Multiple criteria network models for project management

Vícekriteriální síťové modely v projektovém řízení

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Abstract: The aim of the paper is to present one possibility of how to model and solve a resource oriented critical path problem. As a starting point, a single criteria model for critical path finding is shortly mentioned. Lately, more criteria functions for this model are defined. If any project task uses more resources for its completion, its duration usually depends on only one of them – other resources are not fully used. In here defined multiple criteria approach, these dependencies are not assumed. Each criteria function is derived from a theoretical task duration based on a number of units of only one resource and on its importance. Using either linear programming model with aggregated criteria function or simple Excel calculation with Microsoft Project software support, a so-called compromise critical path can be found. On this path, some resources are overallocated and some are underallocated but the total sum of all underallocations and all overallocations is minimized. All resources are used as effectively as possible and the project is as short as possible too.

Key words: network modeling, agricultural project management, multi criteria programming, compromise critical path, resource management, Excel

Abstrakt: Cílem tohoto příspěvku je představit jeden z postupů, řešících otázku stanovení kompromisní kritické cesty v modelech projektového řízení vzhledem k optimálnímu využití zdrojů disponibilní zdrojové základny. Výchozím předpokladem modelu je nepřímá úměrnost mezi délkou trvání činnosti a intenzitou čerpání zdroje, konstantní intenzita čerpání po dobu trvání činnosti a konstantní ohodnocení vazeb mezi jednotlivými činnostmi. Základní struktura síťového modelu je popsána s využitím omezujících podmínek modelu celočíselného programování a zdrojové zabezpečení ve vztahu k časové náročnosti jednotlivých činností je formalizováno za pomoci kriteriálních funkcí. Počet kriteriálních funkcí je totožný s počtem zdrojů zabezpečujících projekt. Každému zdroji v projektu je navíc možno přiřadit určitou váhu, reprezentující jeho důležitost, či ocenění pro příslušnou činnost či pro celý projekt. Vzniklou vícekriteriální úlohu lze potom převést například s využitím vážené agregace kriteriálních funkcí na klasickou maximalizační úlohu celočíselného lineárního programování. Výsledkem řešení je kompromisní kritická cesta, na které je souhrn nevyužití a přetížení jednotlivých zdrojů minimální. Po aplikaci následné analýzy optimálního řešení je možno stanovit optimální počet zdrojů nutný k zabezpečení daného projektu vzhledem k vypočtené kritické cestě. Analýza citlivosti optimálního řešení dále umožňuje stanovit většinu klasických ukazatelů činností jako například časové rezervy či lhůtové ukazatele.

Klíčová slova: síťové modely, projektové řízení v zemědělství, vícekriteriální programování, kompromisní kritická cesta, management zdrojů, Excel

INTRODUCTION

Agricultural project management is the process that assures every aspect of product development and actions taken to minimize both time and cost in bringing a product to the marketplace. At present time, the modern project management uses a lot of sophisticated mathematical methods for project duration minimization, for cost minimization, for optimal allocation of resources etc. Many of these methods are based on network modeling techniques, especially critical path methods, and mathematical programming algorithms. Nevertheless, when working with resources, many practical approaches are based on empirical or heuristic procedures, because the exact mathematical methods are quite hard to use and so they are not included in the majority of software standards. In this text, I would like to present one simple approach, which will help us to effectively allocate and manage project resources.

METHODOLOGY

Recently, practically all project management formalization tools were based on the Activity on Arc (AOA) network graphs where all project tasks (activities) are represented by arcs and nodes are used only for milestones. Actually the majority of project management software uses activity on node (AON) network graphs. It means that all project tasks (activities) are represented by nodes and their relationships by arcs. When finding critical path, each project task can either be critical or noncritical. Let us remind that critical path is very important for some project terms and deadlines because the delaying critical task will cause delay of the project finish date. My further approach is primarily based on mathematical model of the AON network model. This model is a simply modified integer programming linear model and all approaches below can be described using this model. In the following text, no such mathematical model will be formalized (for more details about this model see Šubrt 2001) but the whole idea and approach will be demonstrated on a practical example from product marketing branch.

The aim of my approach is to show how to use resources in project management projects in a better way, more effectively. This approach deals mostly with the resources "type of work" (labor forces) because material resources have different features not suitable for the type of calculations mentioned below.

As a starting point, let us suppose a project formalized in a network graph. Theoretically, no matter if we shall use an Activity on Arc graph or an Activity on Node graph but AON one is more wide-spread and allows to model more complicated project management situations (for example various types of task dependencies etc.). All later notations will assume AON graphs. Network graphs in project management we usually use to calculate length of the project, analyze structure of critical path; to analyze noncritical tasks (paths) slacks etc. For most of this calculation, we have to assume fixed duration of a task (usually deterministic), which in every real situation depends on resources assigned to it and on the amount of available resource work. (Note that no task in the universe can be completed without a resource!)

Let us assume there is s resources available for the project and s_i , $s_i \leq s$, resources assigned to i-th task. Classical methods assume that the task duration depends only on a critical resource, it means on a resource which amount of work and number of assigned units determines these duration. The duration of a task can be defined as

$$t_i = \max_{k=1,2,\ldots,s_i} \left(\frac{v_i^k}{r_i^k}\right)$$

where t_i is a i-th task duration, v_i^k is an amount of work for one k-th resource unit needed for i-th task completion, r_i^k is a number of k-th resource units assigned to i-th task and s_i is a number of resources assigned to i-th task. The r-th resource is called critical when

$$\frac{v_i^r}{r_i^r} = \max_{k=1,2,\ldots,s_i} \left(\frac{v_i^k}{r_i^k}\right)$$

All other resources assigned to this task are not fully used and we call them underallocated. It means that practically on each task in a project, one or more resources are not effectively used. Of course after completion of the adequate part of work, these resources can be assigned to another task and lag times can be minimized but in practice this policy does not work so properly (for more information see Critical Chain Approach Goldratt 1999). Calculating the project (using mathematical programming model or by modified CPM method) we obtain critical path, where critical resources are optimally (100%) allocated and all other resources are underallocated. On noncritical paths critical resources are 100% allocated too but due to slacks on noncritical tasks 100% allocation of these resources cannot be guaranteed on the whole length of any noncritical path. Applying cost coefficient sensitivity analysis (or task slack analysis), a detailed result of the possible infectiveness of resource usage can be done. This classical approach can be called "single criteria" because while searching critical path, the mathematical programming model with only one criteria function in the form of "project duration maximization" is used.

Multiple criteria approach – compromise critical path

In a multiple criteria approach let us assume theoretical dependencies of task duration on each resource separately. It means that each task has as many theoretical durations as the number of resources assigned to it and the whole model has as many theoretical critical paths as the number of all resources defined.

The idea of methods mentioned below is to aggregate all these "individual" critical paths into one "compromise" critical path where no resource can be defined as critical and where the sum of resource overallocations is minimized and the sum of resource underallocations is minimized too. Analyzing these over (under) allocations by the application of sensitivity analysis methods, each tasks assignment can be minimally modified to obtain a solution with resource assignments as good as it gets.

For solving this kind of multiple criteria model, many algorithms are available e.g. single criteria optimization with others as constraints, weighted goal programming, criteria function aggregation, etc. The most suitable approach seems to be the last one – criteria functions aggregation. First two are not very efficient because in fact only one critical path exists and the decision maker has no reason to limit the theoretical duration of nonexistent paths. There are many ways of how to aggregate these criteria functions. The most effective one for this purpose seems to be a convex linear combination of criteria functions coefficients. In fact these convex linear combination of coefficients – in the meaning of criteria functions normalized weights – represent a relative importance of each resource for the project or for the individual task where the concrete resource is assigned. This relative importance of a resource can be expressed for instance by its cost, by its credibility, by risk of its use etc.

Solving mathematical programming model, we obtain a compromise critical path and its length determining project duration. On this path (and on some other paths too), some resources are overallocated (some task have not enough resources for their realization) and some of them are underallocated (these ones have free capacities) but the total sum of under allocation and over allocation is minimized and relatively balanced. The higher a resource's weight was set the lover its over (under) allocation is. The theoretical duration of each critical task is

$$t_i' = \sum_{k=1}^{s} w^k \frac{v_i^k}{r_i^k}$$

where w_k is a weight of k-th resource. From this formula, the theoretical amount of resources needed for task completion could be defined as

$$r_i'^k = w^k \frac{v_i^k}{t_i'}$$

If the difference $\Delta r_i^k = r_i'^k - r_i^k$ is positive then the k-th resource is underallocated and Δr_i^k of its units is free. If the difference Δr_i^k is negative then the k-th resource would be overallocated and the number of its units assigned must increase by $\left|\Delta r_i^k\right|$ to complete the task in time. If the majority of resources from resource pool are assigned to all tasks, we can use global resource weights (w^k) for the whole project, i.e.

$$\sum_{k=1}^{s} w^k = 1$$

When there are many different resources working on separate tasks it is better to use individual weights for each task (w_i^k), it means to use weights where

$$\sum_{k=1}^{s} w_i^k = 1, \ i = 1, 2, ..., n$$

RESULTS

Let us have a small part of large marketing project called "Evaluate Business Approach, Potential Risks and Rewards". This project part has been formalized in an AON network. It has 8 tasks (nodes) and 6 resources assigned to them. Each node in the graph includes the following information: Task ID, Task Name and Assigned Resources Names. Table 1 shows the amount of resource work needed for task completion.

Firstly let us apply classical approach where each task duration is defined as

$$t_i = \max_{k=1,2,...,s_i} \left(\frac{v_i^k}{r_i^k}\right)$$

so $t_1 = 12, t_2 = 10, t_3 = 12, t_4 = 24, t_5 = 24, t_6 = 24, t_7 = 8$ and $t_8 = 12$. Using this approach, we have to keep in mind that Parker is assigned only to 66.7% of first task's duration. It should mean either that be utilises only 66.7% of his capacity, that his intensity of work shows 33.3% decrease, or that he can start 4 hours later than Johnson, or that he can be finished 4 hours before Johnson etc. Applying either CPM method or mathematical programming model with single criteria function

$$12t_1 + 10\ t_2 + 12t_3 + 24t_4 + 24t_5 + 24\ t_6 + 8\ t_7 + 12\ t_8 \ldots \max$$

we obtain critical path 1–3–4–5–7–8 and its length (project duration) 92h. Analyzing resources on this path, we can see that that the efficiency of its usage is relatively poor but no resource is overallocated (see Table 3).

Now let us apply multiple criteria approach. Theoretically we can define as many individual critical paths (criteria functions) as the number of resources in the project is and a multiple criteria-programming problem arise. For instance CP for Smith can be described by criteria function

$$10t_2 + 18t_4 \ldots \max$$

its structure will be 2–4 and its length 28h. Let us assume each resource has individual weight depending on its importance for the concrete task (see Table 2). For effective solving of this problem, we have to find the way of how to aggregate these individual critical paths into one.

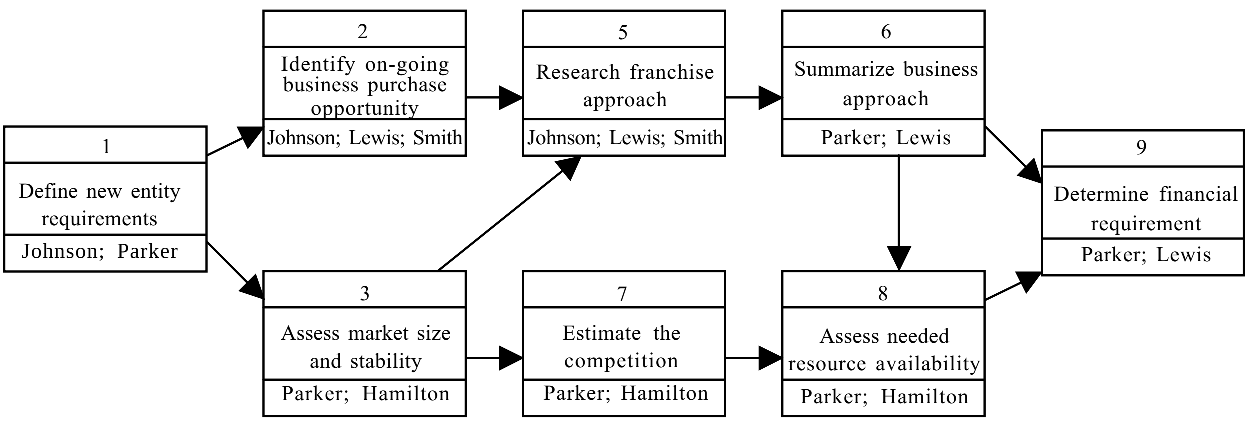

Figure 1. Corresponding part of the marketing project AON graph

Table 1. Amount of resource work needed for each task completion (hours)

Task ID	1	2	3	4	5	6	7	8
Resource name								
Parker	8		12		24	24	8	12
Hamilton			8			12	8	
Lewis		10		24	18			12
Smith		10		18				
Johnson	12	8		24				

Table 2. Individual resource weights

Task ID	1	2	3	4	5	6	7	8
Resource name								
Parker	0.4		0.2		0.5	0.7	0.5	0.8
Hamilton			0.8			0.3	0.5	
Lewis		0.3		0.4	0.5			0.2
Smith		0.6		0.3				
Johnson	0.6	0.1		0.3				

Table 3. Important tasks characteristics obtained by classical approach

Task ID	1	2	3	4	5	6	7	8
Task duration (h)	12	10	12	24	24	24	8	12
Total slack (h)	0	2	0	0	0	24	0	0
% of resource allocation								
Parker	66.67		100		100	100	100	100
Hamilton			66.7			50	100	
Lewis		100		100	75			100
Smith		100		75				
Johnson	100	80		100				
Critical path length (h)					92			

Table 4. Important tasks characteristics obtained by compromise critical path approach

Task ID	1	2	3	4	5	6	7	8
Task duration (h)	10.4	9.8	8.8	22.2	21	20.4	8	12
Total slack (h)	0	0	1	0	0	23.8	0	0
% of resource allocation								
Parker	76.92		136		114	118	100	100
Hamilton			90.9			58.8	100	
Lewis		102		108	85.7			100
Smith		102		81.1				
Johnson	115.4	81.6		108				
Critical path length (h)					83.4			

According to the algorithm mentioned earlier, we apply the weighted additive aggregation, where task duration will be defined as

$$t_i = \sum_{k=1}^{s} w_i^k \frac{v_i^k}{r_i^k}$$

and so $t_1 = 0.4 \times 8 + 0.6 \times 12 = 10.4$; $t_2 = 9.8$, $t_3 = 8.8$, $t_4 = 22.2$, $t_5 = 21$, $t_6 = 20.4$, $t_7 = 8$ and $t_8 = 12$. On the first task, Johnson is now about 15% overallocated and Parker about 23% underallocated (see Table 4). Increasing Parker's weight, his overallocation will decrease and decreasing Parker's weight, his overallocation will increase.

Changing weights, the project manager can effectively control resource usage on a task. Practical experiences demonstrate that 10–15% of over allocation is tolerable (especially when more than one resource unit is assigned to a task – for instance in case of manual workers) and many resources can allow such increase intensity of work. Applying mathematical programming model with aggregated criteria function

$$10.4\,t_1 + 9.8\,t_2 + 8.8\,t_3 + 22.2\,t_4 + 21\,t_5 + 20.4\,t_6 + 8\,t_7 + 12\,t_8 \ldots \max$$

we obtain different critical path 1–2–4–5–7–8 and its length (project duration) 83.4h. On this critical path, the sum of over- and underallocation is balanced (dependent on resource weights) and on all noncritical paths, we can additionally manage overallocation by consuming task slacks. Tables 3 and 4 compare both approaches. First one contains characteristics acquired by "classical approach", second one by "compromise critical path" approach.

CONCLUSION

There are several methods of incorporating multiple objectives in project management problems. One of them deals with the possibility of critical path finding using mathematical programming models. Applying multi objective approach, a compromise critical path with optimum resource allocation can be found. If each criteria function represents one single resource, theoretically as many critical paths as the number of project resources can be defined. After assigning a relative resource importance number to each resource, one aggregated objective function arises and the model can be solved using single objective programming method.

Remember that this approach does not substitute resolving resource conflicts using resource-leveling algorithms. It is mostly focused on resource management within a single task and consequently on finding compromise critical path. Sharing resources among two or more parallel task is an objective of subsequent algorithm.

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