Approaches to assess the group consensus in Yes-or-No type experts' group decision making

Přístupy k hodnocení skupinové shody v binárním Ano/Ne expertním skupinovém rozhodování

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Abstract: Group consensus indicators provide an important insight and information about how to combine a group of expert judgments. This paper is concerned with the development of a set of indicators to be used in analyzing the group consensus in evaluating Yes-or-No type's decision problems. The opinions of the experts are in the form of a real number between 0 and 10 expressing the degree of answers Yes or No (0 for sharp No and 10 for sharp Yes). Two methods for obtaining the consensus indicators are developed. The first of them is based on configuring the one previously developed by (Ngwenyama et al. 1996), which is reviewed in this paper. The other one is an improved one that does not rely on the existence of the known or desired similarity significance levels or thresholds. A new measure of consensus is introduced, the standard deviation. An experiment is conducted to get acquainted with the relationship between the standard deviation of group decisions and one of the developed group consensus indicators, which measures the agreement level within the group of decisions. This research is intended to develop more consistent indicators and measures group consensus and position of each individual relative to others for Yes-or-No type group decisions. This is aimed at the exploitation of such important and relevant consensus information for developing a new consensus-based heuristic algorithm to combine the multiple experts' judgments or to be able to select the adequate combining criteria. Finally, the presented approach could be usefully utilized in critical "Yes – or – No" GDM problems in business and industry.

Key words: group decision making, analyzing consensus, binary decision making

Abstrakt: Indikátory skupinové shody poskytují důležitý vhled a informaci o hodnocení skupinou expertů. Článek se zajímá o vývoj sady indikátorů, které by se daly použít pro analýzu skupinové shody při rozhodovacích problémech s hodnocením Ano/Ne. Názory expertů ve tvaru reálných čísel v rozsahu 0–10 vyjadřují stupeň odpovědí Ano a Ne (0 znamená určitě Ne, 10 znamená určitě Ano). Byly vytvořeny 2 metody pro získání indikátorů shody. První metoda, jejíž přehled článek uvádí, je založena na konfiguraci původní metody Ngwenyamy. Druhá, zlepšená metoda, odstranila závislost na existenci známých nebo žádoucích prahů nebo úrovní podobnosti. Jako nová míra shody je zavedena směrodatná odchylka. Byl proveden experiment pro posouzení souvislosti mezi směrodatnou odchylkou skupinového rozhodování a jedním z indikátorů shody, který měří stupeň souladu v rámci skupiny rozhodnutí. Účelem tohoto výzkumu bylo vyvinout konzistentnější indikátory a míry skupinové shody a porovnání každého jednotlivého experta s ostatními ve skupině. Smyslem je též využití důležité informace o shodě pro vytvoření nového, na shodě založeného heuristického algoritmu, který by dal dohromady hodnocení více expertů tak, aby bylo možné vybrat vhodná agregační kritéria.

Klíčová slova: skupinové rozhodování, analýza shody, binární rozhodování

Consensus relevant information should constitute an important guide in combining or aggregating the decisions of multiple experts or expert systems. This information provide a clearer picture about the differences and similarities among the decisions made by the multiple experts, and that can help on either

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developing a decision making procedure based on the evaluation of such information or developing a method that can be used in the selection of adequate decision combination or aggregation criteria. The main intention of such analysis is to improve the decision quality of the finally reached consolidated decision.

The primary focus of this paper involves the situation in which a group of experts is cooperatively evaluating a Yes-or-No type decision making problem, in which each expert provides his/her decision about whether the answer is Yes or No. The decision of each expert is in the form of a crisp numerical value within the range from 0 to 10, a scale corresponding to the degree of decisiveness of Yes and No decision answers, where the value 0 means sharp degree of "No", and the value 10 means a sharp degree of "Yes". Intermediate values reflect the degree of bias to either decision answers, and of course the value 5 represents an overlap between the two classes of decision, and means no bias. Then, given such multiple experts' decisions, we are interested in assessing the level of group consensus. This is through determining the level of group agreement, and identifying which individual decision has the highest level of agreement with other group members, and which one can be considered as a problematic decision or outlier.

Consensus is one major topic in the group decision making (GDM) (Shih et al. 2004). Analyzing consensus has drawn a considerable attention in the past; for instance see (Fedrizzi and Kacprzyk 1988; Cook and Seiford 1978). Bryson in 1996 (Bryson 1996), considered the GDM problem in which every decision maker provides his/her opinion about a given set of decision alternatives or objects utilizing the analytical hierarchy process (AHP) (Saaty 1989) to obtain a preference vector or weight vector containing the weights of AHP ranking. Given such preference or weight vector of each decision maker, Bryson proposed a framework for assessing the current level of group consensus, and described a decision procedure for the consensus building. In 1996, he and together with Ngwenyama et al. (Ngwenyama et al. 1996) proposed three indicators related to the level of agreement, and another three individual indicators related to the measure of the position of each individual to the group. In 2004, Shih et al. proposed geometric distance-measure based consensus indicators.

In this paper, we review the approach developed by Ngwenyama et al. to measure the group consensus. Then, we reconfigure such approach to the situation of the binary decision problem; that is of the Yes-or-No type. The binary preference vector is defined, and accordingly the adequate values of agreement and disagreement significance levels are set. We argue that the difficulty of identifying reasonable value of significance levels remains an obstacle to the consistent and reliable implementation of such procedure. Consequently, we propose another improved version that is independent of the value of significance levels. The advantages offered by the new developed set are described. Finally, the standard deviation, as a measure of dispersion among a group of numerical values, is introduced as a new added measure of group consensus, which can be utilized too. The mathematical relationship between the standard deviation of numerical judgments and their group agreement level is studied.

PREVIOUSLY DEVELOPED APPROACH

Ngwenyama et al. in 1996 described an approach to assess the group consensus given a set of preference vectors of each decision maker belonging to the group. This preference vector can be in form of scores, ranks or weights of multiple decision options or alternatives. The preference vector: $V^i = (w_1, w_2, ..., w_n)$, denotes the vector of the i^{th} group member or decision makers, out of g members belonging to the decision making group G, and assessing the rank of n alternatives; $i \in <1, g>$. For instance, such weights can be obtained via the application of the AHP. For example, in evaluation of the submitted papers for publishing in a journal, each decision maker (reviewer) has 3 decision alternatives: to accept paper without changes, to accept after minor changes, to reject. Consequently, the preference vector here consists of three components; the ranks or weights of every decision option.

Ngwenyama has proposed six indicators:

(1) Group Strong Agreement Quotient (GSAQ): measures the level of agreement in the decisions made by the group members.

(2) Group Strong Disagreement Quotient (GSDQ): measures the level of disagreement in the decisions made by the group members.

(3) Group Strongest Disagreement Indicator (GSDI): measures the breadth of decision opinions in the group.

(4) Individual Strong Agreement Quotient (ISAQ): measures for each individual's decision how much it is in concordance with other members' decisions.

(5) Individual Strong Disagreement Quotient (ISDQ): measures for each individual's decision how much it is in dis-concordance with other members' decisions.

(6) Individual Strongest Disagreement Indicator (ISDI): gives the ultimate disagreement of the individual's decision with any one of the other members' decisions. It helps to identify which individual has the greatest disagreement with other group members.

The first three indictors identify the agreement, disagreement, and breadth of opinions characteristics of the group. The last three indicators give the estimates of the position of each individual's decision relative to the other group decisions. The computation of these indicators is reviewed below.

Ngwenyama et al. used the cosine of the angle between two vectors to express the similarity between any two preference vectors. A vector consists of one or more components and takes the following form: $V = (x_1, x_2, ..., x_n)$. For our decision making problem, the vector components can be scores, ratings, or weights in form of real numbers assigned by an expert to the decision alternatives. In a decision making problem, where there are three decision alternatives to evaluate, we require a preference vector of three components.

As the cosine of the angle between two vectors increases, the similarity or agreement between them increases. Then, the similarity between two preference vectors *k*, l is mathematically formulated as:

$$Sim^{k,l} = \frac{V^k \bullet V^l}{\left\|V^k\right\| \times \left\|V^l\right\|} \qquad (1)$$

where:

$V^k \bullet V^l$ = the scalar product of vectors V^k and V^1

$\left\|V^k\right\|$ = the magnitude of the i^{th} vector

If $Sim^{k,l} = 1$, then the two vectors have the same direction, which corresponds to the angle θ = 0°. If $Sim^{k,l} = 0$, then the two vectors are said to be completely dissimilar, it corresponds to θ = 90°, which is the largest possible angle between two vectors. As an example, suppose that there are two experts who are evaluating three alternatives through assigning their preferences in the form of weights within [0, 1]. Suppose that their preference vectors were as follows:

V^1= (0.7, 0.05, 0.25)

V^2= (0.8, 0.1, 0.1)

Then the similarity value is computed as follows:

$$Sim^{1,2} = \frac{(0.7\times0.8)+(0.05\times0.1)+(0.25\times0.1)}{(\sqrt{0.7^2+0.05^2+0.25^2})\times(\sqrt{0.8^2+0.1^2+0.1^2})} = 0.975$$

In addition, Ngwenyama et al. specified two thresholds for strong agreement and disagreement, α, δ respectively. Then, the group members are said to have strong agreement or similarity if $Sim^{k,l} \geq \alpha$, and are said to have strong disagreement or dissimilarity if $Sim^{k,} \leq \delta$. The six indicators were mathematically formulated as follows:

(1) $GSAQ_\alpha$:

$$GSAQ_\alpha = \sum_{i=1}^{g-1} \sum_{j=i+1}^{g} 2\times\Gamma(i,j) \Big/ g(g-1) \qquad (2)$$

$$\Gamma(i,j) = \begin{cases} 1 & \text{if } Sim^{i,j} \geq \alpha \\ 0 & \text{otherwise} \end{cases}$$

where:

g = the total number of members (decision markers) in the group *G*

(2) $GSDQ_\delta$:

$$GSDQ_\delta = \sum_{i=1}^{g-1} \sum_{j=i+1}^{g} 2\times\Phi(i,j) \Big/ g(g-1) \qquad (3)$$

$$\Phi(i,j) = \begin{cases} 1 & \text{if } Sim^{i,j} \leq \delta \\ 0 & \text{otherwise} \end{cases}$$

(3) GSDI:

$$\text{GSDI} = \underset{\substack{\forall i,j\in G,\\ i\neq j}}{\text{Min}} \left\{Sim^{i,j}\right\} \qquad (4)$$

(4) $ISAQ^i_\alpha$:

$$\text{ISAQ}^i_\alpha = \sum_{j=1, j\neq i}^{g} \Gamma(i,j) \Big/ (g-1) \qquad (5)$$

$$\Gamma(i,j) = \begin{cases} 1 & \text{if } Sim^{i,j} \geq \alpha \\ 0 & \text{otherwise} \end{cases}$$

(5) $ISDQ^i_\delta$:

$$\text{ISDQ}^i_\alpha = \sum_{j=1, j\neq i}^{g} \Phi(i,j) \Big/ (g-1) \qquad (6)$$

$$\Phi(i,j) = \begin{cases} 1 & \text{if } Sim^{i,j} \leq \delta \\ 0 & \text{otherwise} \end{cases}$$

(6) $ISDI^i$:

$$\text{ISDI}^i = \underset{\substack{\forall j\in G,\\ j\neq i}}{\text{Min}} \left\{Sim^{i,j}\right\} \qquad (7)$$

It should be noted that the chosen values for significance levels α, δ, influentially determine the values of 4 indicators: GSAQ, GSDQ, ISAQ and ISDQ. Ngwenyama has suggested two possible values for α and δ; 0.985 for α, which corresponds to the cosine of 10°, and surprisingly 0.966 for δ, which corresponds to the cosine of 15°. Unfortunately, there is no clear rule to help in setting the values for α, δ, or to describe the relation between these values and the indicators' values. Moreover, the threshold δ is set un-logically very high, without any justification. This are considered limitations of the Ngwenyama's approach.

BINARY PREFERENCE VECTOR

In order to reconfigure the Ngwenyama's consensus indicators for the binary decision making problem, first we need to define the adequate form of the preference vector. There are only two decision options or alternatives: either Yes or No. As we have said before, independently of the method or approach used, all decision makers or group members should provide a crisp numerical values within the range 0 to 10 that expresses the degree of their bias either toward Yes or No decision answers. The value of 10 corresponds to sharp Yes, and value of 0 corresponds to sharp No. These numerical values could be obtained using the AHP, expert systems, or single judgments by experts or decision makers. Here is not our object of concern how these values are obtained, but how to measure the consensus of a group of binary preference vectors composed of real-valued components. A binary preference vector consists of two scores or any other numerical values that express the ranks of two decision alternatives. For instance, if the i^{th} decision maker assigns 8 for Yes decision option; it means that he/she assigns 2 for No. Then, the i^{th} preference vector, V^i, is $V^i = (8, 2)$. Mathematically, the decision made by every i^{th} member, which belongs to the decision making group G, is represented by the preference vector $V^i = (x^i_1, x^i_2)$, $x^i_2 = 1 - x^i_1$, where:

V^i = preference vector of the i^{th} group's member
x^i_1 = score, rank, or priority of Yes decision option
x^i_2 = score, rank, or priority of No decision option

Remark: In order to further preserve the possibility of using similarity measures between vectors, we keep using the vector notation in spite of the fact that the binary preference vectors have only one independent component.

GROUP AND INDIVIDUAL INDICATORS TO ASSESS CONSENSUS IN THE BINARY GDM

In order to define the six consensus indicators for a group of binary preference vectors, it is important to look carefully over the inherent feature of such type of the vector, and the used meaningful numerical scale. Ngwenyama et al. defined two thresholds of agreement and disagreement α, δ described previously. For the binary group decision problem, in order to assign meaningful values for the similarity and dissimilarity significance thresholds, it is necessary to grasp the nature of the scale used. Every group member provides a numerical value within the range [0, 10]. Values above the middle 5 express bias toward Yes option, whereas values below 5 express bias toward the No answer. Exact 5 value means that the decision cannot be attributed neither to Yes or No; it is an overlapped, non-biased decision. Here, a special attention should be made that the existence of the middle value, 5, is not an indication of the third decision option, but it merely expresses the overlap between the Yes and No decision, as in all other binary classification techniques which use a similar range of numerical values to express the degree of each class or decision.

In order to describe how to set the threshold levels of agreement and disagreement, let us explain it through examples. According to our used meaningful scale, if two numbers lay both above 5 or if they lay both below 5, they are referring to same decision direction; that is they are similar. For instance, the numerical values: 6&7, 7&8, 9&10, 2&4, 3&2, 0&1, ... etc, are considered relatively similar pairs; and the numerical values: 2&6, 3&9, 10&0, 8&4, ... etc, are considered relatively dissimilar pairs, since the two pair's values lie always in a different decision direction. The value 5 is common between the two decision directions, that is, it can be similar for any close values to it regardless of their directions.

The cosine of angle will be used to compute the similarity value between any two preference vectors, except for the case when there is a disparity in the decision directions between the corresponding two numerical independent components. The cosine of angle in this case will be divided by 2 (i.e., half-discounted) to reflect the difference in the two decision directions, "Yes" and "No".

Generally, setting the values of α and δ depends on the analyst's vision about which value of similarity can be considered the threshold of strong agreement or disagreement. Increasing values of α and δ restricts the assessment process to stronger threshold levels, which assign more similarities to the disagreement classification. This increases the confidence about the estimated indicators, but can lead to a non-representative assessment of consensus due to the loss of information or miss-classification, when the similarity should be viewed at less strict level. On the other hand, decreasing both values leads to a loss in classifying similarities into agreement or disagreement categories. More similarities are classified into strong agreement, and less similarities values are classified into the strong disagreement level. With a high amount of difference between α and δ, it leads to some similarities which are neither classified as agreement nor disagreement, and this of course leads to the information loss, since the indicators values are unable in this case to convey any information about these unclassified similarities in the group decisions. So, first we recommend that the better way to set the values of α and δ is to adhere to the adequate view of the analyst, when the similarity between two numerical judgments can be considered as a threshold of agreement or disagreement. Second, it is preferred to have only a single threshold value, say α, which is used to indicate, whether the similarities are agreement or disagreement, and without having unclassified values.

NEWLY DEVELOPED CONSENSUS LEVEL INDICATORS

We are concerned with developing the six consensus indicators independent of the significance threshold values. In the previous approach, that was based on Ngwenyama et al, previous work, 4 out of 6 indictors depend on the values of strong threshold of agreement and disagreement, or in other meaning of the significance levels. The problem here is which values of α, δ are considered optimum in the sense that they closely mirror the consensus level group's judgment values into a representative and consistent indicators values. The determination of the threshold values remain a vague issue, since it is difficult to assess the effect of restricting or loosening these values on the obtained consensus indicators' values. This holds, unless there was a considerable experimentation conducted in order to optimize these values. However, setting these values should be based on the designer's or analyst's opinion about which value is considered as a threshold for strong agreement or disagreement, and this also can sometimes be considered a non-exact issue. In order to avoid the inherent vagueness about the values of these thresholds, and in order to be independent from such values, we propose other six consensus indicators that do not require specifying values of significance levels. We redefine new sets of indictors and mathematically formulate them as follows:

(1) GASI (Group Average Similarity Indicator):

It expresses the average similarity of all pairs of group members. It measures the level of agreement or similarity in the group. As this indicator value reaches 1; this means a complete agreement, and as it reaches 0, it means a complete disagreement. As the value of GASI increases over 0.5; this means that the agreement level within the group decisions is higher than the disagreement level in them. GASI is mathematically expressed as follows:

$$\text{GASI} = \sum_{i=1}^{g-1} \sum_{j=i+1}^{g} 2 \times \text{Sim}(i,j) \Big/ g(g-1) \qquad (8)$$

where: g = the total number of members in the group G

(2) GADI (Group Average Dissimilarity Indicator):

$$\text{GADI} = \sum_{i=1}^{g-1} \sum_{j=i+1}^{g} 2 \times [1 - \text{Sim}(i,j)] \Big/ g(g-1) \qquad (9)$$

where: g = the total number of members in the group G

Or simply, GADI = 1 – GASI

is the average dissimilarity of all pairs of group members. It measures the level of disagreement in the group.

(3) GMDI (Group Maximum Dissimilarity Indicator):

Same like eq. (4), it has not changed, since it does not depend on the value of thresholds. We have given it a more vivid name. It measures the breadth of opinions in the group.

$$\text{GMDI} = \underset{\substack{\forall i,j \in G,\\ i \neq j}}{\text{Min}} \left\{ \text{Sim}^{i,j} \right\} \qquad (10)$$

(4) IASI^i (Individual Average Similarity Indicator):

It measures the average amount of agreement, every i^{th} individual bears with other group members.

$$\text{IASI}^i = \sum_{j=1, j \neq i}^{g} \text{Sim}(i,j) \Big/ (g-1) \qquad (11)$$

The individual who has highest value of IASI^i is said to have the maximum agreement with other group members, IASI^{max}. It has been found that this group member always is the median in the case of odd group members' number.

(5) IADI^i (Individual Average Dissimilarity Indicator):

It measures the average amount of disagreement, every i^{th} individual bears with other group members.

$$\text{IADI}^i = \sum_{j=1, j \neq i}^{g} [1 - \text{Sim}(i,j)] \Big/ (g-1) \qquad (12)$$

Or simply, IADI^i =1 – IASI^i

(6) IMDI^i (Individual Maximum Dissimilarity Indicator):

This indicator is same like in eq. (7), it has not changed, since it does not depend on the value of thresholds. We have given it a more vivid name. It helps to identify, which individual decision has an ultimate disagreement with other group's members' decisions.

$$\text{IMDI}^i = \underset{\substack{\forall j \in G, \\ j \neq i}}{\text{Min}} \left\{ \text{Sim}^{i,j} \right\} \tag{13}$$

Based on the definition of the proposed modified indicators, it holds that the value of $\text{IASI}^{\max}$ is always greater than the value of GASI.

It should be noted that the above introduced new set of indicators, which do not require the specification of thresholds for agreement or disagreement, provides a relief from the previously mentioned vagueness of Ngwenyama's indicators. In addition, this new set takes account of all values of similarity, not only the count of it, as it was previously.

A NEW MEASURE OF CONSENSUS: THE STANDARD DEVIATION

We introduce the standard deviation as another measure of consensus based on statistical characteristics of the incoming experts' numerical judgments, which measures the amount of dispersion from the centre of group of numerical values. As it was defined for a group of experts' numerical judgments, the formula is as follows:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{g} (x_i - \bar{x})^2}{g-1}} \tag{14}$$

where:

x_i = the i^{th} expert's numerical judgement, $i = 1, 2, ..., g$

$\bar{x}$ = the arithmetic mean of the expert's judgement

g = the total number of experts in the judging group

The standard deviation as a measure of dispersion provides new information about the consensus level of the group. It differs from GASI in that GASI quantifies the average of the total pair-wise similarity of the group, whereas the standard deviation measures the dispersion from the centre or mean of the group. In order to become acquainted with the nature of relation between both measures, an experiment was conducted to understand the relation between GASI and σ. Because of the lack of the actual real data, this experiment was conducted on random sets of data. Nine data sets were created, each of which contains a different number of experts participating in judging the binary decision making problem utilizing the same numerical judgmental scale that we referred to previously. Thirty tests –judgment problems (30 points) were created uniformly randomly for each set. For each set, in every point there is computed both standard deviation and GASI. Then the values of GASI were plotted versus those of σ in scatter graphs (see Figures 1. through 9). In order to get more insight about the nature of the relationship between both measures, the regression analysis was used to fit the relationship. The polynomial regression was proved to be the most fitting function amongst all experimented ones: Linear, Logarithmic, Power, Polynomial Regression, and Exponential, according to its attributed highest correlation coefficient values for the whole number of experts. The polynomial regression equations are attached to every figure. It is noticed that the relationships throughout all experts' numbers exhibit an inverse proportionality, as was logically expected, and a small amount of non-linearities detected by small coefficients' magnitudes in the non-linear term in the polynomial regression function. The main intention of this experiment was to comprehend the relationships between the two variables for the purpose of discovering or mining the more embedded information within the numerical judgments group to be able to exploit it in either reaching the consensus or helping to select the suitable combining criteria or algorithm to be used in reaching the finally consolidated decision.

After describing the relationship between the developed GASI indicator and the standard deviation, it should be noted that these new indicators can be utilized by the researchers in further development of new indicators and in developing the new group technique or combining criteria. For the moment, the result of this relationship may not be exploited, but in future it could represent additional information used in the group consensus analysis.

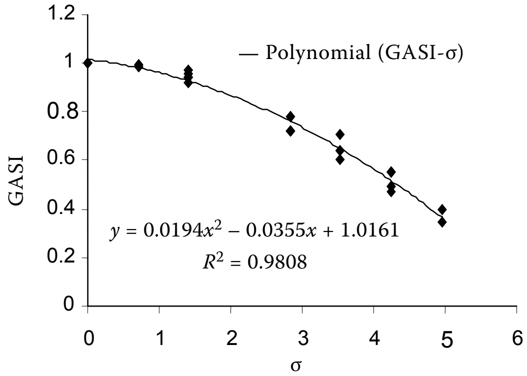

Figure 1. Relationship between GASI and σ on 2 experts' problems

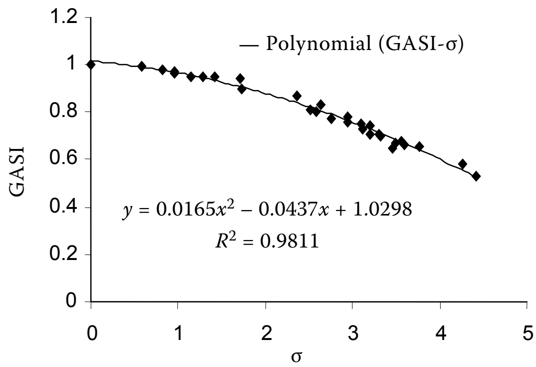

Figure 2. Relationship between GASI and σ on 3 experts' problems

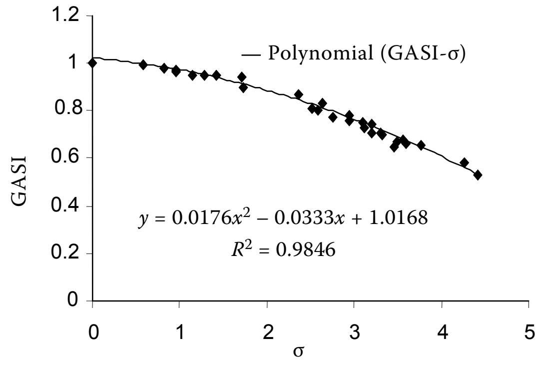

Figure 3. Relationship between GASI and σ on 4 experts' problems

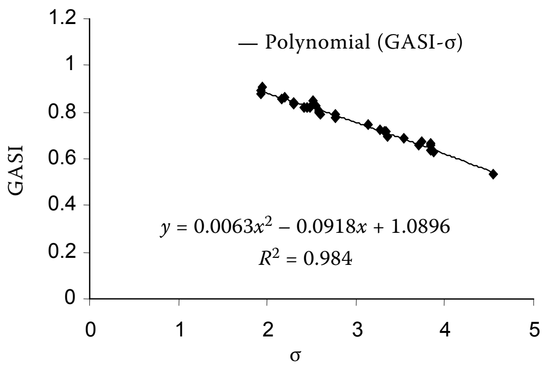

Figure 4. Relationship between GASI and σ on 5 experts' problems

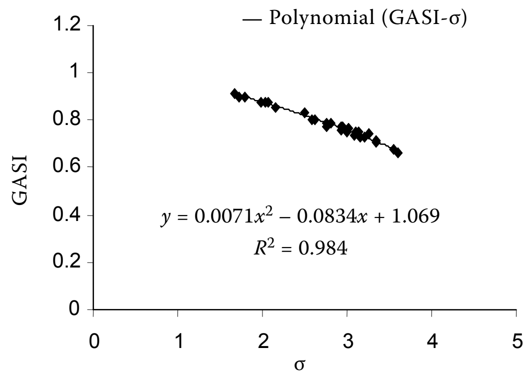

Figure 5. Relationship between GASI and σ on 6 experts' problems

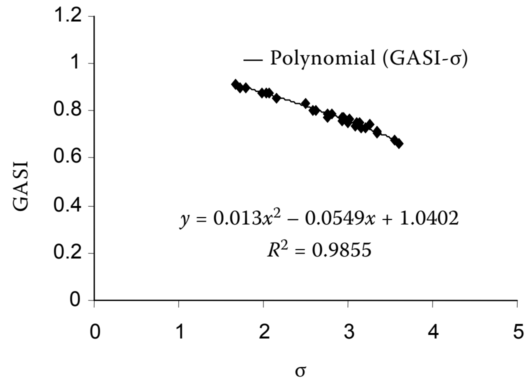

Figure 6. Relationship between GASI and σ on 7 experts' problems

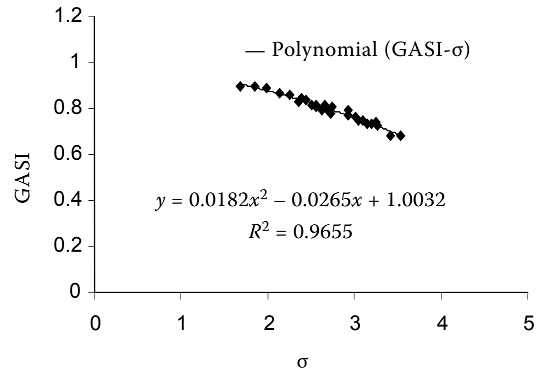

Figure 7. Relationship between GASI and σ on 8 experts' problems

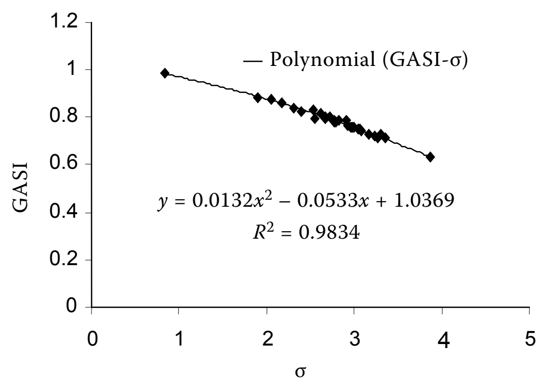

Figure 8. Relationship between GASI and σ on 9 experts' problems

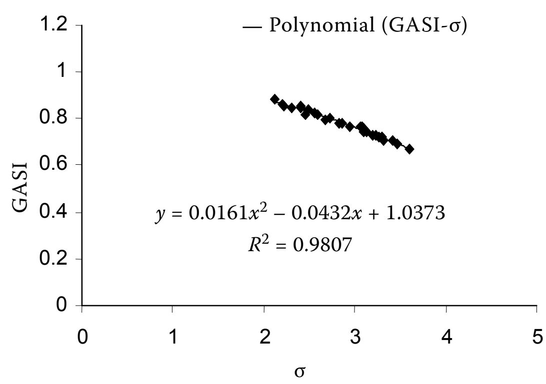

Figure 9. Relationship between GASI and σ on 10 experts' problems

CONCLUSION

The main purpose of this research is to present a detailed consensus-based GDM approach to handle the complex and critical binary or "two-alternative" decision making problems confronted within business and engineering applications situations. This is done through reconfiguring the previously developed Ngwenyama et al.'s consensus indicators for the binary decision making problem. We have shown how we can measure the similarity between any two experts' numerical decisions according to the suggested numerical scale that represents the decisive degree of Yes or No decision answers. Also, we have provided for contradictory numerical judgments, and used half-discounting to express this disparity. The consensus indicators were then reconfigured based on the estimated adequate thresholds of agreement and disagreement. We argued that the dependency on the two thresholds introduces some vagueness to computations of indicators, and suggested generally how the values of the two thresholds should be selected, which was not included in Ngwenyama. Further, we have developed a new set of indicators that do not require the specification of significance levels or thresholds. Four of the original indicators, which were depending on threshold values, were changed. These new sets of indicators are based on the average, and provide more information than the previously developed ones in the sense that they take into account the magnitude of similarity, not only the counts of similar decisions satisfying thresholds as it was previously. Moreover, it is considered more compact information in the sense that four out of the six indicators can be expressed only in terms of two, the GSAQ and ISAQ. Finally, we have introduced a new and simple measure of group decision similarity; the standard deviation. The relationship between the newly developed indicator GASI and the standard deviation was addressed using the regression analysis technique. The polynomial regression best fitted the relationship which has a logically inverse proportionality between the two variables. These consensus indicators and measure of dispersion can be used in future researches to either help selecting the adequate combing criteria or developing new heuristics based on consensus to combine the experts' judgments.

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