

# Mathematical function for the transition curves description

R. CHOTĚBORSKÝ<sup>1</sup>, D. HERÁK<sup>2</sup>, V. BEZOUŠKA<sup>3</sup>, P. HRABĚ<sup>1</sup>, M. MÜLLER<sup>1</sup>

<sup>1</sup>*Department of Material and Engineering Technologies, Technical Faculty, Czech University of Agriculture in Prague, Prague, Czech Republic*

<sup>2</sup>*Department of Mechanics and Engineering, Technical Faculty, Czech University of Agriculture in Prague, Prague, Czech Republic*

<sup>3</sup>*Department of Electrotechnical Engineering and Automation, Technical Faculty, Czech University of Agriculture in Prague, Prague, Czech Republic*

**ABSTRACT:** Toughness is one of important material properties. At present steel is still the most used material. Owing to the temperature this material can fail both brittly and ductily. Therefore we look for the temperature above which the material will fail ductily. The Charpy impact test is one of methods how to determine the ductility by the temperature – transition access in the determined temperature range. The measured data file is large and it requires the interlay by a curve which presents the relation between the impact energy and the temperature.

**Keywords:** transition curve; transition temperature; impact energy

Sometimes the tensile stressed construction materials fail brittly also at much lower stress than it corresponds to the tensile strength determined at the smooth test piece. This instable brittle fracture conduced to very rapid and unexpected fracture and therefore it is very dangerous. It goes out from cracks or other defects which effect the stress concentration which is present in real machine parts at any time. At the design of constructions resistant to the brittle failure the so-called transition access is often used. It secures that the transition temperature is enough under the temperature range when the machine part will be used (HERTZBERG 1989; KETTUNEN, KUOKKALA 2003).

One of the most used tests of the transition temperature determination is the Charpy impact test (ČSN EN 10 045-1) when the test specimen with the notch is impact stressed by means of the pendulum edge (SIH 1973, 1978). The tests are carried out in the sufficient temperature range and the relation between the measured impact energy values ( $KV - J$ ) and the temperature ( $T - ^\circ C$ ) is called the transition curve. Its behaviour using the materials of the b.c.c lattice shows the well-known "S" character. The transition curve can be divided into three significant ranges:

1. the low temperature range where the brittle failure occurs – the impact energy values are low,
2. the transition range – the significant increase of the impact energy values and as a rule with the high spread of measured values (caused by the different failure mode),
3. the higher temperature range where the ductile failure occurs – the impact energy values are high.

The typical property of transition curves is the high spread of the measured impact energy values, above all in the transition range, which is for us the most important, but in the higher temperature range, too. However the values do not fall under a definite limit and the failures are always ductile. The representation of measured values using the continuous curve demands a suitable method of smoothing. Except the nonparametric methods of smoothing (splines – excel etc.) the possibility of regression exists using the suitable regression functions which contain the regression parameters. Their values together with the regression function type describe unambiguously the transition curve (OWEN, FAWKES 1983; SIH 1968; HERÁK, CHOTĚBORSKÝ 2004).

Most often the transition curve is described as the hyperbolic tangent function in the form

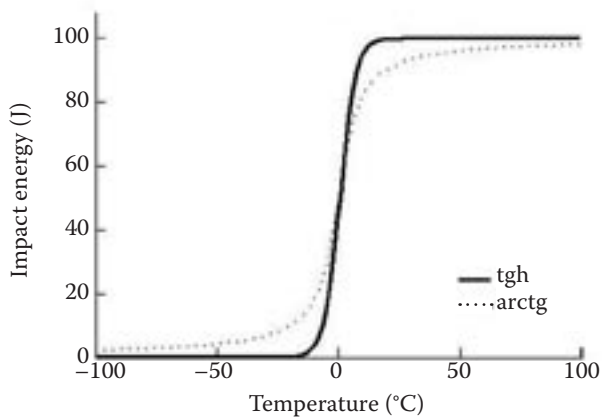


Fig. 1. Course of the hyperbolic tangent ( $KV_{tgh(t)}$ ) and arcustangent ( $KV_{arctg(t)}$ )

$$KV = f(T) = a + b \times \operatorname{tgh} [c \times (T - d)] \quad (1)$$

where:  $T$  – the temperature,  
 $a, b, c, d$  – the parameters as the results of the regression calculations,  
 $a$  – the mean of the upper and lower levels of the transition curve,  
 $b$  – a half of upper and lower level difference,  
 $c$  – the slope of the transition curve in the transition range,  
 $d$  – the suitable chosen parameter.

It presents the temperature of the curve asymmetry midpoint. It can be identified with the transition temperature as the curve inflexion point.

Then the expression (1) is possible to written:

$$KV = \frac{L + H}{2} + \frac{H - L}{2} \times \operatorname{tgh} \frac{2 \times (T - T_{tr})}{\Delta T} \quad (2)$$

This equation contents already the clear and simple parameters:

$H$  – the upper impact energy level,  
 $L$  – the lower impact energy level,  
 $T_{tr}$  – the transition temperature,  
 $\Delta T$  – the transition range width.

The further function of the comparable curve from is the arcustangent function (3)

$$KV = \frac{L + H}{2} + \frac{H - L}{\pi} \times \operatorname{arctg} \frac{\pi \times (T - T_{tr})}{\Delta T} \quad (3)$$

These functions are in possession of a definite limiting. It is the possibility of the curvature choice in the transition curve bending (Fig. 1).

The curvature can be simply changed – using the powers of the initial function  $\operatorname{tgh}(x)$  and  $\operatorname{arctg}(x)$  when the exponent determinates the curvature bending.

$$KV = \frac{L + H}{2} + \frac{H - L}{2} \times \operatorname{sgn} (T - T_{tr}) \times$$

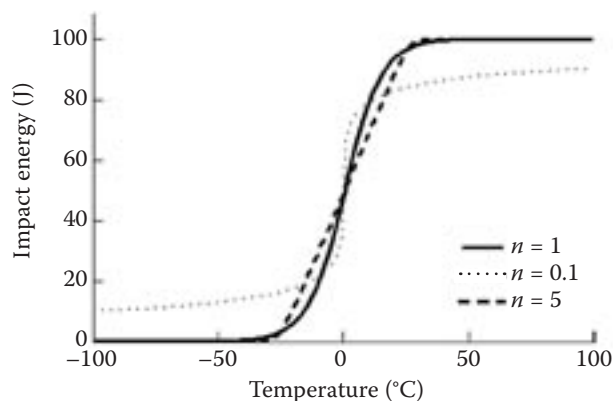


Fig. 2. Signum function for various exponents

$$\times \left[ \operatorname{tgh} \left( \frac{2 \times |T - T_{tr}|}{\Delta T} \right) \right]^{\frac{1}{n}} \quad (4)$$

where the signum function is defined by the relations  $\operatorname{sgn}(x) = -1$  for  $x < 0$ ,  $\operatorname{sgn}(0) = 0$ ,  $\operatorname{sgn}(x) = 1$  for  $x > 0$ .

In the equation (4) three variables can be changed for the suitable description:

$T_{tr}$  – the inflexion point position,  
 $\Delta T$  – the transition range width,  
 $n$  – the exponent.

In this case the exponent determines the bend sharpness of the curve bending. The sharp bend is in the case if  $n < 1$ . If  $n > 1$  the curve bending is milder as is shown in Fig. 2.

In some cases the measured values  $KV$  cannot be interlined using the above presented transition curves function. In this case the function with the off-centre position of the inflexion point is possible to use, namely the function of the type  $\exp(x)$ .

$$KV = L + \frac{H - L}{1 + p} \times \exp \left[ \frac{1 + p}{\Delta T} \times (T - T_{tr}) \right] \text{ for } T < T_{tr}$$

$$KV = H - \frac{H - L}{1 + p} \times \exp \left[ \frac{1 + p}{p \times \Delta T} \times (T - T_{tr}) \right] \text{ for } T > T_{tr} \quad (5)$$

where:  $p$  – the suitable selected parameter.

In the case of the equation (5) we look for the „ $p$ “ parameter, the inflexion point position and the transition range width. Seeing that the equation of the transition curve description is composed from two parts, in this case it is possible to look for two parameters „ $p$ “ particularly for each part, when the upper and the lower bend must be described by different way. Fig. 3 shows the function (5) course for different parameters „ $p_L$ “ and „ $p_H$ “, where „ $p_L$ “ affects

Table 1. Chemical composition of the steel 21MnTiB (weight percentage)

C	Mn	Si	B	Cu	Ti	Al	P
0.2	0.89	0.09	0.005	0.057	0.015	0.03	0.015

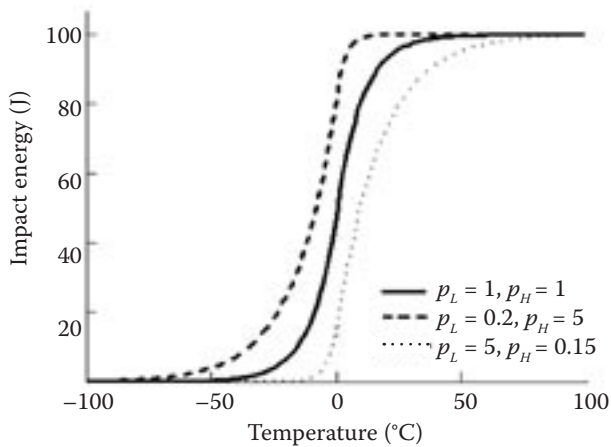


Fig. 3. Function (5) with the off-centre position of the inflexion point for various parameters „ $p$ “

the curve bending under the inflexion point, „ $p_H$ “ over the inflexion point.

#### MATERIAL AND METHODS

The steel 21MnTiB with a ferritic-pearlitic structure (Fig. 4) and with the chemical composition (see Table 1) was used as the material for the experimental description of the transition curve. This steel was cold formed using the broaching of the wire. The amount of the deformation was 10%. From this semi-product the specimens with V-notch (ČSN EN 10 045-1) for the Charpy impact test were made. The specimens were temperature treated in the bath of ethanol cooled using the liquid nitrogen on test temperature. Then the specimens were broken using the Charpy impact machine and the impact energy was read. The measured values are presented in Table 2. This values used for transition curve description.

Table 2. Measured values of the impact energy

Test temperature (°C)	Impact energy value (J)
-75	10, 25, 15, 20
-60	36, 30, 20, 25
-50	50, 30, 40, 25
-40	70, 50, 40, 60
-20	70, 90, 80, 100
0	110, 90, 100, 70
20	95, 110, 90, 80

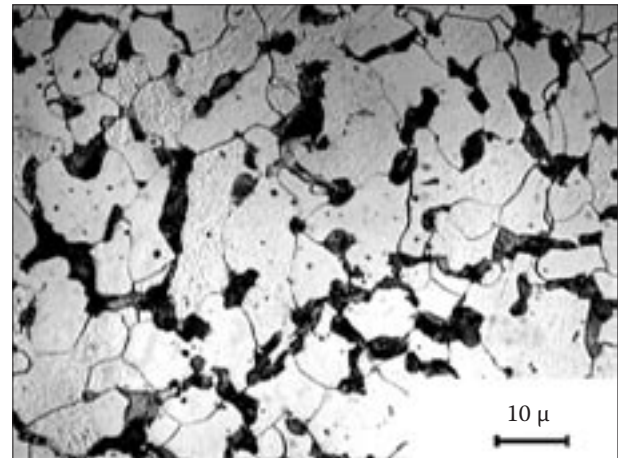


Fig. 4. Ferritic-pearlitic structure, fine-grained laminated to transition pearlitic

For the transition curve description the function (2), (3), (4), (5) were selected. The function parameters were calculated in MathLab.

#### Algorithm notation in MathLab

For compiling of the algorithm in MathLab the arithmetic means of the impact energy measured values of single sets of measuring are necessary. This arithmetic mean values were used as “entry of impact energy values”. For upper and lower limits of the curve the arithmetic mean of the impact energy value at the lowest “ $L$ ” or highest “ $H$ ” test temperature was used. The transition curve equation must be in such a form and such coefficients must be used, when the MathLab software operate. The parameter “ $C$ ” is the arithmetic mean of the highest and the lowest impact energy. The parameter “ $D$ ” is a half of

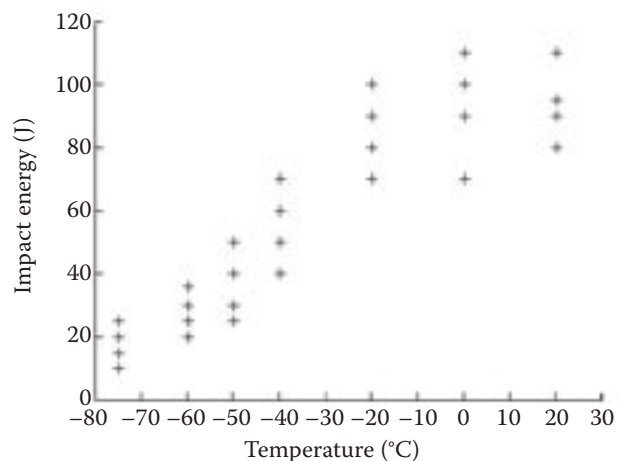


Fig. 5. Graphic representation of the measured values

the highest and the lowest impact energy difference. (parameters “a” and “b”) which we determine using the least-squares method (KUNCICKY 2004). Then the curve equation contains only two unknowns

```

L = 18%h           – the lowest impact energy value
H = 94%h           – the highest impact energy value

% deviations calculation
C = (L + H)/2      – calculation of the parameter „C“
D = (H - L)/2      – calculation of the parameter „D“
T = (-75 -60 -50 -40 -20 0 20) – entry of the temperatures
Y = (18 28 36 55 85 93 94) – entry of the impact energy values
a = -50            – estimation of the parameter „a“ (transition temperature)
b = 0              – estimation of the parameter „b“ (transition range width)
consA = a; %Ttr    – inflexion point of the curve
consB = b; %delta t – transition range width
for j = 1:700      – number of loops for the parameter „b“ calculation
b = b + 0.1        – step value for the parameter „b“ calculation
a = -50
for i = 1:700      – number of loops for the parameter „a“ calculation
a = a + 0.1        – step value for the parameter „a“ calculation
for k = 1:7
Kv(k) = C + D × tanh(2 × (T(k)-a)/b) – the curve equation
end
odch(i,j) = ((Y(1) - Kv(1))^2 + (Y(2) - Kv(2))^2 + (Y(3) - Kv(3))^2 + (Y(4) - Kv(4))^2 + (Y(5) - Kv(5))^2 + (Y(6) - Kv(6))^2 + (Y(7) - Kv(7))^2)
end
end

%determing of min
minim1 = min;
minim2 = min(minim1);

%determining of the minimum ordinates
for j = 1:700
for i = 1:700
if odch(i,j)==minim2
pozA = i;
pozB = j;
end;
end
end

%coefficients calculation
b = pozB × 0.1 + consB – value of the sought parameter „b“
a = pozA × 0.1 + consA – value of the sought parameter „a“

%graph drawing
tep = -80:1:20 – entry of the range and axis step
vyst = C + D × tanh(2 × (tep - a)/b);

plot(tep,vyst)
axis(-90 30 10 100) – adjustment for the axis range representation
hold on
plot(T,Y,'r')

```

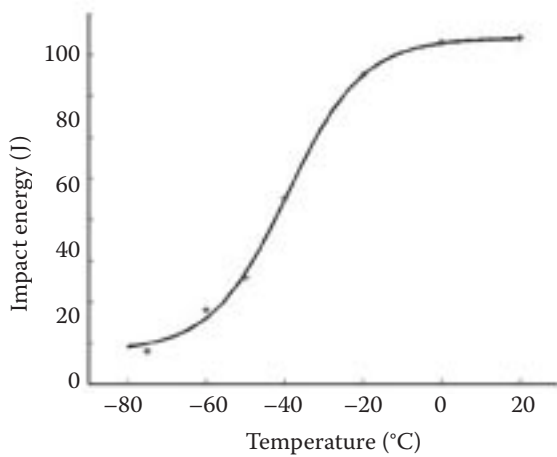


Fig. 6. Course of the hyperbolic tangent function after the measured values approximation

### RESULTS

From the calculation of looking for the unknown parameters of the equation we get the unambiguous description curve, where mere by a introducing of  $KV = 27$  J value it is possible to get the transition temperature using the method of the minimum energy. For the further method of looking for the transition curve equation namely using the method of the curve inflexion point, the transition temperature value is one if the demanded parameters already at the calculation.

When we look for another function than the hyperbolic tangent type used by us in Matlab, which describes better the relation between the impact energy and the temperature, we can use the functions (3), (4), (5). These functions can be suitably chosen for the curve description or the algorithm can be modified by looking for such functions parameters, which have the minimal deviations from the measured values.

Fig. 7 presents the functions (2), (3), (4), (5), which describe the measured values of the impact energy. For these functions the same values of the upper ( $H$ ) and lower ( $L$ ) limits were entered in the same way as for the model program. At the arcustangent function (3) the same parameters were looked for as at the function (2). But the arcustangent function demands to look not only for the inflexion point and the transition range width, but in addition to look for the limits  $H$  and  $L$ . The signum function has the same curve as the hyperbolic tangent function and is unambiguous for the description. The exponential function (5) is of the deficient curvature in the curve bendings.

### CONCLUSIONS

The interlay of the impact energy values using the function which unambiguously describes the

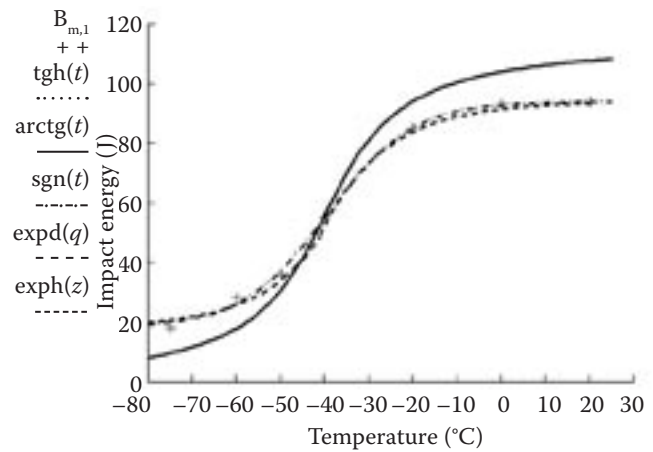


Fig. 7. Functions (2) –  $tgh(t)$ , (3) –  $arctg(t)$ , (4) –  $sgn(t)$ , (5) –  $expd(q)$ , and  $exp(z)$  describe the measured impact energy values.  $B_{m,l}$  are the impact energy arithmetic means

transition curve is one of possibilities. By a simple entry of energy limits we get the transition temperatures, which are the guide for the construction design.

The suitable chosen function for the transition curve description or for the entry in the algorithm of a suitable function from many offered gives the possibility to define the measured values evaluation with more precision. In our opinion the values entry in MathLab program and the curve plotting inclusive the transfer in the text editors is more simply and the results are more accurate than when using the accessible tabulator processor excel. In addition the searched results can be automated, as the transition temperature determination using the minimum energy method, which is used for the making of the constructional steels truncated symbols, or if need be, for the transition temperature as the inflexion point searching. In MathLab calculation the inflexion point of demanded parameters.

The transition curves and transition temperatures determination is one of the most used tests. In this paper we attempted to outline the possibilities for the more effective and accurate transition curves determination and for required the data. The model presented in this paper is possible to be used for the hyperbolic tangent function parameters determination. For the other functions the notation must be modified in the appurtenant form.

### References

- HERÁK D., CHOTĚBORSKÝ R., 2004. The mathematical descriptions transitions curve. In: 9. medzinárodné sympóziu jakost a spoľehlivosť. Nitra, SPU: 89–92.

- HERTZBERG R.W., 1989. Deformation and Fracture Mechanics of Engineering Materials. New York, Wiley: 680.
- KETTUNEN O., KUOKKALA V., 2003. Plastic Deformation and Strain Hardening. Trans Tech Publications. Zurich, Uetikon: 410.
- KUNCICKY D., 2004. MatLab Programming. Upper Saddle River, Prentice Hall: 267.
- OWEN D.R.J., FAWKES A.J., 1983. Engineering Fracture Mechanics, Numerical Methods and Application. Swansea, Pineridge Press Ltd.: 305.
- SIH G.C., 1978. Mechanics of fracture – stress analysis of notch problems. Alphen aan den Rijn, Noordhoff International Publishing: 198.
- SIH G.C., 1973. Handbook of Stress Intensity Factors. PA, Lehigh University.
- SIH G.C., LIEBOWITZ H., 1968. Mathematical Theories of Brittle Fracture. In: LIEBOWITZ H. (ed.), Fracture, Vol. II. New York, London, Academic Press: 67–190.

Received for publication July 1, 2005

Accepted after corrections September 12, 2005

## Matematické funkce pro popis tranzitních křivek

**ABSTRAKT:** Jednou z důležitých vlastností materiálů je houževnatost. Nejpoužívanějším konstrukčním materiálem je v současné době stále ocel. Tento materiál se v závislosti na teplotě může porušit jak křehce, tak i houževnatě, a proto hledáme teplotu, nad kterou se bude porušovat houževnatě. Jednou z metod, jak zjistit houževnatost teplotně-tranzitním přístupem, je zkouška rázem v ohybu podle Charpyho v určitém teplotním intervalu. Soubor takto naměřených dat je velký a vyžaduje proložení křivkou, která nám reprezentuje závislost nárazové práce na teplotě.

**Klíčová slova:** tranzitní křivka; tranzitní teplota; nárazová práce

---

*Corresponding author:*

Ing. ROSTISLAV CHOTĚBORSKÝ, Ph.D., Česká zemědělská univerzita v Praze, Technická fakulta, katedra materiálu a strojírenské technologie, 165 21 Praha 6-Suchbát, Česká republika  
tel.: + 420 224 383 274, fax: + 420 234 381 828, e-mail: choteborsky@tf.czu.cz

---