Development and evaluation of full-vehicle vibration model of MF 285 tractor

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Abstract


The vibration transmitted to the tractor driver not only leads to the driver health problems, but also reduces the driver working efficiency. One of the methods utilized to reduce driver vibration is the seat suspension system. Development of the tractor vibration model is the first step toward the implementation of the tractor seat suspension. In this research development and evaluation of full-vehicle vibration model of MF 285 tractor was considered. At validation phase, the developed model simulated the predictable outputs correctly. The modified parameters of the model reduced the simulation error by 30% compared to the model with primary parameters. Furthermore, considering the operation status of tractor engine in the simulated tractor, the simulation error was reduced from 45% to 27%.

Keywords: step function; simulation; transmitted vibrations; Matlab software

Vibration transmitted to the drivers of agricultural tractors has been widely studied by many researchers over the last 50 years (MARSILI et al. 2002). Furthermore, it was considered that the drivers of agricultural tractors are exposed to a high level of vibrations during farm operations (SCARLETT et al. 2007). The high level of vibrations experienced by a driver has harmful effects on his health and creates an early fatigue that results the reduction of the driver’s efficiency. Employment of suspension systems seems to be a suitable solution to overcome against this problem; however, design of a new suspension system needs a thorough understanding about the transmitted vibrations to the driver. Development of a vibration model is considered as one of the methods utilized to increase the understanding of the researcher about the transmitted vibrations to the driver. Traditionally two types of vibration model are used to meet this goal, namely: quarter car vibration model (ELMADANY, ABDUL-JABBAR 1999), and full-vehicle vibration model (CROLLA, ABDEL-HADY 1991).

The main objective of this study is development, evaluation and validation of the full-vehicle vibration model of a tractor (tractor model MF 285). The developed vibration model can later be utilized in order to simulate the transmitted vibrations to this tractor, which can be used in the design process of the suspension systems.

MATERIAL AND METHODS

Simulation of the transmitted vibrations to the tractor chassis. In this study full-vehicle vibration model of MF 285 tractor (Iranian Tractor Manufacturing company, Tabriz, Iran) was developed for the simulation of the transmitted vibrations to the tractor chassis. One of the specifications of the examined tractor is that this tractor has not any...
conventional suspension system; however, stiffness and damping of tractor tires and driver’s seat, provides a kind of suspension for tractor driver. Furthermore front axle of this tractor is pivoted to the centre of the tractor chassis, therefore these specifications should be considered in the development of the tractor vibration model.

Development of mathematical equations. Stiffness ($k$) and damping ($c$) coefficients of tractor tires control the process of vibration transmission from the ground to the tractor chassis. In other words, parallel arrangement of a spring and a damper is routinely utilized instead of tractor tire in the tractor suspension model (Taylor et al. 2000); therefore, full-vehicle vibration model of tractor can be schematically presented as shown in Fig. 1a.

Since spring force is proportional to the variation of the spring length, and damper force is proportional to the time rate variation of the damper length (Khajavi, Abdollahi 2007), the vibration force transmitted from each wheel to the contact point between wheel hub and axle can be formulated as $k(z_a - z_g) + c \frac{dz}{dt}(z_a - z_g)$. This vibration force is substituted for spring-damper combination as shown in Fig. 1b, in other words, the vibration forces transmitted to the tractor chassis are summarized as presented in mathematical system of Eq. (1):

$$\begin{align*}
F_{f1} &= k_{tf}(z_{af1} - z_{gf1}) + c_{tf} \frac{dz}{dt}(z_{af1} - z_{gf1}) \\
F_{f2} &= k_{tf}(z_{af2} - z_{gf2}) + c_{tf} \frac{dz}{dt}(z_{af2} - z_{gf2}) \\
F_{r1} &= k_{tr}(z_{ar1} - z_{gr1}) + c_{tr} \frac{dz}{dt}(z_{ar1} - z_{gr1}) \\
F_{r2} &= k_{tr}(z_{ar2} - z_{gr2}) + c_{tr} \frac{dz}{dt}(z_{ar2} - z_{gr2})
\end{align*}$$

where:

- $F_{f1}$, $F_{f2}$ – transmitted force to the front left and right corner of tractor chassis (N)
- $F_{r1}$, $F_{r2}$ – transmitted force to the rear left and right corner of tractor chassis (N)
- $k_{tf}$, $k_{tr}$ – stiffness of front and rear tire of tractor (N/m)
- $z_{af1}$, $z_{af2}$ – vertical displacement of front left and right corner of tractor chassis (m)
- $z_{ar1}$, $z_{ar2}$ – vertical displacement of rear left and right corner of tractor chassis (m)
- $z_{gf1}$, $z_{gf2}$ – vertical undulation of ground under front left and right wheel of tractor (m)
- $z_{gr1}$, $z_{gr2}$ – vertical undulation of ground under rear left and right wheel of tractor (m)
- $c_{tf}$, $c_{tr}$ – damping ratio of tractor front wheels and rear wheels, respectively (N.s/m)
- $\frac{dz}{dt}$ – first order differentiation sign

Pivoted assembly of the front axle of the examined tractor to the tractor body makes it possible to substitute the left and right wheel forces of the front axle with a force exerted to the pivot point (Fig. 1c), because equation of the static equilibrium of tractor front axle free body diagram in Z direction (Fig. 1d) results:

$$F_f = k_{tf}(z_{af1} - z_{gf1}) + c_{tf} \frac{dz}{dt}(z_{af1} - z_{gf1}) + k_{tf}(z_{af2} - z_{gf2}) + c_{tf} \frac{dz}{dt}(z_{af2} - z_{gf2})$$

where:

- $F_f$ – resultant force exerted to the pivot point of tractor front axle (N)

Moreover; kinetic equations of the tractor body (Fig. 1c) results in the linear acceleration of tractor’s centre of gravity in Z direction $\left(\frac{dz}{dt^2}, \frac{dz}{dt} \right)$ and angular accelerations of tractor body about X and Y axes which cross the centre of gravity of tractor $\left(\frac{d^3z}{dt^3}, \frac{d^2z}{dt^2} \right)$. These accelerations can be calculated using the mathematical system of Eq. (3):
\[
\begin{align*}
\frac{d^2}{dt^2} z_{c.g.} &= -\frac{1}{m_t} (F_{r1} + F_{r2} + F_{f1}) \\
\frac{d^2}{dt^2} \theta &= \frac{1}{I_{xx}} (F_{f1} l_f - (F_{r1} + F_{r2}) l_r) \\
\frac{d^2}{dt^2} \phi &= \frac{1}{I_{yy}} (F_{f2} l_f - F_{r2} l_r)
\end{align*}
\] (3)

where:
- \( m_t \): tractor mass (kg)
- \( \frac{d^2}{dt^2} z_{c.g.} \): vertical linear acceleration of tractor centre of gravity (m/s²)
- \( I_{xx} I_{yy} \): mass moment of inerts of tractor about X and Y axes which cross centre of gravity of tractor (kg²)
- \( l_r l_f \): dimensions that specify the location of tractor centre of gravity from rear and front portions of chassis (m)
- \( t_1 t_2 \): dimensions that specify the location of tractor centre of gravity from left and right portions of chassis (m)
- \( \frac{d^2}{dt^2} \theta \): roll and pitch angular accelerations of tractor body during tractor movement (radian/s²)

Calculation of the linear vertical acceleration (heave) and angular accelerations (roll and pitch) of tractor body leads to the components of tractor driver’s seat base acceleration. With reference to the mathematical system of Eq. (4), seat base acceleration components can be calculated on condition that the distance between tractor centre of gravity and tractor seat base \( R \) is known.

\[
\begin{align*}
\frac{d^2}{dt^2} z_{\text{base}} &= \frac{d^2}{dt^2} z_{c.g.} \\
\frac{d^2}{dt^2} \theta_{\text{base}} &= \frac{R}{d^2} \frac{d^2}{dt^2} \theta \\
\frac{d^2}{dt^2} \phi_{\text{base}} &= \frac{R}{d^2} \frac{d^2}{dt^2} \phi
\end{align*}
\] (4)

where:
- \( \frac{d^2}{dt^2} z_{\text{base}} \): linear acceleration of tractor seat base in \( z \) direction (m/s²)
- \( \frac{d^2}{dt^2} \theta_{\text{base}} \): linear acceleration of tractor seat base in \( \theta \) direction (m/s²)
- \( \frac{d^2}{dt^2} \phi_{\text{base}} \): linear acceleration of tractor seat base in \( \phi \) direction (m/s²)

Moreover using the mathematical system of Eq. (5), linear accelerations of the force application points to the tractor body (including the acceleration of the front axle pivot point and acceleration of the joints of the tractor rear wheels to the tractor body) can be calculated:

\[
\begin{align*}
\frac{d^2}{dt^2} z_{af1} &= \frac{d^2}{dt^2} z_{af2} = l_f \frac{d^2}{dt^2} \theta + t_1 \frac{d^2}{dt^2} \phi \\
\frac{d^2}{dt^2} z_{af1} &= \frac{d^2}{dt^2} z_{af2} = l_r \frac{d^2}{dt^2} \theta + t_2 \frac{d^2}{dt^2} \phi
\end{align*}
\] (5)

where:
- \( \frac{d^2}{dt^2} z_{af}\): instantaneous vertical linear acceleration of the front axle pivot point (m/s²)
- \( \frac{d^2}{dt^2} z_{af1} \): instantaneous vertical linear acceleration of rear left corner of tractor chassis (m/s²)
- \( \frac{d^2}{dt^2} z_{af2} \): instantaneous vertical linear acceleration of rear right corner of tractor chassis (m/s²)

To calculate the linear accelerations of the joints between front axle and front wheels, kinetic equation of the front axle of tractor Eq. (6) should be utilized (Fig. 1d):

\[
\frac{d^2}{dt^2} \alpha = \frac{1}{I_{xxa}} (-F_{f1} t_1 + F_{f2} t_2)
\] (6)

where:
- \( \frac{d^2}{dt^2} \alpha \): angular acceleration of tractor front axle (radian/s²)
- \( I_{xxa} \): mass moment of inertia of tractor front axle about the pivot point (kg/m²)

And finally, linear accelerations of the front axle-wheel joints can be calculated from the mathematical system of Eq. (7):

\[
\begin{align*}
\frac{d^2}{dt^2} z_{af1} &= \frac{d^2}{dt^2} z_{afc} + t_1 \frac{d^2}{dt^2} \alpha \\
\frac{d^2}{dt^2} z_{af2} &= \frac{d^2}{dt^2} z_{afc} + t_2 \frac{d^2}{dt^2} \alpha
\end{align*}
\] (7)

**Implementation of the mathematical equations in the MATLAB software.** Fig. 2 shows the computer model of the developed mathematical equations of the tractor full-vehicle vibration model created by the MATLAB-Simulink program (ver. 7.12.0.635; MathWorks, Natick, USA). To execute the tractor full-vehicle vibration model, it is essential to enter model parameters into the computer. Primary list of the utilized parameters for executing the model which are obtained from the specifications of an MF 285 tractor is given in Table 1 (Lines, Murphy 1991; Vitas et al. 1998).

Inputs of the developed model are the \( z \) displacements that describe road and farm field irregularities, and outputs of the model are linear accelerations and displacements of the tractor seat base along \( X \).
Y and Z directions. Step functions with the height of 10 cm were selected as the \( z \) inputs. The \( z \) inputs were applied to the front and rear wheels of the modelled tractor at times \( t_0 \) and \( t_1 \), respectively. The time period of \( (t_0, t_1) \) was considered with regard to the distance between the front and rear axle (WB) and the velocity \( (v) \) of the tractor \( (t_1 - t_0 = WB/v) \); moreover, this forward velocity \( (v) \) must be used to validate the model as well. To evaluate the similarity between the results of the simulation and real tests, a vibration measuring apparatus (Fig. 3) was installed on an MF 285 tractor. This apparatus was able to measure the instantaneous vibration along X, Y and Z axes with the frequency of 10 Hz. Furthermore, this system is able to store the acquired accelerations in the form of three text files into the memory card of the system. These acceleration time histories can later be transferred into the computer for further analysis.

To perform practical tests, the sensor of the vibration measuring apparatus was installed on the contact point between tractor seat base and body, and vibration induced by passes of tractor with the speed of 2.7 km/h over the step like irregularities with the height of 10 cm along X, Y and Z directions were recorded. To assess the effect of the engine-induced vibration on the output of the vibration measuring sensor; at the first stage, data logging was done when the tractor was turned off, and at the second stage, when the tractor was turned on.

**Validation of the developed model.** Accurate performance of the developed model was checked by the evaluation of the rationality of the model outputs, and the assessment of the similarity between the results obtained from the simulation and the experimental tests. In order to evaluate the rationality of the outputs, some inputs of the model which have predictable outputs were applied to the model, and the obtained and predictable results were compared. In this stage of validation process, the utilized inputs applied to the model, and their accompanied predictable outputs were as shown in Table 2.

![Fig. 2. Presentation of the computerized model of mathematical equations in Simulink environment of the Matlab software](image)

![Fig. 3. Tractor seat base vibration measuring apparatus](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{\text{of}} )</td>
<td>90,000 (N/m)</td>
<td>( t_2 )</td>
<td>0.87 (m)</td>
</tr>
<tr>
<td>( c_{\text{of}} )</td>
<td>1,000 (N·s/m)</td>
<td>( l_r )</td>
<td>0.91 (m)</td>
</tr>
<tr>
<td>( k_{\text{tr}} )</td>
<td>90,000 (N/m)</td>
<td>( I_f )</td>
<td>1.35 (m)</td>
</tr>
<tr>
<td>( c_{\text{ir}} )</td>
<td>2,000 (N·s/m)</td>
<td>( I_{\text{ext}} )</td>
<td>600 (kg·m(^2))</td>
</tr>
<tr>
<td>( m_t )</td>
<td>2,800 (kg)</td>
<td>( I_{\text{pyt}} )</td>
<td>2,000 (kg·m(^2))</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>0.87 (m)</td>
<td>( I_{\text{ext}} )</td>
<td>5 (kg·m(^2))</td>
</tr>
</tbody>
</table>
A sinusoidal function with amplitude of 1 and frequency of 0.01 rad/s was selected as a displacement function with gradual variations; furthermore, quantitative comparison of the simulation and real test results was carried out using the Root Mean Square (RMS) value of the obtained acceleration data (Duke, Goss 2007). The RMS value of the acceleration data is calculated from the following formula:

$$\text{RMS}(a) = \sqrt{\frac{1}{T} \int_0^T a^2(t) dt}$$

where:
- $T$ – sampling time period (s)
- RMS$(a)$ – Root Mean Square (RMS) value of the obtained acceleration data (m/s$^2$)
- $a^2(t)$ – squared value of acceleration data as a function of time (m$^2$/s$^4$)

This formula is utilized for calculating the RMS value of the data recorded in analogue (continuous) domain. The digital (discrete) form of RMS formula can be implemented using the computer code written in code writing environment of the MATLAB software:

```matlab
% measured data must be entered in this vector
a = []; % measured data
N = numel(a)
a = [a.^2]
A = 0
for i = 1:N
    A = A + a(i)
end
A = A/N
RMS = A^(1/2)
```

Knowing the RMS values of the real and simulated accelerations, the error of the simulation can be calculated with the aid of the following formula:

$$\text{Error} = \frac{\text{RMS}_{\text{real}} - \text{RMS}_{\text{simulated}}}{\text{RMS}_{\text{real}}} \times 100\%$$

### Table 2. Some of the inputs of the developed vibration model which have predictable outputs

<table>
<thead>
<tr>
<th>Input</th>
<th>Predictable output</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) application of an arbitrary displacement function to front wheels of tractor</td>
<td>$\ddot{x}_{\text{seat base}} = 0$</td>
</tr>
<tr>
<td>(b) application of two arbitrary displacement functions with the same value and opposite directions to the rear wheels of tractor</td>
<td>$\ddot{y}<em>{\text{seat base}} = 0, \ddot{z}</em>{\text{seat base}} = 10$</td>
</tr>
<tr>
<td>(c) application of a displacement function with gradual variation to all of the tractor wheels</td>
<td>$\ddot{x}<em>{\text{seat base}} = 0, \ddot{y}</em>{\text{seat base}} = 0, \ddot{z}_{\text{seat base}} = 10$</td>
</tr>
</tbody>
</table>
RESULTS AND DISCUSSION

The outputs obtained by the application of the functions presented in Table 2 as inputs of the developed model, are shown in Fig. 4.

Fig. 4 indicates that the model could be able to accurately simulate the predictable outputs.

In order to simulate the pass of a pulled tractor with the speed of 2.7 km/h, over a 10 cm step-like obstacle, a three seconds time delay was considered for the time of application of the step-like function to the rear wheels compared to the front wheels, because the wheel base of the examined tractor is 2.25 cm, which results in a 3-second time delay \[ t = \frac{WB}{v} = \frac{2.25 \text{ (m)}}{0.75 \text{ (m/s)}} = 3 \text{ s} \]. The results of the simulated vibration of the tractor seat base along the Z axis in the time domain is shown in Fig. 5.

The simulation error of the linear acceleration of the tractor seat base along the Z direction was about 38.3%; however, since the trend of the acceleration variations and the stimulus times were accurately simulated, the obtained result is solely an indication of the scale dissimilarity between the simulated and real values of the seat base acceleration. Inspection of the input parameters of the model showed that among the parameters of the developed model, the stiffness of tractor wheel \( k_t \) is the parameter that significantly controls the outputs of the model; however, because the inflation pressure of the utilized tires was lower than the standard inflation pressure, in the next stage of the simulation, new values for the parameter of \( k_t \)

were considered in order to increase the similarity between simulated and real results. The new values of the parameters are given in Table 3.

New outputs of the seat base acceleration along the X, Y and Z axes with their accompanied experimental outputs are shown in Fig. 6.

After modification of the input parameters, the simulation error of the vibration along X, Y and Z directions were 5.1%, 4.3% and 6.2% respectively, which is an indicator of the reliability of the model in simulation of the seat base acceleration induced by passing a pulled tractor over a step-like irregularity. Calculation of the RMS value of acceleration and estimation of the simulation error for evaluation of the developed vibration models were also used by other researchers (Adams 2002; Sarami 2009), and the accuracy of the developed models of those studies were comparable to the results obtained in this study. The value of majority of simulation errors was lower than 10%. To complete the model with regard to the effect of the engine running with the angular velocity of 1,800 rpm, a sinusoid stimulus with the amplitude of 1 mm and the frequency of 30 Hz was applied to the model as a new input. Results obtained from the execution of this stage of the simulation with its accompanied experimental acceleration along the Z axis are shown in Fig. 7.

The simulation error value of the acceleration along the Z axis while the tractor was turned on, was about 27%; however, ignoring the sinusoidal function with the frequency of 30 Hz as an input of the model, resulted in a simulation error of 45%; therefore, in order to have a realistic vibration model, working conditions of tractor should be considered, too.

Table 3. Modified parameters utilized in simulation of vibration model of an MF 285 tractor

<table>
<thead>
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<th>Parameter</th>
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<th>Value</th>
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</thead>
<tbody>
<tr>
<td>( k_{gf} )</td>
<td>30,000 (N/m)</td>
<td>( t_2 )</td>
<td>0.87 (m)</td>
</tr>
<tr>
<td>( c_{gf} )</td>
<td>1,000 (N/s/m)</td>
<td>( l_r )</td>
<td>0.91 (m)</td>
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<td>( k_{gr} )</td>
<td>30,000 (N/m)</td>
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<td>2,000 (N/s/m)</td>
<td>( l'_{axx} )</td>
<td>600 (kg·m²)</td>
</tr>
<tr>
<td>( m_t )</td>
<td>2,800 (kg)</td>
<td>( I_{yy} )</td>
<td>2,000 (kg·m²)</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>0.87 (m)</td>
<td>( I_{xxa} )</td>
<td>5 (kg·m²)</td>
</tr>
</tbody>
</table>

RMS error simulation = \[ \frac{\text{RMS of simulated acceleration} - \text{RMS of real acceleration}}{\text{RMS of real acceleration}} \times 100 \]
CONCLUSION

In this study, development of the full-vehicle vibration model of a tractor (model MF 285) with regard to the structural specifications of this tractor, such as pivoted assembly of tractor front axle to the tractor body, was considered. From the obtained results, it can be concluded that the developed model was able to accurately simulate the terrain irregularity-induced vibrations transmitted to the tractor driver seat base; therefore, this model can be utilized in order to simulate the tests needed for designing the tractor driver seat suspension system.

References


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