the beech, where all series of forest stands were more mixed, the divergences were noticeable.

The use of GS SILVA 2.2 when processing the FMP is at the moment facing one great obstacle, and that is the contemporary methodology of FMP, specifically the determination of the mensurational characteristics for the given forest stand groups (ZEZULA 2003). This fact does not allow the GS to become a substitute for the prevailing way of processing the FMP because this method is very time-consuming. But due to the generators of the initial situation it is possible to use GS SILVA 2.2 for making the FMP more accurate even from the current data (composition, mid-diameter, mid-height, stock density, number of stems and mix description) obtained from the FMP (ĎURSKY 2003). The growth simulator SILVA 2.2 is also a suitable instrument for planning the tending fellings.

The need and effort of people to know the behaviour of the system in future opens the door for such programs as SILVA 2.2 and with the present development of information technologies it is possible to expect further and further significant progress.

References

Využití matematických modelů a růstových simulátorů pro tvorbu lesních hospodářských plánů a plánování výchovných zásahů

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ABSTRAKT: Článek se zabývá modelováním vývoje lesa a jeho využitím pro lesní hospodářské plánování, konkrétně modelováním růstových taxačních veličin dvou hlavních hospodářských dřevin, smrku a buku. Dále popisuje využití růstového simulátoru SILVA 2.2 při modelování výchovných zásahů, a to získávání dat a hlavní výstupy modelování. Jako příklad využití studované metodiky je použit majetek Městských lesů Kroměříž u obce Racková.

Klíčová slova: matematické modelování vývoje taxačních veličin; smrk; růstové simulátory; SILVA 2.2; výchová těžba


Pro nejvíce zastoupené dřeviny smrk a buk byly vybrány porosty odpovídající požadavkům nepravé časové řady. V nalezených porostech nepravé časové řady byly pro jednotlivé věkové stupně zmenšeny statistické soubory hodnot těchto veličin: tloušťka (d_{1,3}), výška (h), korunové parametry (výška nasazení koruny, její délka a šířka). Statistické soubory hodnot veličin byly zpracovány v programech Adstat a Excel, kde byly vypočítány základní statistické charakteristiky a proveden test na shodu teoretického rozdělení s rozdělením z experimentálního souboru; jednalo se o test výběrové šikmotnosti a špičatosti. Test na autokorelaci byl proveden pomocí poměru von Neumana.

Dále byly vypočítány modely pro předpověď vývoje d_{1,3} a h a rovněž pro zásobu. Pro všechny veličiny je vhodným modelem Korfova funkce. Hodnota zásoby 630 m^3/ha (tab. 2b) řadí toto stanoviště do stanovišť s vysokou produkci, což potvrzuje názor, že HS 45 patří mezi optima smrku (PRŮŠA 2001).

Pro simulace RS SILVA 2.2 byly vybrány porosty z dané časové řady a v nich provedeno měření vstupních veličin. V RS SILVA 2.2 byly poté provedeny simulace pro jednotlivé typy výchovných zásahů a ty byly mezi sebou porovnány. Z výsledků simulace je patrné, že z produkčního hlediska je nejlepší metoda cílových tlouštěk. Jejím nedostatkem je snížení koeficientu štíhlosti a nepravidelné rozmístění stromů po ploše. Nejlepší alternativou výchovy je pro daný majetek výběrná probírka, která při pouhé 8% ztrátě na COP zajišťuje hodnotu štíhlostního koeficientu 80. Pro vlastník je důležitý fakt, že smrkové porosty na zkoumaném majetku patří mezi porosty s vysokou produkcí dřeva a že nejlepší variantou výchovy pro daný majetek je výběrná probírka, která poskytuje zásobu 660 m^3/ha s nejlepší hodnotou štíhlostního koeficientu.

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