Mathematical Investigation of the Effects of Slicing on the Osmotic Dehydration of Sphere and Cylinder Shaped Fruits

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Abstract

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The dehydration kinetics of the fruits with special geometries, i.e. spherical and cylindrical (e.g. apple, peach, banana, pineapple, etc.), were studied based on mathematical methods. The influence of the size reduction (slicing) of these fruits into smaller rings was also investigated. The mathematical modelling was performed based on the Fick's second law. The results showed that increasing the value of the water diffusion coefficient in fruit (for instance, via increasing the process temperature) promotes faster water migration from the fruit. Mathematical modelling also showed that the characteristic length of fruits (radius) is in an inverse relation to the dehydration kinetics. Comparing the results obtained with both the sphere- and cylinder-shaped fruits revealed that slicing the fruit into more thin rings makes a better condition for operating the osmotic dehydration process with a higher efficiency and a shorter duration.

Keywords: osmotic dehydration; slicing; mass transfer; mathematical modelling

Osmotic dehydration is a very gentle method to reduce water activity in foods (Segu *et al*. 2006). It is a water removal process, which is based on immersing foods, such as fruits and vegetables, in a hypertonic (osmotic) solution (Ohnishio & Miyawaki 2005; Chenlo *et al*. 2006). The hypertonic solutions are concentrated aqueous solutions of soluble solids, e.g. sugar, salt, sorbitol, glycerol, etc., having a higher osmotic pressure and a lower water activity (Kaymak-Ertekin & Sultanoglu 2000; Vega-Mercado *et al*. 2001). This method enables the storage of the product for longer periods, preserves flavour and nutritional characteristics, and prevents microbial damage (Segu *et al*. 2006). Osmotic dehydration of fruits and vegetables is gaining attention due to its important role in the food processing industry because this method inhibits the enzymatic browning and improves the colour, flavour, and texture of the final product; it is also a less energy intensive process as no phase change takes place (Beristain *et al*. 1990; Sutar & Gupta 2007). The rate of dehydration (water loss) during osmotic dehydration depends upon factors such as: solution concentration, immersion time, solution temperature, size and geometry of the food, solution to food mass ratio, and the level of agitation or circulation of the osmotic solution. A large number of recent publications have described both mathematically and experimentally the influence of these variables on the mass transfer rates during osmotic dehydration (Beristain *et al.* 1990; Rastogi & Raghavarao 2004; Falade *et al.* 2007; Moreira *et al.* 2007; Sutar & Gupta 2007).

In spite of the numerous mathematical models presented to describe the kinetics of the osmotic dehydration of foods, it is still difficult to establish general rules about the variables that affect this process (Spiazzi & Mascheroni 1997). Mathematical modelling of the water loss during the osmotic dehydration of foods is a difficult issue due to the complexity and heterogeneity of the biological material; consequently, the models commonly make use of a macroscopic approach in which the tissue of fruit is assumed to be homogeneous. Fick's second law is usually applied to find an approximate solution, and the effective diffusivity of water in the tissue is used to account for the variations of the physical properties of the tissue as well as for the influence of the solution characteristics and process variables (Kaymak-Ertekin & Sultanoglu 2000; Segu *et al.* 2006; Garcia *et al.* 2007).

In the models based on the Fick's second law, the diffusion of water from the inside of the food to the surrounding hypertonic solution plays the major role in the osmotic dehydration. In this case, the concentration gradient of water between the inside and outside of the food acts as the driving force for the mass transfer of water and the rate of moisture loss. Some equations have been developed by the mathematical modelling of the osmotic dehydration process for the fruits having simple geometric configurations, i.e. slab, cylinder, and sphere (Sutar & Gupta 2007). The osmotic dehydration of fruits can be performed by placing them, whole or in pieces, in hypertonic solutions (Evans *et al.* 2002). It is clear that the dehydration process will be done in a desired manner (with a higher rate of water loss) if we reduce the size of the fruit by slicing it into smaller pieces (e.g. slicing a cylinder-shaped fruit such as banana or pineapple to smaller rings). Although this procedure (slicing of fruit) is used in numerous practical osmotic dehydration experiments (Conway *et al.* 1983; Rastogi & Raghavarao 1995, 1997; Mavroudis *et al.* 1998; Khin *et al.* 2007), it was not, however, investigated from the viewpoint of mathematical analysis and theoretical models.

The main aim of this work is to investigate the dehydration kinetics of the sphere- and cylinder-shaped fruits during the osmotic dehydration process, based on the mathematical modelling. The effect of the size reduction (slicing) of the fruits on the osmotic dehydration kinetics was also studied using the diffusion model based on the Fick's second law.

Mathematical Modelling

Let us consider a sphere-shaped fruit (e.g. apple, tomato, etc.) having uniform radius of a, and a cylinder-shaped one (e.g. banana, pineapple, etc.) with a radius of b and a height of h, which we want to dehydrate using the osmotic dehydration technique. We can perform the process maintaining these fruits initial geometries (sphere and cylinder) or reducing their size into more small pieces. Parallel slicing the sphere-shaped fruit into k pieces creates k rings with nearly identical thicknesses ($2a/k$) and different cross sections, while slicing the cylinder-shaped fruit creates k rings with exactly equal thicknesses (h/k) and identical cross sections (Figure1).

In order to study the dehydration kinetics of the sphere- and cylinder-shaped fruits in the osmotic dehydration process, we must analyse the Fick's second law in spherical and cylindrical coordinates (Figures 2a, b). Furthermore, the effects of the fruit slicing will be obtained by analysing the Fick's second law in Cartesian coordinates, for the ring-shaped fruit slices (Figure 2c). The following assumptions were used in the modelling of all systems:

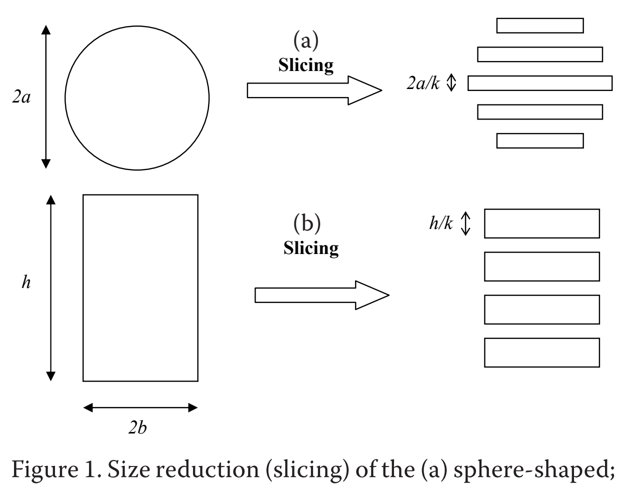

Figure 1. Size reduction (slicing) of the (a) sphere-shaped; (b) cylinder-shaped fruits

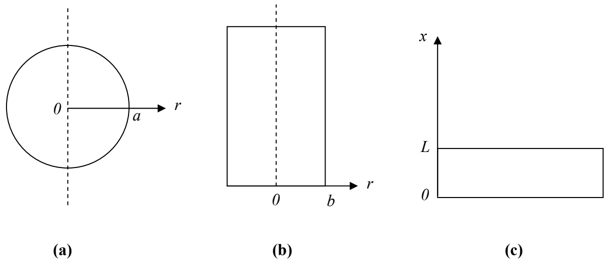

Figure 2. The geometries of the (a) sphere-shaped; (b) cylinder-shaped; (c) ring-shaped fruits

– Water diffusion from fruit into the osmotic solution is the dehydration rate controlling step rather than other phenomen, e.g. tissue swelling and so on.
– Water diffusion occurs in one dimension, i.e. in the sphere- and cylinder-shaped fruits in the direction of r and in the ring-shaped pieces in the direction of x (Figure 2).
– The diffusion coefficient of water in the fruit tissue (D) is constant and independent of concentration.
– The concentration of water in the hypertonic solution (C_e) is constant during the dehydration process.
– The initial concentration of water in fruit is C_0.

Sphere-shaped fruit (spherical coordinates)

In this case, the diffusional release of water from a fruit with spherical geometry (Figure 2a) into a hypertonic solution is considered. The concentration of water inside the fruit ($0 \le r \le a$) is a function of both time (t) and position variable (r) and is determined by transient diffusion according to the Fick's second law for spherical coordinates, as follows:

$$\frac{\partial C}{\partial t} = \frac{D}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial C}{\partial r}\right) \quad (1)$$

where: C – concentration of water in the tissue of fruit

The initial and boundary conditions are:

$$C(r,0) = C_0 \quad (2)$$

$$C(a,t) = C_e \quad (3)$$

$$C(0,t) = \text{finite} \quad (4)$$

Eqs. (2) and (3) were written based on our assumptions and Eq. (4) is right because we know that the concentration of water in the center of the fruit ($r = 0$) is not infinite and has a finite value.

Eq. (1) is a linear partial differential equation (PDE) having a non-homogeneous boundary condition (Eq. (3)). Eq. (1) can be solved using the method of variable separation (product method) (Kreyszig 1979) by inserting the given initial and boundary conditions (Eqs. (2) and (3)).

The final solution of this initial and boundary value problem gives function C as follows:

$$C(r,t) = C_e - \frac{2a(C_0 - C_e)}{\pi r}\sum_{n=1}^{\infty}\frac{\cos n\pi}{n}\, e^{-(n\pi/a)^2 Dt}\sin\left(\frac{n\pi r}{a}\right) \quad (5)$$

The rate of water diffusion from the fruit (J) is expressed based on the Fick's first law:

$$J = \frac{dM_t}{dt} = -SD\left[\frac{\partial C(a,t)}{\partial r}\right] \quad (6)$$

where:
M_t – cumulative amount of water released from the fruit at any time t
S – surface area of the sphere

$$S = 4\pi a^2 \quad (7)$$

Inserting Eqs. (5) and (7) in Eq. (6) and integrating from 0-t gives M_t as follows:

$$M_t = \frac{8a^3(C_0 - C_e)}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}(1 - e^{-(n\pi/a)^2 Dt}) \quad (8)$$

The total initial amount of water or the amount removable at infinite time (M_∞) is given by:

$$M_\infty = \frac{4\pi a^3 C_0}{3} \quad (9)$$

Finally, the fractional release of water at time t in terms of fruit and osmotic solution characteristics becomes:

$$\frac{M_t}{M_\infty} = \frac{6(C_0 - C_e)}{\pi^2 C_0}\sum_{n=1}^{\infty}\frac{1}{n^2}(1 - e^{-(n\pi/a)^2 Dt}) \quad (10)$$

Cylinder-shaped fruit (cylindrical coordinates)

In this case, the concentration of water inside the fruit ($0 \le r \le b$) as a function of r and t can be determined by transient diffusion based on the Fick's second law for the cylindrical coordinates:

$$\frac{\partial C}{\partial t} = D\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) \quad (11)$$

The initial and boundary conditions governing this system are:

$C(r,0) = C_0$ (12)

$C(b,t) = C_e$ (13)

$C(0,t) = \text{finite}$ (14)

Eq. (14) can be explained based on the reason of Eq. (4). The above initial and boundary condition problem (Eqs. (11)–(14)) can be solved on the basis of the method of variables separation. The final solution of this problem is:

$$C(r,t) = C_e + 2(C_0 - C_e)\sum_{n=0}^{\infty} \frac{J_0(\beta_n r/b)}{\beta_n J_1(\beta_n)} e^{-(\beta_n/b)^2 Dt} \quad (15)$$

where:

J – Bessel function of the first kind

β_n – constants and roots of the equation given below:

$J_0 = (\beta_n) = 0 \quad n = 0, 1, 2, 3, ...$ (16)

The rate of water diffusion from the fruit (J) and the cumulative amount of water released (M_t) are given by the Fick's First law:

$$M_t = 4\pi h b^2 (C_0 - C_e)\sum_{n=0}^{\infty} \frac{1 - e^{-(\beta_n/b)^2 Dt}}{\beta_n^2} \quad (17)$$

where: S – the area of the side of the cylinder:

$S = 4\pi h$ (18)

The combination of Eqs. (15), (17), and (18) and integration from Eq. (17) from 0–t gives the function M_t as follows:

$$J = \frac{dM_t}{dt} = -SD\left[\frac{\partial C(b,t)}{\partial r}\right] \quad (19)$$

The total initial amount of water (M_∞) in this cylinder-shaped fruit is expressed as:

$M_\infty = 4\pi b^2 h C_0$ (20)

At last, the fractional release of water at time t can be obtained by dividing Eq. (19) by Eq. (20):

$$\frac{M_t}{M_\infty} = \frac{4(C_0 - C_e)}{C_0}\sum_{n=0}^{\infty} \frac{1 - e^{-(\beta_n/b)^2 Dt}}{\beta_n^2} \quad (21)$$

Ring-shaped fruit (Cartesian coordinates)

Consider a sliced ring of either sphere- or cylinder-shaped fruit having a thickness of L (L is equal to $2a/k$ and h/k for the rings obtained from the sphere-shaped and cylinder-shaped fruits, respectively).

For all rings, we suppose that water diffusion occurs only in the direction of x and we neglect the radial diffusion, because of the small value of the ring thickness to its diameter ratio. Therefore, the concentration of water inside the ring ($0 \le x \le l$) can be found by transient diffusion according to the Fick's second law for Cartesian coordinates (slab or flat systems):

$$\frac{\partial C}{\partial t} = D\,\frac{\partial^2 C}{\partial x^2} \quad (22)$$

The initial and boundary conditions for this system are:

$C(x,0) = C_0$ (23)

$C(0,t) = C_e$ (24)

$C(L,t) = \text{finite}$ (25)

Applying the method of variables separation to Eq. (22) and using the given initial and boundary conditions (Eqs. (23)–(25)), give the concentration of water inside the ring-shaped fruit in terms of time (t) and position variable (x):

$$C(x,t) = C_0 + \frac{2(C_0 - C_e)}{\pi}\sum_{n=1}^{\infty}\left(\frac{1-\cos n\pi}{n}\right) e^{-(n\pi/L)^2 Dt} \sin\left(\frac{n\pi x}{L}\right) \quad (26)$$

Using the Fick's first law, we can express the rate of water diffusion (J) and the cumulative amount of water released (M_t) from both sides of the system (x=0 and x=L) as follows:

$$J = \frac{dM_t}{dt} = -2SD\left[\frac{\partial C(0,t)}{\partial x}\right] \quad (27)$$

where: S – surface area of the diffusion section. introducing Eq. (26) in Eq. (27) and integrating from 0–t, give M_t as follows:

$$M_t = \frac{4SL(C_0 - C_e)}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{1-\cos n\pi}{n^2}\right)\left(1 - e^{-(n\pi/L)^2 Dt}\right) \quad (28)$$

The total initial amount of water inside the ring (M_∞) can be expressed as:

$M_\infty = SLC_0$ (29)

Therefore, the fractional release of water at time t from any sliced ring of the fruit (either sphere or cylinder-shaped) becomes:

$$\frac{M_t}{M_\infty} = \frac{4(C_0 - C_e)}{\pi^2 C_0}\sum_{n=1}^{\infty}\left(\frac{1-\cos n\pi}{n^2}\right)\left(1 - e^{-(n\pi/L)^2 Dt}\right) \quad (30)$$

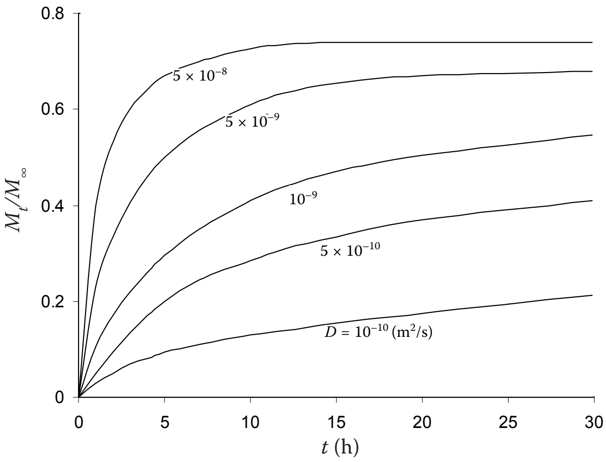

Figure 3. The effect of the water diffusion coefficient on the dehydration kinetics of the sphere-shaped fruits

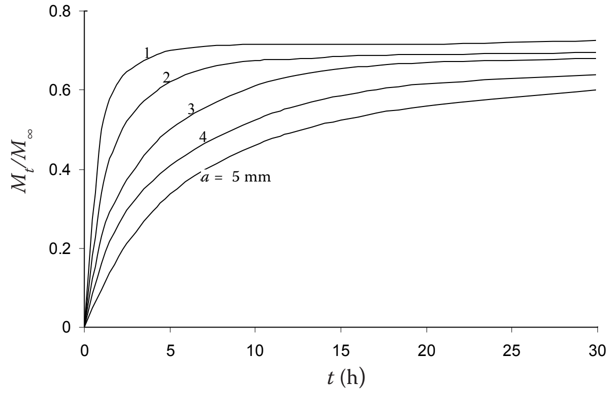

Figure 4. The effect of size (radius) on the dehydration kinetics of the sphere-shaped fruits

RESULTS AND DISCUSSION

Based on the equations obtained from the past section (Eqs. (10), (21) and (30)), we can describe the dehydration kinetics of the sphere- and cylinder-shaped fruits during the osmotic dehydration process, i.e. the effects of the characteristics of the fruit and hypertonic solution (e.g. water diffusion coefficient and the characteristic length of the fruit), as well as the effects of the size reduction (slicing) of the fruit. In all plotted graphs, a constant value has been chosen for the initial concentration of water in the fruit, as well as the concentration of water in the hypertonic solution (C_0=50 Kmol/m^3 and C_e=15 Kmol/m^3).

Figure 3 shows the effect of the diffusion coefficient of water in the fruit on the dehydration kinetics of a sphere-shaped fruit having the radius of 3 mm. It can be seen that the fraction of water removed from the fruit (M_t/M_∞) strongly depends on the diffusion coefficient, in such manner that increasing the value of D (which can be achieved by increasing the temperature of process) causes the fruit to reach the higher equilibrium levels in a shorter period. On the other hand, Figure 3 reveals that the water migration in more permeable fruits (fruit having higher value of D) occurs faster that in those with a lower permeability.

Figure 4 demonstrates the fractional water release curves versus time for the sphere-shaped fruits having the water diffusion coefficient of 5×10^{-9} m^2/s and various radii between 1 mm to 5 mm. As can be seen, the size (radius) of the fruit has an inverse effect on its dehydration kinetics. For instance, the fraction of water removed from a sphere-shaped fruit having a radius of 1 mm after 5 h of initiating the osmotic dehydration process is 0.7, while this value for the same fruit with a radius of 5 mm under identical conditions and the same duration is 0.34. Because of this fact,

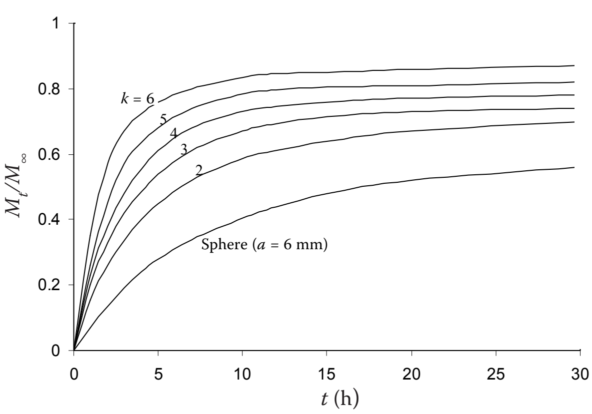

Figure 5. The influence of the slicing on the osmotic dehydration kinetics of the sphere-shaped fruits

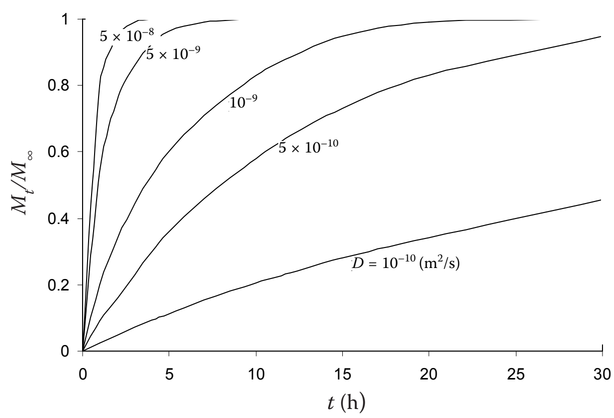

Figure 6. The effect of the water diffusion coefficient on the dehydration kinetics of the cylinder-shaped fruits

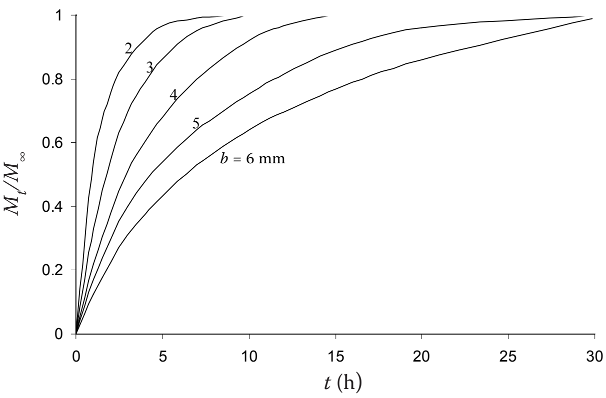

Figure 7. The effect of size (radius) on the dehydration kinetics of the cylinder-shaped fruits

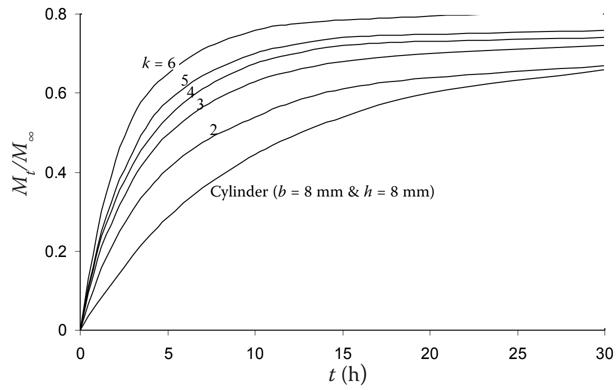

Figure 8. The influence of the slicing on the osmotic dehydration kinetics of the cylinder-shaped fruits

we should reduce the size of fruits to promote their dehydration levels and rates in practical applications.

Figure 5 was plotted to investigate the effects of the sphere-shaped fruits slicing on their osmotic dehydration kinetics. In Figure 5, the dehydration curve of a typical sphere-shaped fruit with a radius of 6 mm and water diffusion coefficient of 5×10^{-9} m^2/s was plotted. Furthermore, five individual dehydration curves were plotted for that fruit when it was sliced into 2, 3, 4, 5, and 6 slices with identical thicknesses (based on the Eq. (30)). Figure 5 shows clearly that the slicing of the fruit drastically affects its dehydration kinetics during the osmotic dehydration process. Increasing the number of slices (k) creates better conditions for the dehydration process by increasing the level of dehydration during a specified time interval and/or decreasing the duration of the process to reach a specified dehydration level. Figure 5 shows that the fraction of water removed from the complete sphere-shaped fruit (a = 6 mm) after 10 h is 0.4, while the values in the case that he fruit was sliced into 3 or 6 slices are 0.67 and 0.84 h, respectively. Figure 5 also shows that slicing the sphere-shaped fruit into 2, 3, 4, 5, and 6 slices increase its ability of water removing up to 61, 93, 118, 143, and 171 % during the early 5 h of the dehydration process.

In Figure 6, the effect of the water diffusion coefficient on the M_t/M_∞ curves against time was shown for a cylinder-shaped fruit having the radius of 2 mm. Similar to the results obtained for the sphere-shaped fruits, the direct dependence of M_t/M_∞ on the diffusion coefficient is evident in Figure 6.

The effect of the geometry and characteristic length of the cylinder-shaped fruits (i.e. radius of cylinder) on their dehydration kinetics was shown in Figure 7. A typical fruit having D of 5×10^{-9} m^2/s was chosen. Like the sphere-shaped fruits, the cylinder-shaped fruits of a smaller radius dehydrate faster and reach the equilibrium level sooner.

The influence of slicing on the water release kinetics of the cylinder-shaped fruits is demonstrated in Figure 8, where the M_t/M_∞ curve of a typical cylinder-shaped fruit having the radius and height of 8 mm ($b = h$ = 8 mm) and water diffusion coefficient of 5×10^{-9} m^2/s was plotted versus time. The same curve was also plotted for the cases that the fruit mentioned had been sliced into k identical slices. Figure 8 proves the fact that the size reduction of the cylinder-shaped fruits accelerates their dehydration rates during the osmotic dehydration process. Figure 8 shows that the time required to remove 40% of the initial stored water from the fruit mentioned above in a hypertonic solution having water concentration of 15 Kmol/m^3 is about 9 h while, by slicing the fruit into 3 or 6 slices, the required time decreases to 3.6 and 1.9 h, respectively.

CONCLUSION

The dehydration kinetics of the sphere- and cylinder-shaped fruits in the osmotic solutions was mathematically investigated. The effect of the size reduction (slicing) on the dehydration kinetics was also studied. The mathematical modelling was performed based on the water diffusion from the fruit (or its slices) into a hypertonic solution according to the Fick's second law in the spherical, cylindrical, and Cartesian coordinates for the sphere-shaped, cylinder-shaped, and ring-shaped (parallel sliced) fruits, respectively. The results have showen that the diffusion coefficient of water and the radius of the sphere- and cylinder-shaped fruits have direct and inverse effects on the dehydration kinetics, respectively. Comparing the dehydration curves of the complete sphere- and cylinder-shaped fruits with those of sliced pieces showed that cutting the fruit into more thin rings accelerates dramatically the dehydration rate. Based on the result obtained, we can predict the dehydration kinetics of the sliced sphere- and cylinder-shaped fruits during the osmotic dehydration process.

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