

# Human capital and modelling of its development

## *Lidský kapitál a modelování jeho rozvoje*

J. TVRDOŇ

*Czech University of Agriculture, Prague, Czech Republic*

**Abstract:** The paper deals with the relation between education, as pivotal characteristics of capital, and efficiency of school work-places with use of production modelling. A starting analytic tool is a determination of school facilities efficiency according to an efficiency matrix from which it results that also schools with a lower volume of resources per a student can significantly contribute to human capital development. Transformation of these sources into knowledge is expressed by a production function of education in which results of students are an endogenous variable in dependence on school resources, qualification level of students' families, school-mates' level, previous results of students and their effort. The course of the production function and its shape depends on many factors and economy of scale determined in the paper.

**Key words:** education economy, human capital, modelling, production function in education

**Abstrakt:** Článek se zabývá vztahem mezi vzděláním jako stěžejní charakteristiky kapitálu a výkonností školských pracovišť s využitím produkčního modelování. Výchozím analytickým nástrojem je vymezení výkonnosti školských zařízení dle matice efektivnosti, z níž vyplývá, že i školy s nižším objemem zdrojů na studenta mohou významně přispívat k rozvoji lidského kapitálu. Transformace těchto zdrojů ve znalosti je vyjádřena produkční funkcí vzdělání, v níž jsou endogenní proměnnou výsledky studentů v závislosti na školských zdrojích, kvalifikační úrovni rodin studentů, úrovně spolužáků, předchozích výsledků studentů a jejich úsilí. Průběh produkční funkce a její tvar závisí na řadě faktorů a ekonomii z rozsahu vymezených v článku.

**Klíčová slova:** ekonomie vzdělání, lidský kapitál, modelování, produkční funkce ve vzdělání

The category human capital and human resources in concurrence with their significance become one of the most often used terms of economists and managers making an effort to increase economic efficiency at both the microeconomic and macroeconomic levels. It results from their pivotal and non-substitutable role in the security and development of competitiveness of enterprises, states and integrated units.

According to the Lisbon Summit, the European Union should become a regional complex with the highest economic efficiency. However, differences between the USA and Europe in GDP per one inhabitant (by 40 percent) and also in productivity not only persists but rather increases. In productivity, they grew from 20 to 30 percents in less than last ten years. The biggest differences are, however, in the dynamics of investment in future growth, it means in education, research and dissemination of new technologies, especially information, and of biotechnology. Only three European countries – Sweden, Finland and Denmark – can compare with the USA in these indicators.

An objective identification of reasons of this situation, which is a starting point of its improvement, is a complex task, however, the insufficient support of education is still more obvious.

The European Investment Bank in this connection draws the attention to the fact that investment in a knowledge society reach almost seven percent the GDP in the USA, while in Europe only about four percent. The lowest investment is in Europe in research and development, in software and higher education. That is why, and also regarding a high rate of return of investment in education, the EIB has just the support of human capital creation among five priorities of its policy.

Europe has an unflattering position also in the evaluation of universities. In a Chinese comparable study devoted to universities, 35 American ones were ranked among the 50 best universities in the world. Among the best 500, there were 169 American, 42 British, 43 German, 35 Japanese and 22 Canadian universities. From the Czech, only the Charles University was on

the 318<sup>th</sup> place, and three Hungarian universities were ranked in the chart.

## AIMS AND METHODOLOGY

In context with the above mentioned aim of the paper is to contribute with use of the theory of educational activity efficiency to an analysis of selected determinants of development of knowledge society which should support optimization of resources allocation in the educational system. The methodological procedure results from the following hypothesis:

- Basic elements of the educational system of a school, a faculty and a university can be characterized analogically to the entrepreneurial sphere as organizational formations transforming inputs to results of educational activity.
- To evaluate their efficiency, economic principles can be used. Hoenack (1994) included knowledge economics directly in the study of organizational behaviour and emphasized the advantages of decentralization with the compensation of information costs, and he introduced stimuli for the improvement of technologies and motivation of workers.
- The analysis of costs efficiency of educational institutions and its increase is a significant tool of knowledge society.
- A starting point of the investigated cost function of education is its inverse production function enabling the application of the theory of educational enterprises and the use of basic economic principles of the firm theory (Varian 1999) in a purposeful modification.

Used methods were predetermined by problems which were investigated according to the above mentioned structure, however, in all parts it was dealt, though in a various rate, with a quantitative analysis, qualitative analysis, synthesis, comparison, the method of analogical judgements, the normative method, the method of questioning, document processing and so on.

## RESULTS OF THE ANALYSIS

### Specifics of educational process of educational institutions

Models of production functions of education, analogically to classical production function, have to express how educational institutions create a vector of results from the given flow of inputs. This “educational

production function” is, of course, a model stylization which supposes that the technologies the educating students were analogical to the technologies used in production of goods. The following statement does not have to correspond: what “a school provides” will result from what is the “input” in it. Even if in the case of an educational production function the relation between these two aspects does not have to be expressed directly, especially, if a real endogenous factor home and an input of student’s output works. On this account, much more substitutable “results” than “outputs” are considered: a result can be an academic education or wages, however, in leaving a school, it can be no education or many particular effects or a summary of profits from education. Therefore, the first task is to model such an objective enterprise function and procedures.

An original study of production functions of education compares variously sized volumes of resources with homogenous units of results with a smaller externalization on how these inputs are created: aspects as educational technologies or forms of school management are not taken into account. One key difference among schools is therefore between the efficiency of educational process and the amount of resources. A good resources-provided efficient faculty should produce better results than a resource-limited and inefficient faculty. However, there is no obvious way of evaluation of the absolute results of resource-limited efficient faculties compared to resource-well-provided insufficient faculties. Therefore, in modelling of a production function as a relation between inputs and outputs, the differences in the level of efficiency has to be considered regarding the context of the total level of resources.

Two dimensions of efficiency refer to that. The first, efficiency of inputs selection, considers the selection of all inputs so that society marginal costs were equal to society marginal profits.

The second, efficiency of outputs selection, is connected with the choice of the right amount of produc-

Table 1. The matrix of efficiency for faculties of four types

	High input of resources per student	Low input of resources per student
Efficient output (high added value)	A	C
Inefficient output (low added value)	B	D

tion from each educational program, again so that social marginal costs were equal to social marginal profits of benefit (from results). In this stage, it is not necessary to prescribe what these outputs should be (results of exams, earnings, welfare etc.).

Here a simple explanation of efficiency of output selection could help, supposing an existence of four types of faculties as it is shown in the Table 1. A faculty of the type A is efficient and well resource-efficient. It should produce the highest absolute results. A faculty of the type D is inefficient and has few resources. That is why it will probably create the lowest absolute results. A faculty of the type B is inefficient but well-provided with resources, while the faculty C is efficient but with a lack of resources. A priori, absolute results of the faculty B and C cannot be evaluated. According to the added value, the types A and C should reach the same or better score than the types B and D. So, the group AB is compared with the group CD, without the presumption of differences in efficiency. The question is whether AB reaches or not higher results than CD and, in a given comparable efficiency, if it can be expected. However, if this estimation is not fulfilled, then the difference between high and low units of resources is irrelevant and “money (inputs resources) does not matter”. Further it also indicates that not all types of faculties are efficient. In fact, the result that “money does not matter” shows necessarily that the type CD is more efficient than the type AB.

A range of conclusions can follow. An activity of the faculty AB can be considered as a simple “wasting” of other input resources which it gets, and the type CD is not “efficient” in fact, but it does not have to be true and it is necessary to analyze them further (maybe continuous and historical evidences can be cited, as Hanushek 1998 reports). The resources given to A and B can be re-measured, although also political costs and costs for transitive adjustments can be included in them here. Resources can be transferred, preferentially to the faculty C – supposing that such faculties can be identified – or at least to faculties of the type CD. But it would need to find out if they were able to repeat their efficiency with more resources (it means that there is no non-economy of scale). However, one possible influence will manifest itself, if the faculty systems cause the equity of added value, than the allocation of a resources unit is not far from the optimum in reality. Transfers from A to D which distribute their inputs better, secure that all students will get the same further education. The most contributonal investigation regarding the evidence that “money does not matter” would be, if we look at how and why the efficient faculty with a low volume

of resources is. For that, a production function of education can be used.

## Determination of production function of education

### *Production function of manufacturing concern*

A knowledge society influences in a fundamental way economic development which in reality means a growth of product potential. Its growth is influenced especially by factors on a supply side. It can be written:

- $y = f(C, L, A)$  (1) Classical production function of Adam Smith
- $y = f(C, L)$  (2) Neoclassical production function
- $y = F(IC)$  (3) Current definition of production function

$y$  = GDP or GNP

$C$  = capital

$L$  = labour force

$IC$  = intellectual capital

Intellectual capital can be divided into:

- market capital...knowledge of market and ability to influence the market
- human capital...knowledge and education of labour force
- structural capital...intra-plant culture and firm operation

The particular parts of the intellectual capital influence mutually, they work in synergy and co-create a knowledge society.

### *Production function of education*

From an individual perspective, education is required as one of the elements of the general function of utility and in such a way it will be worked out in dependence on how fast it can increase the level utility in comparison with other factors. If an analysis of industrial organization is used, a starting production of education can be written more formally in the sense:

$$A_t = h(R_{t-1}, F_{t-1}, P_{t-1}, A_{t-1}, Z_{t-1}) \quad (1)$$

$A_t$  are achievements of a student in time  $t$ ;  $R_{t-1}$  are school input resources;  $F_{t-1}$  are inputs of a family and a household during the previous period;  $P_{t-1}$  are inputs of peers. Previous achievements of the

student  $A_{t-1}$  should be included as an expression of previous ability or a starting context for learning, so the achievement in time  $t$  depends on resources devoted to these achievements. Finally, the effort of the student  $Z_{t-1}$  should be also included in the function. This variable, which can be determined endogenously by other elements of production function, is crucial for the efficiency of the provider of education. This function can be then modelled with an implicit presumption that individuals maximize  $A$  as their result. This maximization can be considered as subject to many limitations:  $R$  can be considered as independently fixed thanks to governmental inputs. For family inputs, the price of time  $F$  cannot exceed expenditures for other goods plus no income from loans and probably it is determined uniformly together with  $R$ .  $P_{t-1}$  and  $A_{t-1}$  can be perhaps considered as exogenous,  $Z$  are probably a function of the previous efficiency and cognitive ability and can be measured with use of consumed time. It is expected that a partial derivation of the result compared to every input will be positive. However, it is less obvious in which relation partial derivations are (measured nominally) (e.g. if  $\partial A/\partial F > \partial A/\partial P$  or vice versa).

For educational enterprises, the purpose function is less obvious. It could be either a sum of values  $A$  of the particular students, an average value  $A$  across  $n$  students or a threshold value  $A$  per student. So more generally, providers of education can be considered as multiproduct enterprises. Universities produce both students and also research results; schools can produce students socialized and with broaden human capital (these multiproduct effects are considered lower). Conventionally for simplicity, a uniform maxim is considered as an aggregation of students achievements. Therefore, education providers have to divide students and resources among schools (and study groups in schools) so that they reach this maxim and subject to the ability and possibility to divide students:

regarding

$$\int_0^{B'} \sum_{j=1}^J n_j A_j(B) dB \quad (2)$$

subject to

$$TC = \sum_{j=1}^J R_j n_j$$

Therefore, the school will maximize the achievement  $A_j$  of  $n_j$  students, subject to the ability  $B$  till the threshold level  $B'$ ; across groups  $j = 1 \dots J$  (Arnott, Rowse 1987). The mentioned maximum of the equation (2) is a conditional budgetary limitation which

is here more simply marked as total costs  $TC$  equal to the amount of resource  $R$  (given in a way of an exogenous pattern of financing) multiplied by the number of students. In the given optimal level of achievement it can be supposed that such an achievement is transferred unambiguously or monotonously into other results of a direct utility, as are higher earnings or better health.

Such a production function is modelled with the use of the Cobb-Douglas function or the function with a constant elasticity of substitution (Hanushek et al. 1996; Figlio 1999, used translog functional form). Some of these inputs, as for example achievements of a family and students, have not, however, marked process which could be allocated to them at least from the school perspective (even if social designers could be able to determine the process). Further the above mentioned discussion and Figlio's models (1999) insist that it is improbable that these inputs were either additive, it means that there is no differential effectiveness, or that the function was homothetic, i.e. that the marginal rate of substitution among inputs depends on shares of inputs and not on the production extent. By use of the equation of production function of an individual, the additivity shows that all interaction terms have zero coefficients:

$$A_t = m_1 (S_{t-1}) + m_2 (F_{t-1}) + m_3 (P_{t-1}) + m_5 (Z_{t-1}) \quad (3)$$

The homothetic production function would require that  $m(\cdot)$  was monotonous for the equations (4) and  $h(\cdot)$  homogenous of the 1st grade:

$$A_t = m(h(S_{t-1}, F_{t-1}, P_{t-1}, A_{t-1}, Z_{t-1})) \quad (4)$$

Public and private activities in education will probably weaken these presumptions. School resources will be perhaps partially dependent on family inputs, but only thanks to local taxes and middle voter preferences; peers' inputs will be probably connected in many resources and achievements of parents and students will be endogenous to achievements of teachers. So generally, presumptions about pedagogues, forms of lectures and school structures, i.e. education technology, should be taken into account in the models. More critically it can be said that the aim of schools is to produce high-quality students, but also their high absolute number, so that a compromise probably happens between these two outputs (which are not fully recorded in the above mentioned optimization).

Other method of efficiency measurement from an enterprise's perspective is the use of approaches which connect production with an imaginary border and do not force a standard form of technology to

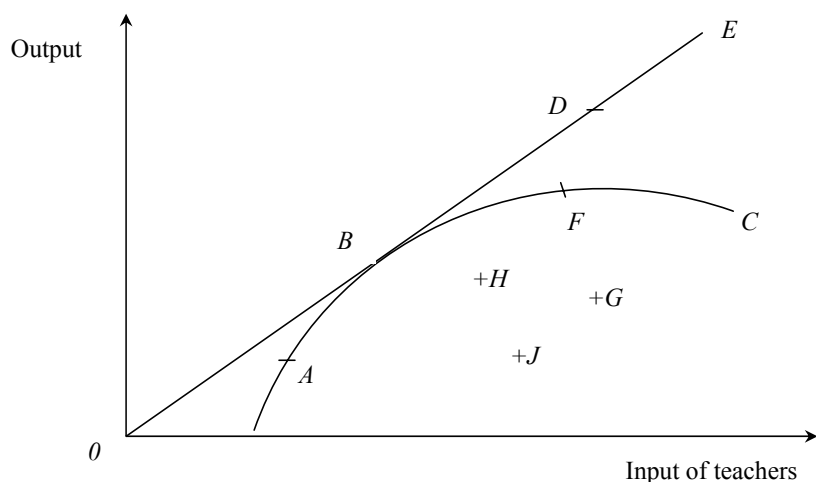


Figure 1. Output function for educational enterprises

all institutions. The Figure 1 shows a simple measurement of efficiency with a relation of one input (teachers) to output, while all other inputs are fixed. By mapping the achievements of seven providers (A, B, F, C, H, G, J), production functions connect results with changes in this common input with admitting differences between technical efficiency and efficiency from scale. The shape of the curve ABFC indicates technically efficient providers and B is also efficient from the viewpoint of extent (tangent to OBE) while A seems as a too small, the provider F is efficient in one domain but seems to be too big. The shape ABFC and the share of schools in this curve are interesting. Insufficient providers are indicated as G, H and J; all show significantly lower outputs than enterprises on the efficient curve.

A similar approach of efficiency measurement with the use of linear programming is an analysis of cover data. In this analysis, schools and faculties are considered as decisive units producing  $s$  outputs with use of  $m$  inputs. The  $k$ -th unit produces  $A_{rk}$  outputs  $r = 1, \dots, s$  with  $G_{ik}$  inputs  $i = 1, \dots, m$ . This  $k$ -th unit aims to maximize its weighted set of output where weights for inputs  $v_{ik}$  and weights for outputs  $u_{rk}$  have to be chosen. These weights have to be elected in such a way so that a ratio of a weighted output to a weighted input was equal to 1 or lower than 1. A weighted sum of inputs should reach a unity and a weight linked to each output has to be non-negative (Johnes 1999). Therefore, for each decisive unit a linear program should be solved in such a way to maximize the weighted sum of outputs. For  $k$ -th unit the problem of maximization is given as:

maximize

$$h_k = \sum_{r=1}^s u_{rk} A_{rk} \quad (5)$$

Result  $h_k$  equal to 1 represents a technical efficiency. Because every unit of a provider is evaluated regarding the endogenously selected outputs, the forms of efficiency are not limited. In fact providers can be identified as efficient, with the given results, which they had set for themselves. This approach can be especially useful when a technology is not well defined or when there is a multiplied output which a school would reach as a part of their mission (for discussion see Johnes, Johnes 1995a, b). For example universities differ according to their teaching versus research mix. Schools can be distinguished by local regulations and examination result. School regional committees or educational instances will have many types of their regulations to optimize. These missions will manage the choice of inputs and the structure of outputs. The analysis of cover data will determine the technical effectiveness of institutions.

## CONCLUSION

Schools, faculties and universities are basically educational enterprises which use resources to reach tutorial results, by analogy like firms produce outputs. The mentioned determination enabled a better understanding of efficiency and effectiveness of costs in provision of education. The realized analysis should be considered as very important for knowledge economy with a considerable volume of time and resources devoted to provision of training and education.

Then a result is an improvement of cost effectiveness of educational organizations. However, the necessary first step is modelling of the production function of education which is a decision point of this paper.

## REFERENCES

- Arnott R., Rowse J.I. (1987): Peer group effects and educational attainment. *Journal of Public Economics*, 32 (3): 193–216.
- Figlio D.N. (1999): Functional form and the estimated effects of school resources. *Economics of Education Review*, 18: 241–252.
- Hanushek E.A. (1998): Conclusions and controversies about the effectiveness of schools. Federal Reserve Bank of New York, *Economic Policy Review*, 4: 1–22.
- Hanushek E.A. et al. (1996): The identification of school resource effects. *Education Economics*, 4: 105–125.
- Hoenack S.A. (1994): Economics, organizations, and learning: research directions for the economics of education. *Economics of Education Review*, 13: 147–162.
- Johnes G. (1999): The management of universities. *Scottish Journal of Political Economy*, 46: 505–522.
- Johnes J., Johnes G. (1995a): Neither poison nor panacea. *Economics of Education Review*, 14: 317–318.
- Johnes J., Johnes G. (1995b): Research funding and performance in UK university departments of economics: a frontier analysis. *Economics of Education Review*, 14: 301–314.
- Varian H.R. (1999): *Intermediate Microeconomics*. W.W. Norton, New York.

Arrived on 1<sup>st</sup> February 2006

---

### Contact address:

Jiří Tvrdoň, Czech University of Agriculture Prague, 165 21 Prague 6-Suchbát, Czech Republic  
tel. +420 224 382 290, e-mail: Tvrdon@pef.czu.cz

---