Particle size distribution of sawdust and wood shavings mixtures

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Abstract

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Particle size distribution of the sample of waste sawdust and wood shavings mixtures were made with two commonly used methods of mathematical models by Rosin-Rammler (RR model) and by Gates-Gaudin-Schuhmann (GGS model). On the basis of network analysis distribution function $F(d)$ (mass fraction) and density function $f(d)$ (number of particles captured between two screens) were obtained. Experimental data were evaluated using the RR model and GGS model, both models were compared. Better results were achieved with GGS model, which leads to a more accurate separation of the different particle sizes in order to obtain a better industrial profit of the material.

Keywords: wood shavings; sawdust; particle size distribution; modelling

Particle size is one of the most important physical properties of solids, which are used in many fields of human activity, such as construction, waste management, metallurgy, fuel fabrication, etc. Sawdust and wood shavings are waste products resulting from wood processing. Very promising is the use of waste products as fuel directly in the place where the waste has been produced. Businesses that are engaged in the processing of wood can become energy self-sufficient. For proper design of combustion devices it is necessary to specify the basic physical parameters of the fuel used. As mentioned above, one of the most important physical parameters is particle size distribution. Statistical distribution of sawdust and wood shavings particles can help to design more efficient combustion space for this type of fuel.

Many different methods were described in literature in order to determine particle size distribution (sieving, microscopy, etc.) (Dierickx et al. 2000; Macias-Garcia et al. 2004). Using different methods to determine particle size distribution can get quite different results (Rosin, Rammler 1933; Ramakrishnan 1994a, b). The kind of method used depends mainly on the characteristics of the analyzed material.

Result of particle size distribution analysis can be expressed in different forms, according to the particle diameter indicating the nominal mesh sizes, or by particle size distribution, in grams, in percentage by weight of each fraction (differential distribution, as the cumulative percentage of sizes below a given value, undersize, and as the cumulative percentage of size above a given value, oversize) (Ballester et al. 2000).

The goal of this work is to obtain a distribution function $F(d)$ (mass fraction) and density function $f(d)$ (number of particles captured between two screens) of the sample of waste sawdust and wood shavings mixtures using Rosin-Rammler (RR) and Gates-Gaudin-Schuhmann (GGS) mathematical models applied to data obtained by network analysis (Diaz-Parralejo et al. 2003).

MATERIAL AND METHODS

Samples of sawdust and wood shavings mixtures were collected from the fuel bunker in the furniture factory. The collection of samples was based on TNI CEN/TR No. 15310-1 (2007). On the day of collection the samples were transported to the laboratory. Afterwards, the waste sawdust and wood shaving mixtures particles were separated into different particle sizes using a 200 mm diameter sifting column whose internal diameters correspond to those set out in the Czech/European standard ČSN EN 933-1 (1999). This column was placed on a vibrating table at 150 vibrations/min for 30 min. The resulting values allow to obtain the experimental particle size distribution curves. These represent the percentages by weight versus particle size. For precise weighting, analytical laboratory balances Radwag AS 220/X (Radwag, Radom, Poland), with readability to 0.0001 g were used.

The RR model distribution function was used to describe the particle size distribution of powders of various types and sizes. The function is particularly suited to represent powders made by grinding, milling, and crushing operations. The general expression of the RR model is:

$$F(d) = 1 - \exp\left[-\left(\frac{d}{l}\right)^m\right] \tag{1}$$

where:
$F(d)$ – distribution function
d – particle size (mm)
l – mean particle size (mm)
m – measure of the spread of particle sizes

Parameters l and m are adjustable parameters characteristic for the distribution. Eq. 1 may be rewritten as:

$$\ln\left\{-\ln\left[1 - F(d)\right]\right\} = m \times \ln d - m \times \ln l \tag{2}$$

If we use the RR mathematical model for particle size distribution, a plot of the first term of Eq. 2, versus natural logarithm d will result in a straight line of slope m if the behaviour of the material fits the RR model. The application of the function to a specific distribution calculation of its parameters is often performed by linear regression data, expressed as $\ln\{-\ln[1-F(d)]\}$, versus $\ln d$, indicating the applicability of RR model for particle size distribution curve. For the regression analysis method of least squares is also frequently used, it can convert

curves to data points. The correlation coefficient is used as a parameter indicating the relevance of the measured data set.

The density function in RR mathematics model will be:

$$f(d) = \frac{m}{l^m} \times d^{m-1} \exp\left[-\left(\frac{d}{l}\right)^m\right] \tag{3}$$

Another popular model for determining particle size distribution is the GGS model; due to simplicity and clarity it has been used in industry since 1940 and is defined as:

$$F(d) = \left[\frac{d}{d_{max}}\right]^m \tag{4}$$

where:
$F(d)$ – the fraction of the sample finer than size d
d – particle diameter (mm)
d_{max} – maximum particle diameter of the distribution (size modulus) (mm)
m – istribution modulus

If we plot logarithm of $F(d)$ versus the logarithm of particle size d, we get a straight line with a slope parameter m, the curve can be described by the mathematical expression:

$$\log F(d) = m \times \log d - m \times \log d_{max} \tag{5}$$

If the particle size distribution curve fits the GGS model, plot of the logarithm of the distribution function versus the logarithm of the particle diameter of the tested material is shown as a straight line. The density function in this model can be written as:

$$f(d) = \frac{m \times d^{m-1}}{d_{max}^m} \tag{6}$$

RESULTS AND DISCUSSION

The values of the weights of different particle sizes of sawdust and wood shavings mixture obtained by network analysis are shown in Table 1, together with the cumulative percentage of weight. Fig. 1 shows the corresponding particle size distribution curve. Fig. 2 shows the distribution function $F(d)$, which was obtained by fitting of measured data of Fig. 1 to Eq. 1. This function shows the different fractions of a waste sawdust and wood shavings mixture by volume, weight or number of particles. Value of the function at a given point is the ratio of the number of the particles (mass or volume), which is lower than a given size.

Table 1. Particle size distribution of sawdust and wood shavings mixtures

Particle size (mm)	Mesh size (mm)	Fraction (g)	Fraction (%)	Cumulative weight under (%)	Cumulative weight over (%)
< 0.1	0.1	3.9479	0.0784	0.9814	0.0186
0.1–0.2	0.2	5.1094	0.1015	0.9030	0.0970
0.2–0.3	0.3	2.553	0.0507	0.8015	0.1985
0.3–0.4	0.4	4.7443	0.0943	0.7508	0.2492
0.4–5.0	0.5	16.5542	0.3289	0.6565	0.3435
0.5–1.6	1.6	9.1431	0.1816	0.3276	0.6724
1.6–3.0	3	4.585	0.0911	0.1460	0.8540
3.0–5.0	5	0.8295	0.0165	0.0549	0.9451
5.0–6.0	6	1.934	0.0384	0.0384	0.9616

Finally, area under the curve between two sizes of particles (e.g. d_1 and d_2) is the number of particles (expressed as particle of mass or volume), whose averages are included in this interval:

$$F(d_2) - F(d_1) = \int_{d_1}^{d_2} f(d)d(d) \qquad (7)$$

The final inclination of distribution function (depending on the diameter of particles, d), at each point indicates the density function $f(d)$, which is defined by Eq. 3 and shown in Fig. 3. This function represents the differential curve, which corresponds to the proportion of particles of a certain

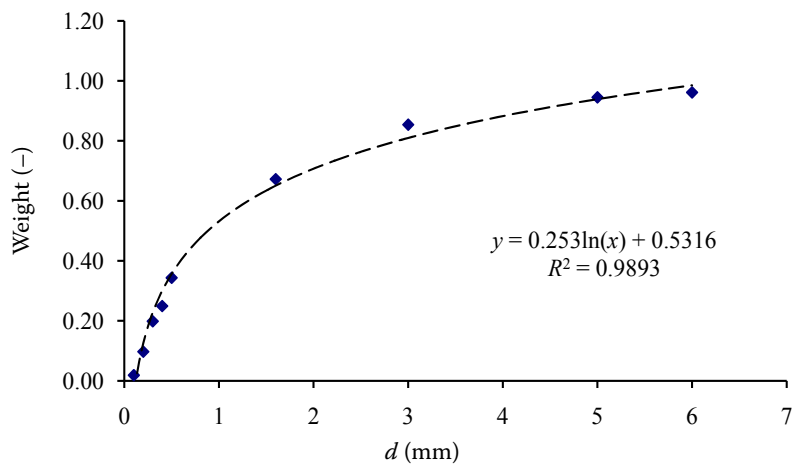$y = 0.253\ln(x) + 0.5316$
$R^2 = 0.9893$

Fig. 1. Particle size distribution obtained by sieving
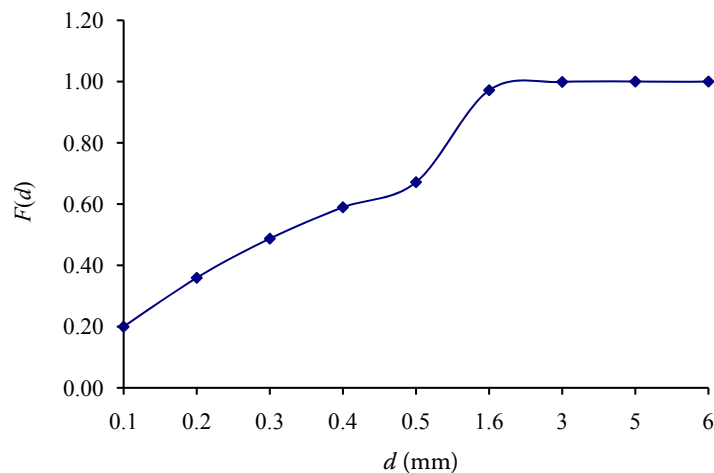

Fig. 2. Plot of the distribution function versus particle size

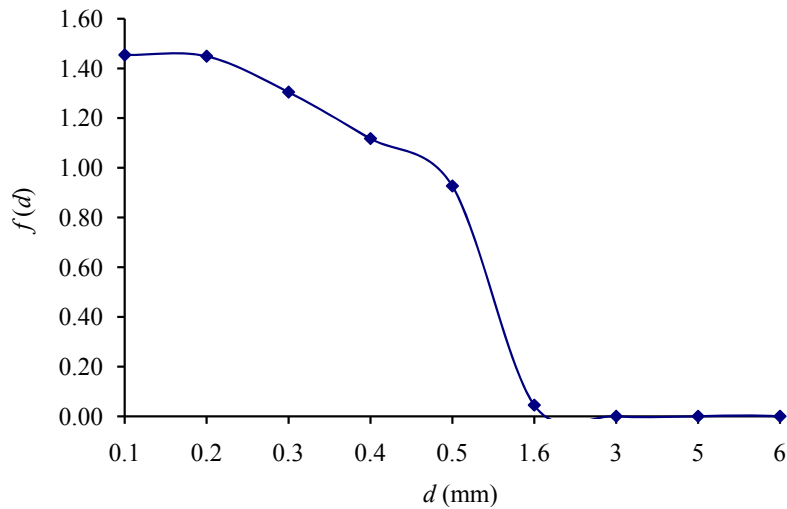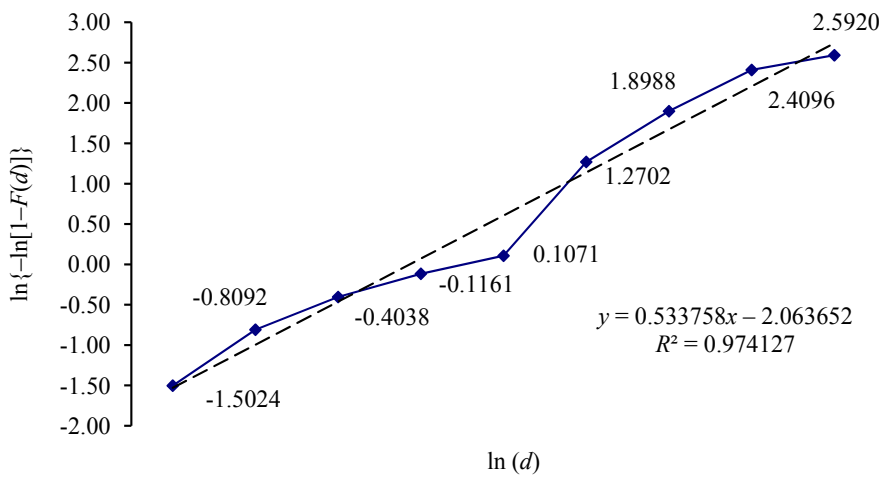Fig. 3. Plot of the density function versus particle size

$$y = 0.533758x - 2.063652$$
$$R^2 = 0.974127$$

Fig. 4. Fit to RR model

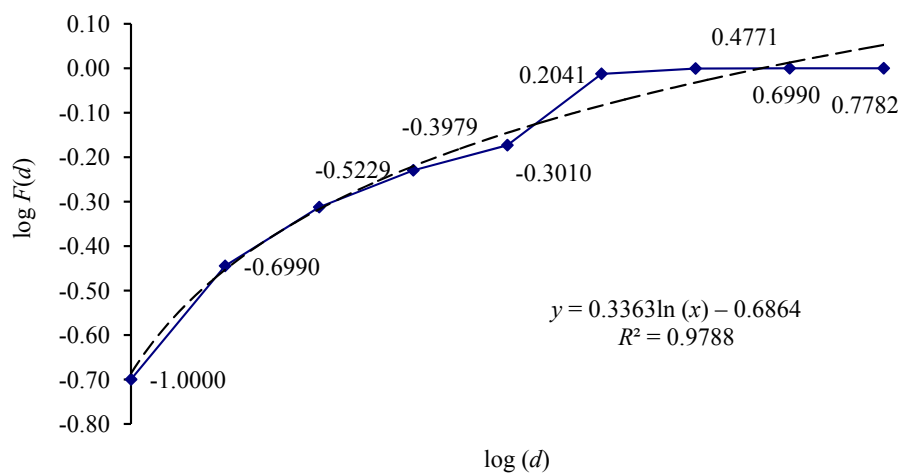$$y = 0.3363\ln(x) - 0.6864$$
$$R^2 = 0.9788$$

Fig. 5. Fit to GGS model

size. Curve analysis of a mixture of waste sawdust and wood shavings is shown in Fig. 1, and is similar to many other materials. Curve in the Figs 2 and 3 is quite unusual (Macias-Garcia et al. 2004). To evaluate the measured data in Table 1, two above described models were used. The results of solu-

tion of the two models are shown in Table 2, Figs 4 and 5 show the compliance of the test sample of waste sawdust and wood shavings mixture with RR and GGS model. From the observation of the two figures and the corresponding linear correlation coefficient, one deduces that the GGS model pro-

Table 2. Fit of tested material to the Rosin-Rammler and Gates-Gaudin-Schuhmann model

Cumulative weight under (%)	$F(d)$ (–)	d (mm)	$f(d)$ (–)	$\log F(d)$ (–)	$\log d$ (–)	$\ln\{-\ln(1-F(d))\}$ (–)	$\ln d$ (–)
0.0186	0.1996	0.1	1.4541	–0.6999	–1.0000	–1.5024	–2.3026
0.0970	0.3593	0.2	1.4488	–0.4445	–0.6990	–0.8092	–1.6094
0.1985	0.4872	0.3	1.3046	–0.3123	–0.5229	–0.4038	–1.2040
0.2492	0.5895	0.4	1.1175	–0.2295	–0.3979	–0.1161	–0.9163
0.3435	0.6714	0.5	0.9268	–0.1730	–0.3010	0.1071	–0.6931
0.6724	0.9716	1.6	0.0449	–0.0125	0.2041	1.2702	0.4700
0.8540	0.9987	3.0	0.000284	–0.0005	0.4771	1.8988	1.0986
0.9451	0.999985	5.0	5.82×10^{-08}	-6.37×10^{-06}	0.6990	2.4096	1.6094
0.9616	0.999998	6.0	5.58×10^{-10}	-6.88×10^{-07}	0.7782	2.5920	1.7918

vides a better fit to the experimental Particle size distribution curve than RR does. However, when testing other materials RR model has usually better fit to the experimental PSD curve (Macias-Garcia et al. 2004).

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