The analytic hierarchy process for the decision tree with multiple criteria

Aplikace analytického hierarchického procesu pro řešení vícekriteriálních rozhodovacích stromů

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Abstract: For choosing the best decision in daily agricultural practice, it is very important to formalize the decision process using systems analysis and mathematical methods. Theory of the decision-making under uncertainty or under risk supposes one criterion. In practice, the best solution is chosen under more decision criteria. Decision problem with *m* alternatives, *n* states of nature and *k* criteria has to be solved in this case. Application of the Analytic Hierarchy Process (AHP) for solution of this problem is described in this paper.

Key words: decision process, Multiple Criteria Decision Tree, Analytic Hierarchy Process, cardinal and ordinal input data.

Abstrakt: V běžné zemědělské praxi je pro volbu nejvýhodnějšího rozhodnutí velmi důležitá formalizace celého rozhodovacího procesu pomocí systémové analýzy a matematických metod. Teorie rozhodování za nejistoty a za rizika předpokládá jediné rozhodovací kritérium. Nejlepší rozhodnutí je však často vybíráno podle více kritérií. V takové situaci musí být řešen rozhodovací problém s *m* alternativami, *n* stavy okolností a *k* kritérii. V tomto článku je ukázáno využití analytického hierarchického procesu (AHP) pro řešení takovéhoto vícekriteriálního rozhodovacího problému.

Klíčová slova: rozhodovací proces, vícekriteriální rozhodovací strom, analytický hierarchický proces, kardinální a ordinální vstupní data

INTRODUCTION

Management of agricultural enterprises, farms and service organizations has daily to solve many problems. For choosing the best decision of these, it is very important to formalize the decision process using systems analysis and mathematical methods.

Decision-making is a process of the choice of one variant from a certain list of real possible variants. If we have one criterion only, this chosen variant is considered to be optimal. However, problems for multiple criteria decision-making are very frequent in everyday life. Unfortunately, because criteria are contradictory and interests of various subject of decision making can be in conflict, we can find a compromise variant only. Multiple criteria decision-making refers to making decisions in the presence of multiple conflicting criteria under certainty.

A similar problem occurs when one criterion is present but the result of each alternative depends on the future states of nature. Decision under uncertainty and decision under risk concerns the methods for choice of the best alternative in this case.

A more complicated situation occurs, when two or more criteria are present in decision under uncertainty or decision under risk. In this case, it is necessary to combine solving algorithms from different branches as multiple criteria decision-making, decision under uncertainty, decision under risk and game theory.

METHODOLOGY

In this paper, we describe the possibility of using of the Analytic Hierarchy Process (AHP) for selection of the best alternative in the decision tree with multiple criteria.

The AHP serves as a mathematical solution method for individual or group decision-making with multiple criteria. With the AHP, the decision-maker constructs the problem hierarchy. Then the decision-maker develops priorities for alternatives and criteria used judgments or comparisons on each pair of elements. Finally, these priorities are expressed as weights. The selection of the best alternative is then based on the synthesis of the weights throughout the hierarchy. We do not need the tangible data, what is the greatest advantage of using of the AHP.

Hierarchy of the multiple criteria decision tree

The first step of the AHP is construction of problem hierarchy. Minimal number of hierarchy levels for multiple attribute decision-makings is three. The first level represents the goal of problem solving. The second level represents the criteria and the third level represents alternatives.

Similar hierarchical structure can be used for decision problems typically defined by decision matrix or decision tree (see Figure 1). The difference between decision tree structure and hierarchy for the AHP consists in sequence of the problem elements. A*lternative – state of nature* sequence is used in decision tree and *state of nature – alternative* in the AHP hierarchy.

When we select the best decision alternative under more criteria in the situation under uncertainty or under risk, the problem has to be described by hierarchy with four levels. The first level has the goal, e.g. the best alternative selection, the second level has *n* states of nature, the third level has *k* criteria and the fourth level has *m* decision alternatives.

When the decision problem is homogenous, described by decision table, its hierarchy is complete. However, the problem hierarchy (not complete) can also be constructed for the non-homogeneous problems when some alternatives are not influenced by all states of nature, when some combinations alternative/state of nature are unreasonable and when more decision modes exist.

Input information of the multiple criteria decision tree

The second step of the AHP is developing of local priorities. These priorities are typically derived based on pair wise assessments using the ratios of measurements from the scale if one exists or judgement if the preference is intangible. Saaty introduced a method of scaling ratios using the principal eigenvector of Saaty's matrix of pair wise comparison $S = (s_{ij})$.

When the precise evaluation w_i of element *i* and evaluation w_j of element *j* exist, Saaty's matrix consists of values $s_{ij} = w_i/w_j$.

When this evaluation is intangible, the elements of Saaty's matrix are considered to be an estimation of importance of *i*-th and *j*-th elements. The scale 1, 3, 5, 7, 9 is often used.

If *i*-th and *j*-th elements are of the same importance $s_{ij} = s_{ji} = 1$

If *i*-th element is slightly preferred to *j*-th $s_{ij} = 3$ and $s_{ji} = 1/3$

If *i*-th element is strongly preferred to *j*-th $s_{ij} = 5$ and $s_{ji} = 1/5$

If *i*-th element is very strongly preferred to *j*-th $s_{ij} = 7$ and $s_{ji} = 1/7$

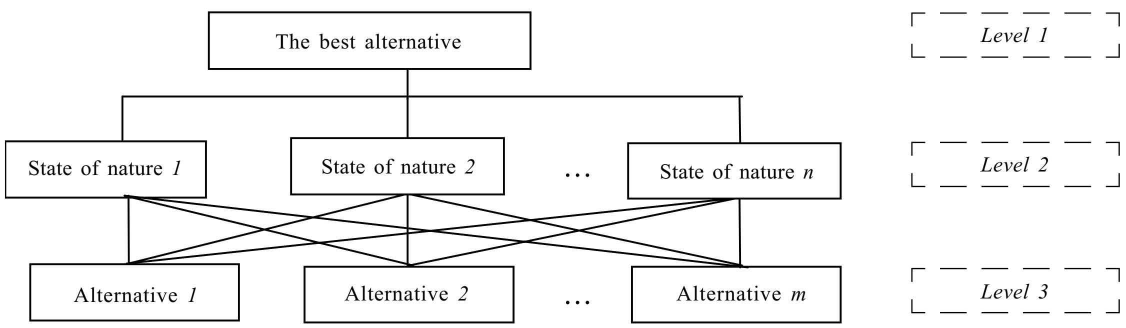

Figure 1. Hierarchy of decision tree

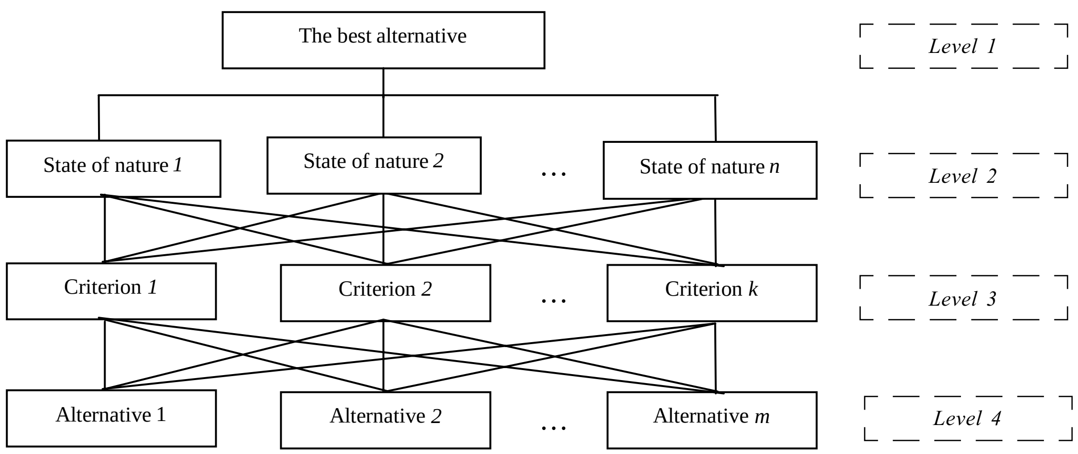

Figure 2. Hierarchical structure of multiple criteria decision tree

If i-th element is absolutely preferred to j-th $s_{ij} = 9$ and $s_{ji} = 1/9$

Intermediate stages expressed by numbers 2, 4, 6, 8 are allowed too.

The input data of the multiple criteria decision tree are:

– preferences of the criteria
– probabilities of the states of nature and
– payoffs for each combination alternative/state of nature and each criterion.

Typically, preferences of the criteria and probability of the states of nature are exactly unknown. The priorities of these elements within a hierarchy can be received by pair-wise comparisons. The priorities of states of nature have to be equal in the situation under uncertainty. If no information about criteria preferences is known, their priorities can be also equal.

On the other hand, the payoffs are often raw data. The actual values of the payoffs can be used and priorities of alternatives can be calculated directly from these data.

RESULTS – THE PROBLEM OF THE DOAGRA

The management of the DOAGRA Ltd. wants to select the optimal organization of machine repairing, because their own repair shop does not seem to work effectively.

There are seven alternatives – three main strategies, the last one can be split into five:

– to keep the repair shop with no organizational changes and offer services for external clients;
– to expand the repair shop and offer services for external clients;
– to sell own repair shop and make a contract with one of five companies, which offer repair shop services
  – Fronk, s. r. o.,
  – Agro Domažlice, a.s.,
  – Karpem, s. r. o.,
  – Bodas, a. s. and
  – ZD Draženov.

Payoffs for two criteria, the costs and revenues for each alternative, depend on the number of repairs of own machines and also on the demand of the external client.

The number of repairs of own machines is the risk factor of the problem. There have been identified four intervals – less than 40, 40–60, 60–80 and more than 80, which represent the expected number of repairs of own machines. Their probabilities have been calculated from the frequency of repairs made in previous years.

Table 1. Decision table – criterion COSTS

Cost		Risk factor – number of internal repair							
Probabilities		< 40				40–60			
		3.70%				11.5%			
		uncertainty factor – number of external repairs							
		<40	40–50	50–60	>60	<40	40–50	50–60	>60
Alternatives – sell repair shop	Fronk, s.r.o.	2 535 167	2 535 167	2 535 167	2 535 167	3 564 397	3 564 397	3 564 397	3 564 397
	Agro Domažlice, a.s.	2 634 864	2 634 864	2 634 864	2 634 864	3 592 282	3 592 282	3 592 282	3 592 282
	Karpem, s.r.o.	2 455 945	2 455 945	2 455 945	2 455 945	3 508 493	3 508 493	3 508 493	3 508 493
	Bodas, a.s.	2 526 578	2 526 578	2 526 578	2 526 578	3 523 493	3 523 493	3 523 493	3 523 493
	ZD Draženov	2 534 211	2 534 211	2 534 211	2 534 211	3 548 715	3 548 715	3 548 715	3 548 715
Alternatives – keep repair shop	capacity keeping	4 961 398	5 631 407	6 301 416	6 435 417	5 966 411	6 435 417	6 435 417	6 435 417
	50% expanding	5 410 067	6 080 075	6 750 084	7 420 093	6 415 080	7 085 089	7 755 097	8 425 106
Cost		Risk factor – number of internal repair							
Probabilities		60–80				>80			
		69.00%				15.80%			
		uncertainty factor – number of external repairs							
		<40	40–50	50–60	>60	<40	40–50	50–60	>60
Alternatives – sell repair shop	Fronk, s.r.o.	4 869 890	4 869 890	4 869 890	4 869 890	6 003 572	6 003 572	6 003 572	6 003 572
	Agro Domažlice, a.s.	4 748 573	4 748 573	4 748 573	4 748 573	6 002 222	6 002 222	6 002 222	6 002 222
	Karpem, s.r.o.	4 911 890	4 911 890	4 911 890	4 911 890	6 186 429	6 186 429	6 186 429	6 186 429
	Bodas, a.s.	4 932 890	4 932 890	4 932 890	4 932 890	6 239 201	6 239 201	6 239 201	6 239 201
	ZD Draženov	4 867 978	4 867 978	4 867 978	4 867 978	6 207 287	6 207 287	6 207 287	6 207 287
Alternatives – keep repair shop	capacity keeping	6 435 417	6 435 417	6 435 417	6 435 417	6 435 417	6 435 417	6 435 417	6 435 417
	50% expanding	7 755 097	8 425 106	9 095 115	9 765 124	9 095 115	9 765 124	9 966 126	9 966 126

Table 2. Decision table – criterion REVENUE

Revenue			Risk factor – number of internal repair							
			< 40				40–60			
Probabilities			3.70%				11.5%			
			uncertainty factor – number of external repairs							
			<40	40–50	50–60	>60	<40	40–50	50–60	>60
Alternatives	sell repair shop	Fronk, s.r.o.	3 775 344	3 775 344	3 775 344	3 775 344	4 780 357	4 780 357	4 780 357	4 780 357
		Agro Domažlice, a.s.	3 775 344	3 775 344	3 775 344	3 775 344	4 780 357	4 780 357	4 780 357	4 780 357
		Karpem, s.r.o.	3 775 344	3 775 344	3 775 344	3 775 344	4 780 357	4 780 357	4 780 357	4 780 357
		Bodas, a.s.	3 775 344	3 775 344	3 775 344	3 775 344	4 780 357	4 780 357	4 780 357	4 780 357
		ZD Draženov	3 775 344	3 775 344	3 775 344	3 775 344	4 780 357	4 780 357	4 780 357	4 780 357
	keep repair shop	capacity keeping	5 829 643	6 740 090	7 649 217	7 830 927	6 826 280	7 463 683	7 463 683	7 463 683
		50% expanding	6 356 828	7 277 090	8 193 849	9 108 164	7 339 615	8 259 726	9 176 863	10 091 807

Revenue			Risk factor – number of internal repair							
			60–80				>80			
Probabilities			69.00%				15.80%			
			uncertainty factor – number of external repairs							
			<40	40–50	50–60	>60	<40	40–50	50–60	>60
Alternatives	sell repair shop	Fronk, s.r.o.	6 120 374	6 120 374	6 120 374	6 120 374	7 460 392	7 460 392	7 460 392	7 460 392
		Agro Domažlice, a.s.	6 120 374	6 120 374	6 120 374	6 120 374	7 460 392	7 460 392	7 460 392	7 460 392
		Karpem, s.r.o.	6 120 374	6 120 374	6 120 374	6 120 374	7 460 392	7 460 392	7 460 392	7 460 392
		Bodas, a.s.	6 120 374	6 120 374	6 120 374	6 120 374	7 460 392	7 460 392	7 460 392	7 460 392
		ZD Draženov	6 120 374	6 120 374	6 120 374	6 120 374	7 460 392	7 460 392	7 460 392	7 460 392
	keep repair shop	capacity keeping	6 974 032	6 974 032	6 974 032	6 974 032	6 484 379	6 484 379	6 484 379	6 484 379
		50% expanding	8 659 850	9 578 978	10 495 762	11 410 732	9 986 436	10 904 380	11 179 387	11 179 387

The demand of the external client is the factor of uncertainty. The number of repairs of external client machines has been assumed in one of four intervals – less than 40, 40–50, 50–60 and more than 60. Probabilities of these intervals could not be estimated, because there is no experience of the previous years.

Combinations of these two factors create sixteen states of nature.

Two decision tables (Table 1 and 2) describe this problem situation.

We have solved this problem using the AHP method and we have used the program AliahThink 4.0, which is software implementation of the AHP method. Five levels complete hierarchy can describe this problem (see Figure 3).

– The first level represents the goal – the best alternative selection.
– The second level represents risk factor – expected number of own machine repairs – and has four elements. Probabilities of number of own repairs express the priorities of elements on this level.
– Uncertainty factor is on the third level; it is expected numbers of machine repairs of external clients. This level has four elements also. Because there is no information about their probabilities, we suppose their priorities are equal.
– The fourth level has two elements – two problem criteria, which are costs and revenue. The priorities of the criteria are supposed equal also.
– The fifth level is the last, it has seven elements – the decision alternatives. The priorities of the alternatives are set as a ratio of their corresponding payoffs.

Figure 4 shows the results of problem solving. The differences among the global priorities of alternatives are not great. However, the alternative Capacity keeping has the greatest priority. The second alternative is Agro Domažlice. This result corresponds with results received by other approaches, see Brožová 2001, Brožová 2002 and Daňsa 2001.

CONCLUSION

Decision problem with *m* alternatives, *n* states of nature and *k* criteria can be graphically represented by the decision tree with one decision node, *m* event nodes, $m*n$

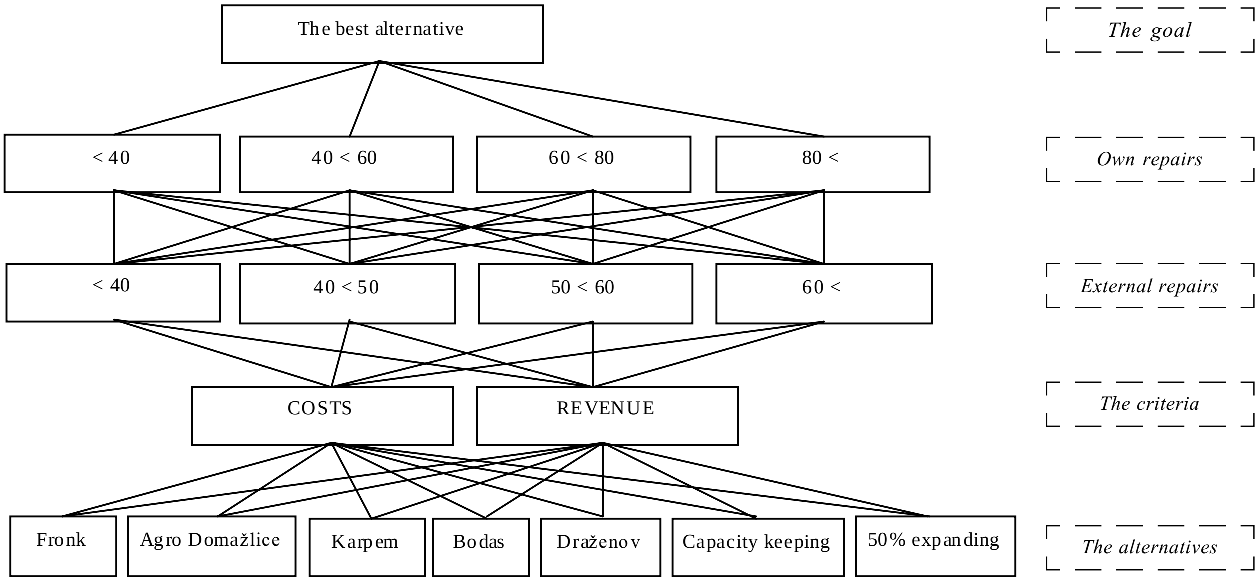

Figure 3. Hierarchy of the problem of Doagra

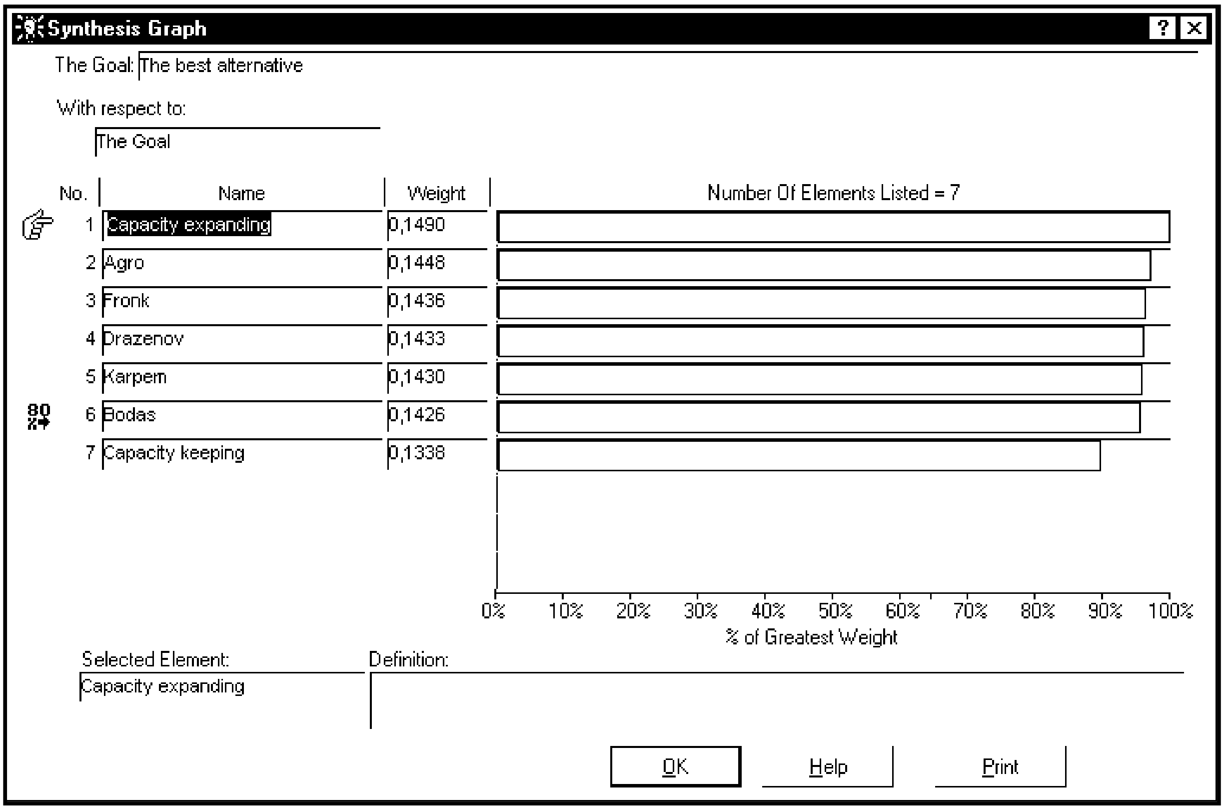

Figure 4. Graphical result of the problem of Doagra (AliahThink 4.0)

criteria nodes and $m*n*k$ payoff nodes. This representation is equivalent to three-dimensional decision matrix, but can be effectively used for the decision problems which have more decision nodes and which are not homogeneous.

When we reorganise this form of the decision tree, we can choose the best alternative using the AHP. The first level of this hierarchy includes the goal – the best alternative solution. The second level consists of the states of nature, the third level of the criteria and the lower level includes the alternatives. The complete hierarchy represents the homogeneous decision problem. However the problem hierarchy can also be constructed for the non-homogeneous problems with more decision nodes.

The AHP needs pair wise comparisons as input information in any form, the decision tree with cardinal as well as ordinal data on payoffs, probabilities of states of nature and criteria preferences can be solved by AHP.

Advantage of this approach is the possibility of solving of the decision trees

– with multiple criteria,
– with more decision points and
– with different type of all kind of input data.

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