

# Applicability of models based on deterministic chaos theory for forecasting of electricity sales in rural areas

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**Abstract:** The applicability of models based on deterministic chaos theory, in particular the self-similar fractal models, logistic models according to Prigogine, logistic models according to Schuster, and heuristic crossing models for one-year-ahead forecasting of the sales of electric energy in the rural areas, is verified in the paper. The model quality assessment was carried out based on the errors analysis of the 10-year ex post forecasts of the electric energy sale to rural consumers by three selected distribution companies. As the electric energy consumption were disturbed by economic crises, the forecasts were developed using not only annual sales value but also using their 2-, 3-, 4- and 5-year totals. The best forecast were obtained when the electric energy sales were predicted using the Schuster and heuristic crossing models, made on the basis of four- and five-year totals of annual sales values.

**Keywords:** electric energy; forecast; deterministic chaos

The annual demand for electric energy in rural areas in Poland is about 15 000 GWh. The statutory responsibility for meeting this demand lies within companies dealing with electric energy transmission and distribution. On the electric energy market, the companies contract to buy electric energy. In particular, their contracts concern annual demands. Inadequate contracting of electric energy increases the costs of customers' service and adversely influences the quality of services rendered by the company. Such contracting may especially concern services rendered to rural consumers as – apart from this problem – the real unit costs of electric energy supply exceed there the prices of electric energy.

To determine the amounts to be ordered, the companies develop forecasts of annual sales of electric energy using various methods. Transformations undergoing in Poland since nineties have considerably lowered the reliability of the forecasting methods used so far because electric energy forecasting was to be carried out under turbulent conditions. The pursuit of forecast improvement has been expressed in the interest in alternative forecasting methods to which deterministic-chaos-based methods are included.

The goal of the paper was to verify the deterministic chaos-based models, especially the self-similar fractal models, logistic models according to Prigogine, logistic models according to Schuster, and

the heuristic crossing models for one-year-ahead forecasting of the sale of electric energy in the rural areas by a three selected companies from the south of Poland.

## MATERIAL AND METHODS

### Model based on fractal dimension

Fractals are objects whose arbitrary small parts are shaped similarly to the shape of the whole. This property is called self-similarity. Analogously self-similar time processes can be defined to which the processes of annual consumption of electric energy can be included. A most simple forecast model for self-similar random time function is of the following form (DOBZANŃSKA 2002):

$$\hat{X}_{t+1} = X_t \frac{\sum_{i=2}^t X_i X_{i-1}}{\sum_{i=2}^t X_{i-1}^2} \quad (1)$$

where:  $\hat{X}_{t+1}$  – forecast of electric energy sale for the year  $t+1$ ,  
 $X_t$  – electric energy sale in the year  $t$ .

The model assumes correlated values of electric energy sales values in years  $t$  and  $t+1$  and was employed in the paper to step forecasting of annual sales of electric energy where  $\hat{X}_{t+1}$  was treated as the best approximation of  $X_{t+1}$  in the sense of the minimum of mean square error (CIEŚLAK 1999).

### Model according to Prigogine

Based on the logistic equation cited by PRIGOGINE (1980), the forecast model for can be written as:

$$\hat{X}_{t+1} = X_t + rX \left(1 - \frac{X_t}{K}\right) \quad (2)$$

where:  $r$  – coefficient of increase of annual sales rate of electric energy,  
 $K$  – upper limit of increase, the other symbols as in equation 1.

The behaviour of the process depends on the value of  $r$  and the ratio  $K$  to  $X$  at the initial time instant. In the paper, parameters  $K$  and  $r$  were determined iteratively based on the annual electric energy sales statistics. The iterations were started inserting  $K$  equal to the largest historical annual sales of electric energy and were ended when the difference of the subsequent values of  $K$  was less than  $1/10^6$ .

### Model according to Schuster

Forecast model of annual electric energy sales based on the Schuster mapping (SCHUSTER 1984) is:

$$\hat{X}_{t+1} = (1 + \alpha_{t+1}) X_0 \quad (3)$$

$$\alpha_{t+1} = r \alpha_t (1 - \alpha_t) \quad (4)$$

$$\alpha_t = \frac{X_t - X_{t-1}}{X_{t-1}} \quad (5)$$

$$\alpha_{t+1} = \frac{X_{t+1} - X_{t-1}}{X_{t-1}} \quad (6)$$

where:  $X_{t-1}$ ,  $X_t$ ,  $X_{t+1}$  – electric energy sales values in years  $t-1$ ,  $t$ , and  $t+1$ , respectively,  
 $X_0$  – annual electric energy sales value taken from the history of the process preceding the forecast starting moment,

$\alpha_t$ ,  $\alpha_{t+1}$  – relative increments of annual electric energy sales,  
 $r$  – external parameter determined from learning series.

### Heuristic crossing model

Heuristic crossing model form is similar to the Schuster mapping (Equation 2) where the relative increment  $\alpha_{t+1}$  described by relationship 4 is replaced by the operator:

$$\alpha_{t+1} = r (\alpha_t - \alpha_{t-1}) + \alpha_t \quad 0 \leq r \leq 1 \quad (7)$$

In the paper, for  $r > 1$  the value of  $\hat{r} = 1$  was adopted in the forecast, and for  $r < 0$  the value of  $\hat{r} = 0$ .

Annual electric energy sales in years 1980–2003 were the learning series for forecasting. Because step forecast of annual electric energy sales was the aim of the forecast, one-year-apart pairs of annual electric energy sales values were taken from the learning series.

## RESULTS AND DISCUSSION

In Figure 1 the time courses of annual electric energy sales rendered by three selected distribution companies (C\_1, C\_2, C\_3) in rural areas in years 1980–2003 are presented. The sales quantities are different as different were the operating areas of particular companies. And thus company C\_1 provided in 2003 electric energy to 354 000 rural consumers, company C\_2 to 220 000, and C\_3 to 140 000 clients only who on average consumed respectively in 2003 about 2100, 2400 and 2200 kWh of electric energy. However, the character of sales changes suggests certain patterns. After the social-economic transformation in Poland began, i.e., after 1990, there occurred a drop in electric energy consumption in the rural areas operated by the selected companies, and the magnitude of this drop and its duration were

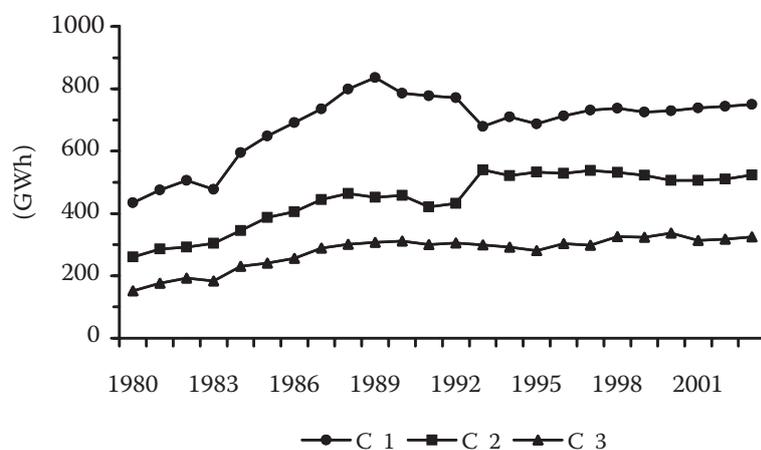


Figure 1. Time courses of annual electric energy sales rendered to rural customers by selected distribution companies in years 1980–2003

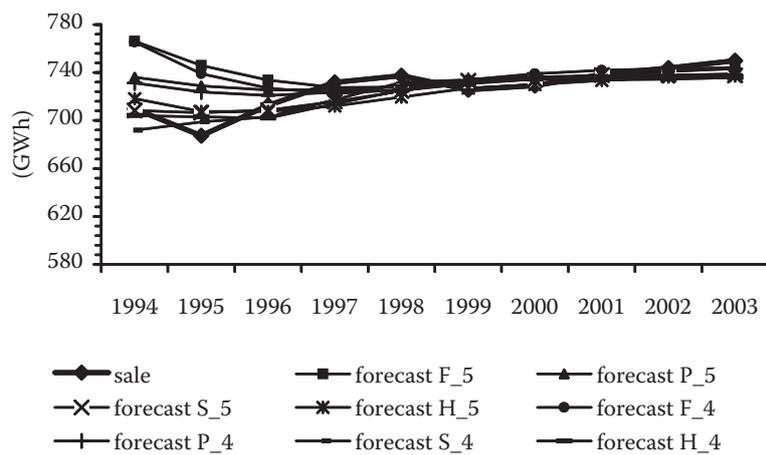


Figure 2. 10-year ex post forecasts of electric energy annual sales to rural customers by a distribution company C\_1

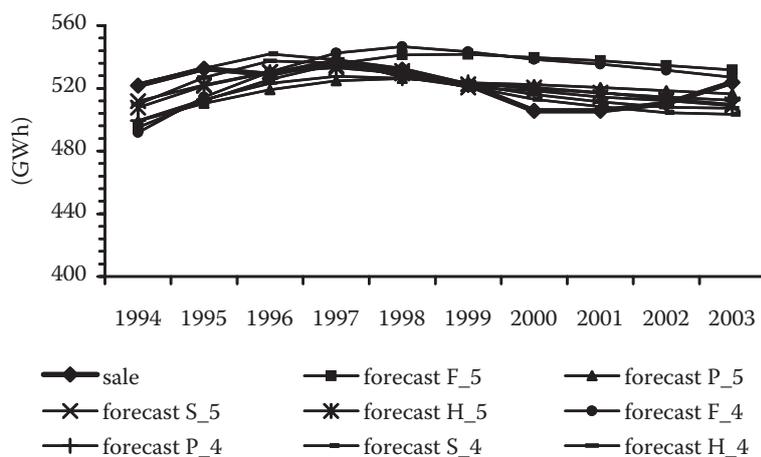


Figure 3. 10-year ex post forecasts of electric energy annual sales to rural customers by a distribution company C\_2

different. After this consumption drop period, the electric energy demand trend in the considered rural areas is rising.

A commonly used technique of forecasting method accuracy assessment is the method of ex post forecasts which can be summed up as the analysis of properties of an error set obtained by comparing the results of the implemented forecasting procedures with real realizations of the process. Based on relationships 1 through 7 ten-year ex post forecasts of electric energy sales in rural areas under consideration were developed. As forecasting models are very sensitive to the dynamics of the process and the electric energy sales time courses were dis-

turbed by economic crises, the forecasts were made not only using annual values of sales but also their two, three, four and five-year totals. The forecasting quality assessment was made based on absolute values of ex post forecast errors (CIEŚLAK 1999) as the most commonly used acceptability measures of the electric energy demand forecasts. For each of the companies, the values of these errors turned out to be the smallest for forecasts based on four and five-year totals of annual sales, being in some cases more than two times smaller than the errors of forecasts determined using annual values. The comparison of real values of sales with their ex post forecasts developed with individual models four and five-year

Table 1. Mean absolute percentage errors of 10-year ex post forecasts of electric energy annual sales to rural customers by the selected distribution company

Company	Fractal model		Prigogine's model		Schuster's model		Heuristic crossing model	
	5-year F_5	4-year F_4	5-year P_5	4-year P_4	5-year S_5	4-year S_4	5-year H_5	4-year H_4
C_1	2.43	2.41	1.78	1.62	1.17	1.38	1.37	1.39
C_2	3.46	3.43	2.33	2.03	1.32	1.10	1.43	1.28
C_3	3.75	3.40	3.18	3.12	2.81	2.42	2.84	2.56

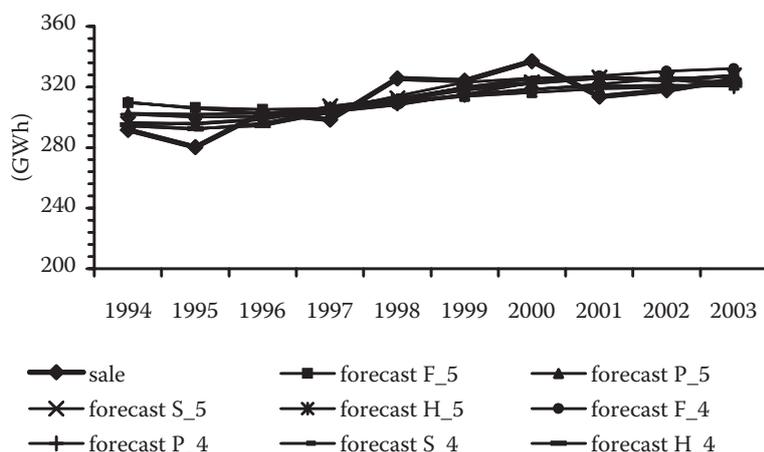


Figure 4. 10-year ex post forecasts of electric energy annual sales to rural customers by a distribution company C\_3

totals are presented in Figures 2–4, and the mean absolute values of the errors of these forecasts are presented in Table 1.

turbed by economic crises, the forecasts may be developed using four- and five-year totals of annual sales values.

## CONCLUSION

1. The calculations made confirmed the suitability of the models of the deterministic chaos origin, and, in particular, the self-similar fractal-dimension based models, logistic models according to Prigogine and according to Schuster, and heuristic crossing models to forecasting the annual sales of electric energy to rural clients, which is supported by a few percent ex post forecast errors of this variable.
2. The best forecast for all the companies were obtained when the electric energy sales were predicted using the Schuster and heuristic crossing models.
3. To improve the forecast accuracy when the time courses of electric energy consumption are dis-

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## Abstrakt

TROJANOWSKA M. (2006): **Vhodnost modelů založených na teorii determinovaného chaosu při sestavování prognóz ročního prodeje elektrické energie na venkově.** Res. Agr. Eng., 52: 25–29.

Cílem práce bylo ověřit vhodnost modelů založených na teorii determinovaného chaosu, zejména sebedobných fraktálních modelů, logistických modelů podle Prigoginea, logistických modelů podle Schustera a modelů heuristického křížení, při sestavování prognóz prodeje elektrické energie na venkově s ročním předstihem. Jakost modelů byla vyhodnocena na základě analýzy chyb ukončených desetiletých prognóz ročního prodeje elektrické energie odběratelům na venkově třemi vybranými distribučními společnostmi. Protože průběhy spotřeby elektrické energie byly

narušovány ekonomickými krizemi, byly tyto prognózy vyhotovovány nejen na ročních hodnotách prodeje, ale také na jejich dvou-, tří-, čtyř- a pětiletých souhrnech. Prognózy nejvyšší kvality byly získány sestavením prognóz prodeje elektrické energie odběratelům na venkově na základě modelů podle Schustera a modelů heuristického křížení, zpracovaných na čtyř- nebo pětiletých souhrnech.

**Klíčová slova:** elektrická energie; prognóza; teorie determinovaného chaosu

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