Characteristics of Extreme Heavy Precipitation Events Occurring in the Area of Cracow (Poland)

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Abstract


The variability of extremely heavy precipitation events with duration of 120 min occurring in the area of Cracow, southern Poland was assessed. The analysis was performed using time series of maximum annual precipitation events with durations $t = 5, 10, 15, 30, 60,$ and 120 min, recorded at the Botanical Garden station at the Jagiellonian University in the period of 1906–1990. The periodicity of precipitation was analyzed using the autocorrelation function and Fourier spectral density analysis. The Probable Maximum Precipitation (PMP) was calculated by Hershfield’s statistical method. The analysis of the autocorrelation function of sequences and the Fourier spectral density revealed a clear periodicity of the maximum precipitation. For precipitation with $t = 60$ min, the maximum values occur every 9 years, but also shorter periods (3-year) may be observed. The PMP values calculated for Cracow differ significantly from the values calculated using the probability distribution, as well as from the ones observed and they increase with increasing precipitation duration. The differences between the PMP and probable as well as observed precipitation tend to decrease with increasing duration of precipitation.

Keywords: autocorrelation; Hershfield’s statistical method; probability distribution; probable maximum precipitation; spectral analysis

In recent decades, thousands of people have lost their lives as a result of floods. As time passes, flood losses increase. This is due not only to the population growth, but also to the fact that the areas, which are subject to flooding, are used both for housing and industry (PALÁT et al. 2010). It can be expected that also European countries may be repeatedly affected by flooding in the next few years. From the perspective of predicting and preventing the effects of floods, it is important to identify the origin of the formation of extremely high precipitation in mountain or in strongly anthropogenic, mainly urban, catchments. The region of the Carpathian Basin of the Vistula is an example of such an area with large losses caused by floods, generated by extremely heavy precipitation – as stated in the paper by MACIEJEWSKI (2000), the flood risk in the Upper Vistula basin is by at least 15% higher than the average flood risk in Poland. Furthermore, as shown by OZGA-ZIELINSKA (2005), precipitation in July 1997 or 1998 and in 2001 in Poland reached the relative values of the same order of magnitude as the maximum precipitation recorded in the world, while its upper boundary is considered as the upper limit of possible maximum precipitation. Similarly high precipitation values have been recorded in other parts of Europe. For example, 1-hour precipitation with estimated return period of 200–500 years and 3–12-hour precipitation with estimated return period of 500–1000 years were recorded on August 29, 2003 in Austria in the Fella river basin. This intensive precipitation led to a catastrophic flash flood affecting an area of approximately 90 km$^2$, which caused significant damage including the death of two people (NORBIATO et al. 2007). On the other hand, research conducted in the Swiss Alps by SCHMOCKER-FACKEL and NAEF (2010) showed a significant increase in the number of floods in the period 1900–2007, resulting from extreme precipitation (over 70 mm within 24 h). This shows that Poland and other European countries are affected by extremely high precipitation, and therefore abrupt and catastrophic floods. These
facts support the adoption of the characteristics of precipitation to estimate the maximum credible flood, which can lead to unprecedented floods. This is also important from the perspective of the projected climate change and thus the threat of the frequent occurrence of extreme precipitation events (ZEVENBERGEN et al. 2011).

Maximum Credible Precipitation (MCP) is derived from the concept of Probable Maximum Precipitation (PMP), which is defined as theoretically the greatest precipitation of specified duration, the occurrence of which is physically possible within the given area, geographical conditions, and season (USDA 1978; DURBÜDE 2008; Bureau of Meteorology 2009). The world literature broadly describes the problem of determining the PMP (WMO 1986; SCHULZE et al. 1994; Bureau of Meteorology 2003; CASAS et al. 2010; HI & HENG 2010). In practice, the mesoscale description of major precipitation events in the given area is applied, which allows to estimate the probable maximum precipitation with over a 12-hour duration – it is a genetic method, and a statistical description of long-term precipitation sequences and application of the Hershfield’s method to determine these characteristics.

The detailed methodology for determining the PMP is described by OZGA-ZIELINSKA et al. (2003), giving a practical example of the calculations in the catchment of the upper Sola and the Small Vistula. Typically, PMP is used to estimate the largest flood that could occur in a given hydrological basin, the so-called probable maximum flood (PMF). On the other hand, PMF is used to determine the design characteristics of flood protection measures (KOUTSOYIANNIS 1999). Despite its widespread acceptance, the concept of PMP has been criticized by many hydrologists. For example, DINGMAN (1994) stated that: “The concepts of PMP and PMF are controversial. Can we really specify an upper bound to the amount of rain that can fall in a given time? (…) we must recognize that the plotted values are only those that have been observed historically at the infinitesimal fraction of the earth covered by rain gauges, and higher amounts must have fallen at ungaged locations at other times and places. And, conceptually, we can always imagine that a few more molecules of water could fall beyond any specified limit”. In turn, BENSON (1973) argues his critical attitude towards PMP in the following way: “The ‘probable maximum’ concept began as ‘maximum possible’ because it was considered that maximum limits exist for all the elements that act together to produce rainfall, and that these limits could be defined by a study of the natural processes. This was found to be impossible to accomplish – basically because nature is not constrained to limits (...).”

Besides the general concept of PMP itself, other issues related to the methodology of determining the PMP amount have been criticized, mainly because there is no unique method for determining the upper bound of rainfall assuming that it really exists (KOUTSOYIANNIS 1999).

The aim of this study is to develop the characteristics of extreme heavy precipitation variability and to estimate the probable maximum precipitation based on long-term measurement sequences. The research was conducted in Cracow using data from the Botanical Garden precipitation station at the Jagiellonian University.

**MATERIAL AND METHODS**

The analysis was based on the sequences of maximum annual precipitation from the multiannual period of 1906–1990 with durations $t = 5, 10, 15, 30, 60$, and 120 min, recorded at the Jagiellonian University precipitation station Botanical Garden. This station is located in the city centre at an elevation of 206 m a.s.l. (50°04’N, 19°58’E). Precipitation data were derived from the study by NIEDZWIEZDZ (1989) and proprietary data sources. The authors did not have more recent precipitation data strings at their disposal. Basic statistical characteristics of precipitation were defined based on the mentioned datasets for different durations. In order to describe the temporal variability of precipitation sequences, the autocorrelation function analysis and Fourier’s spectral analysis were applied. Both types of analyses were performed only for the precipitation duration $t = 60$ min. The results of calculations for other durations were not presented, as the results were similar to the mentioned 60-min duration. Moreover, the highest precipitation so far in Cracow was recorded for the rainfall with this particular duration. The autocorrelation allows to investigate the correlation of variables with the neighbouring variables.

Spectral analysis is used for analyzing the harmonic structure of the time series. The aim of this analysis is to decompose the complex time series, containing cyclic components, into several basic sinusoidal functions (sine and cosine) with specific wavelengths. The analysis may reveal a few periodic cycles of different wavelengths, which at first appeared in the tested time series to be more or less random walking. A general model of the spectral function may be described by the multiple regression function (BLOOMFIELD 1976; ELLIOTT & RAO 1982):
\[
X_t = a_0 + \sum [a_k \cos(\lambda_k \times t) + b_k \sin(\lambda_k \times t)]
\]
for \( k \in (1, q) \) (1)

where:
- \( X_t \) – random variable in time \( t \)
- \( a_0 \) – constant term
- \( a_k, b_k \) – regression coefficients
- \( \lambda_k \) – frequency
- \( t \) – time
- \( q \) – number of variables in the model related to the number of data within the set

The results of the spectral analysis are presented as a periodogram, which was smoothed by the transformation of the weighted moving average using Hamming’s method to remove random fluctuations. The periodogram shows how the variance of the time series is decomposed into individual frequencies (Haan 2002; Węglarczyk 2010). The spectral analysis was performed using STATISTICA Version 10.0 software.

The next stage of the analysis was to determine the function of probability distribution of precipitation recorded at the afore-mentioned station. To describe the empirical curve, the Fisher-Tippett (type III min) and type I max) distribution was adopted with maximum likelihood parameter estimation. These two distributions were selected because they are commonly used in Poland to describe the precipitation distribution and they are recommended by the WMO.

For each theoretical distribution the hypothesis about its compatibility with the empirical distribution was verified using the Kolmogorov test. Finally, the Akaike information criterion (AIC) and Schwartz criterion (BIC) (Konishi & Kitagawa 2008) was used to select the distribution. The model, for which the AIC and BIC obtain the lowest value, was considered the best.

The calculated amount of precipitation at a certain probability of exceedance was compared to the probable maximum precipitation. This feature was determined using Hershfield’s method. Hershfield adopted the Chow’s formula (Chow 1951) initially aimed to determine the precipitation frequency and he developed the following formula that allows to estimate the probable maximum precipitation (Hershfield 1961, 1965)

\[
PMP = \bar{x} + k_m \times \sigma_n
\]
where:
- \( \bar{x}, \sigma_n \) – mean and standard deviation of precipitation with varying duration
- \( k_m \) – incidence rate, calculated from the following equation:

\[
k_m = (X_{\text{max}} - \bar{x}_{n-1}/\sigma_{n-1})
\]
with:
- \( X_{\text{max}} \) – maximum observed value of precipitation within a sequence
- \( \bar{x}_{n-1}, \sigma_{n-1} \) – mean and standard deviation of precipitation sequences observed after removing the maximum value

For this purpose, decreasing statistical strings of precipitation were determined for each duration. Then, the arithmetic mean value and standard deviation \( \sigma_n \) were calculated for all observations. Subsequently, the highest values for each duration were removed from the data strings, and and \( \sigma_{n-1} \) were calculated. This allowed to determine the incidence rate \( k_m \) and then the sought values of PMP.

RESULTS AND DISCUSSION

Basic statistical characteristics of precipitation sequences with the analyzed durations are presented in Table 1. Precipitation series with duration \( t = 60 \text{ min} \) was characterized by the greatest variability – the coefficient of variation was nearly 65%, while the lowest variability was found in the case of precipitation with duration \( t = 120 \text{ min} \). The maximum value of precipitation was less than 99 mm and occurred during an episode of precipitation, which took place on September 9, 1963. A hail-producing thunderstorm occurred that day. This was also the largest precipitation in the history of observations at the Jagiellonian University “Botanical Garden” precipitation station and represented approximately 14% of the long-term average annual precipitation (TWARDOSZ 2005). In Cracow, thunderstorms with hail account for about 12% of days with precipitation and 7.7% of all storms occurring during the year. According to TWARDOSZ et al. (2010), in the multi-year period of 1963–2008 in September there were 5.9% of days with storm precipitation. Hence, the recorded maximum precipitation is a typical phenomenon in the climatic region. Heavy thunderstorm is the most frequent and the most probable in cyclonic conditions, partly under the effect of cyclonic trough (Bc type). The occurrence of abnormally high precipitation, including the afore-mentioned episode of 1963, is related to the synoptic situation. As given by TWARDOSZ and NIEDZWIEDZ (2001), high averages of daily precipitation occur in the advective movement of air masses from the north, north-east, and north-west. Distributions of maximum precipitation for each of the durations show a clear right-side asymmetry,
as evidenced by the skewness. Table 1 also shows the empirical probability of extreme values in the string (min and max) for each duration. Empirical probability of the highest precipitation amounts was slightly more than 1% (except for precipitation with duration of 120 min, where the probability was equal to 1.8%), while for the lowest amounts in the string it was equal to 100%.

The southern Poland region (where Cracow is located) is the area characterized by very different conditions in terms of orography, topography, and geomorphology which also influences the climate-affecting factors. Mountain barriers constitute obstacles to air masses, hence the higher precipitation in the mountains (Starkel 2008). Mountain ranges enhance the intensity of precipitation not only due to the elevation, but also because of their dynamic impact on the flowing air masses, as well as due to the adequate exposure of slopes, which plays an important role in shaping the precipitation intensity (Lorenc et al. 2012). A common phenomenon in the discussed area is the overlapping of different types of precipitation, especially of downpours and widespread rains. This causes the formation of extremely high precipitation amounts, leading to the occurrence of catastrophic floods and landslides. In the region neighbouring Cracow, i.e. the Kielce Upland, which is characterized by a diverse topography, the maximum daily rainfalls are frequently caused by the inflow of warm tropical air from East Europe (Suligowski 2010a). Studies by Siwek (2010) in the Lublin region (eastern part of Poland) confirm the fact of the substantial difference of the discussed area in terms of the extreme precipitation formation. It was found that the occurrence of daily precipitation totals exceeding 100 mm was mostly associated with the impact of low pressure from the centre to the south or south-east of Poland. The analysis conducted by Kotowski et al. (2010a), concerning the maximum amount of precipitation with durations from 5 min to 72 h, in the selected European countries in relation to Polish climatic conditions, indicates that the possibility of similar amount of precipitation needs to be expected. Research conducted by Müller et al. (2009) on the formation of heavy precipitation in Central Europe from 1951–2002 proved the relation between the amount of precipitation with the cyclonic situation above the Mediterranean region.

Figure 1 shows the time series distribution of precipitation for each duration. The results confirm previous observations about high variability of precipitation. In each of the analyzed cases we are dealing with a strong right-sided asymmetry.

Figure 2 presents the example of maximum annual precipitation sequence with duration $t = 60$ min from the period of 1906–1990. The precipitation demonstrates a certain periodicity and significant fluctuations in time. As previously shown, the highest precipitation reached 99 mm and was recorded in 1963. Higher precipitation occurred in 1921, 1932,

![Figure 1. Time series distribution of precipitation](image-url)

<table>
<thead>
<tr>
<th>Duration of precipitation (min)</th>
<th>Mean (mm)</th>
<th>Median (mm)</th>
<th>Min (mm)</th>
<th>Empirical probability of $P_{\text{min}}$ (%)</th>
<th>Max (mm)</th>
<th>Empirical probability of $P_{\text{max}}$ (%)</th>
<th>SD (mm)</th>
<th>Coef. of deviat. (%)</th>
<th>Skew</th>
<th>Kurtosis</th>
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<td>8.80</td>
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<td>98.70</td>
<td>1.5</td>
<td>13.73</td>
<td>64.68</td>
<td>3.424</td>
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<tr>
<td>120</td>
<td>24.58</td>
<td>21.00</td>
<td>10.90</td>
<td>100</td>
<td>66.10</td>
<td>1.8</td>
<td>11.11</td>
<td>45.21</td>
<td>1.552</td>
<td>3.15</td>
</tr>
</tbody>
</table>

$P_{\text{min}}$ – minimum value of precipitation; $P_{\text{max}}$ – maximum value of precipitation; SD – standard deviation.

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Original Paper


Table 1. Statistical features of the analyzed precipitation sequences

<table>
<thead>
<tr>
<th>Duration of precipitation (min)</th>
<th>Mean (mm)</th>
<th>Median (mm)</th>
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<th>Max (mm)</th>
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<td>45.21</td>
<td>1.552</td>
<td>3.15</td>
</tr>
</tbody>
</table>
1963, 1971, and 1986. The ratio of the maximum precipitation to the median values ranged from 315% for duration $t = 120$ min to 592% for $t = 30$ min.

On the other hand, the autocorrelation analysis confirmed the lack of a significant trend (Figure 3). The formulated “zero hypothesis” stated that the autocorrelation coefficients are statistically insignificant towards the alternative hypothesis, that the individual values of the sequence are significantly correlated. The zero hypothesis was verified at the significance level $\alpha = 0.05$. Autocorrelation coefficients are not statistically significant at the specified level of significance. This autocorrelation course shows that it is a stationary process, which is not a “white walking”, as the test probability $P$ value was greater than $\alpha = 0.05$ and thus showed a certain regularity. The course of the correlogram may suggest that there is a certain periodicity of short-term maximum precipitation. It may be assumed that the maximum precipitation with $t = 60$ min occurs with periodicity of 9 years.

This is confirmed by the periodogram (Figure 4), where the maximum hourly precipitation occurs about every 9 years – maximum spectral density occurred in the period equal to 8.44. There are also shorter cycles.
of 3 years and a long span of over 75 years. Summer cycles 3 and 9 have the significance level \( \alpha = 0.05 \).

This last cycle is probably typical of extremely high precipitation, such as the episode of September 9, 1963. Twardosz (1999) suggested the existence of a 70-year high precipitation fluctuation period in Cracow. In Cracow there is a strong concentration of precipitation in the warm half of the year (May to October). This distribution is a manifestation of the pluvial continental climate which is typical for this part of Europe. Empirical distributions of precipitation series with specified durations were described by two distributions commonly used in meteorology: Fisher-Tippett type \( \text{I}_\text{max} \) and \( \text{III}_\text{min} \). The distribution parameters for precipitations with specified durations, estimated by a maximum likelihood method, are presented in Table 2. This table also shows the values of the Kolmogorov \( \lambda \) function, based on which the hypothesis about the correctness of the description of torrential rainfall random variable was verified by the mentioned theoretical distributions. The Kolmogorov test results indicate that there is no basis to reject the proposed theoretical distributions for each duration. A question arises in this situation: Which statistical distribution will be the basis for further studies?

The answer to this question is provided by the values of information criteria AIC and BIC (Table 3). Finally, the distribution for which the values of information criteria were the smallest was adopted for further analyses. The analysis showed that for all precipitation durations, except for \( t = 5 \) min, lower values of AIC and BIC were obtained for the Fisher-Tippett type \( \text{I}_\text{max} \) distribution and this distribution was the basis for further analysis. Figure 5 presents the curves of the Fisher-Tippett type \( \text{I}_\text{max} \) empirical distribution for the precipitation with a specified duration.

A similar distribution was applied to determine the probable precipitation for Wroclaw (Kotowski et al. 2010b). Table 4 presents the values of the calculated precipitation amounts for the analyzed durations.

Table 2. Values of \( \text{I}_\text{max} \) and \( \text{III}_\text{min} \) type of Fisher and Tippett distribution parameters and \( \lambda \) Kolmogorov test for precipitations with different time of duration

<table>
<thead>
<tr>
<th>Duration of precipitation (min)</th>
<th>Fisher-Tippett type ( \text{III}_\text{min} )</th>
<th>Fisher-Tippett type ( \text{I}_\text{max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \varepsilon )</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>0.118</td>
</tr>
<tr>
<td>10</td>
<td>0.1</td>
<td>0.081</td>
</tr>
<tr>
<td>15</td>
<td>0.1</td>
<td>0.067</td>
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<td>0.044</td>
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<tr>
<td>120</td>
<td>0.1</td>
<td>0.037</td>
</tr>
</tbody>
</table>

\( \varepsilon \) – lower limit of precipitation; \( \alpha \) – location parameter; \( \beta \) – scale parameter; \( \lambda \) – value of Kolmogorov test; \( \mu \) – scale parameter

Table 3. Values of Akaike information criterion (AIC) and Schwartz criterion (BIC) for analyzed distributions of precipitation with different time of duration (distributions recommended based on the used criteria are marked in bold)

<table>
<thead>
<tr>
<th>Duration of precipitation (min)</th>
<th>Fisher-Tippett type ( \text{III}_\text{min} )</th>
<th>Fisher-Tippett type ( \text{I}_\text{max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AIC</td>
<td>BIC</td>
</tr>
<tr>
<td>5</td>
<td>4.730</td>
<td>4.823</td>
</tr>
<tr>
<td>10</td>
<td>6.169</td>
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<td>6.785</td>
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<tr>
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<td>7.378</td>
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<tr>
<td>120</td>
<td>7.406</td>
<td>7.537</td>
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</table>

Figure 4. Periodogram of maximum precipitation sequence with duration \( t = 60 \) min
based on the Fisher-Tippett (type $I_{\text{max}}$) distribution and the PMP values for given precipitation durations using the statistical method.

It is apparent that the PMP values are significantly higher as compared to the probable precipitation. The smallest differences can be noticed between the PMP and probable precipitation determined with statistical distribution for precipitation with duration $t = 120$ min. Those differences range from 8\% for the probability $P = 0.1$\% to 28\% for $P = 1$\%. On the other hand, the largest differences can be seen between the PMP and $P = 1$\% and they reach from nearly 28\%, for $t = 120$ min, to nearly 68\%, for $t = 60$ min. At the same time, the overall regularity may be noticed, that the differences between the PMP and the probable precipitation decrease with increasing precipitation duration. The PMP values increase significantly with increasing precipitation duration and they reach maximum at $t = 60$ min, while at $t = 120$ min they are slightly decreased. The PMP values are significantly affected by the $k_m$ coefficient. Changes in its values do not show any clear correla-

<table>
<thead>
<tr>
<th>Duration of precipitation (min)</th>
<th>Precipitation probability (%)</th>
<th>PMP</th>
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<tbody>
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<tr>
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<tr>
<td>30</td>
<td>0.1</td>
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<td>51.3</td>
</tr>
<tr>
<td>120</td>
<td>1</td>
<td>57.6</td>
</tr>
</tbody>
</table>

PMP – Probable Maximum Precipitation

Figure 5. Curves of empirical (points) and theoretical (solid line) distributions obtained for the analyzed precipitation sequences in the “Botanical Garden” precipitation station at the Jagiellonian University in Cracow for precipitation duration: (a) $t = 5, 10, \text{ and } 15$ min, (b) $t = 30, 60$ and $120$ min

Figure 6. Relationship between $k_m$ value and duration of storm

$k_m = 2E - 0.5t_2 - 0.0045t_2 + 0.247t + 6.7939$

$R^2 = 0.7789$

$r = 0.882$

$k_m = 1%$.
reported that in the Anas river catchment located in north-western India the value of $k_m$ parameter for daily precipitation reached 2.50. Ghahraman (2008) stated that the values of this parameter in the Atrak catchment in Iran usually do not exceed 5.0, but in some stations the values exceeding 9.0 are also reported. Ozga-Zielinska et al. (2003) indicated, that the $k_m$ values calculated for several precipitation stations in Poland are much lower than the ones given in the literature. This prevents their direct use in our conditions. Based on the values of $k_m$ calculated from the equation in Figure 6 it is possible to determine this parameter as a function of the precipitation duration. The calculated correlation coefficient $r = 0.882$ is statistically significant at $\alpha = 0.05$ and $\alpha = 0.02$. A significant increase in the value of the parameter $k_m$ in the case of less duration of precipitation (up to $t = 30$ min) and the decrease in the chance of a longer duration can be observed. Casas et al. (2010) based on research conducted at 145 stations with at least 55 years of data series approximated the $k_m$ value by logarithmic function as a function of the average precipitation of specified duration.

The calculated PMP value for the precipitation with 120-min duration corresponds to the repeatability of $10^6$ years, which suggests that the PMP is a very extreme value as compared to the observed ones. According to the National Research Council (1994) the return period of the PMP in the USA was estimated to be $10^5–10^9$ years, while Foufoula-Georgiou (1989), based on the reference data, specified it at $10^5–10^6$ years. Similarly, Austin et al. (1995) reported that the PMP specified for Great Britain corresponds to precipitation of the average repeatability of every 200 000 years.

Figure 7 shows the variability of rain intensity for different durations calculated using various methods. The differences between the observed maximum precipitation intensities and the ones calculated based on the probability distribution are clear – the intensities of the observed precipitation are higher than the ones obtained from the Fisher-Tippett (type I) distribution for durations from 5 to 60 min (on average by 35% as compared to $P = 0.1\%$). The intensity of PMP also exceeds the values obtained not only from the Fisher-Tippett (type I) distribution, but also the ones calculated based on the observed values. However, in the latter case, the difference clearly decreases with increasing duration of precipitation to $t = 30$ min. Therefore, for $t = 5$ min the PMP intensity is by 25% higher than the one observed, and for $t = 30$ min – it is only by 16% higher. After $t = 30$ min the difference between the afore-mentioned intensities increases, to reach the maximum of over 31% for $t = 60$ min.

Statistical methods for calculating the PMP can supplement and verify the results of meteorological methods. In the case of short duration precipitation, they may form the basis for the determination of the probable maximum precipitation, but they require the availability of a long measurement series (Desphande et al. 2008). According to Jagannath and Shrivastava (2006), using the statistical method to determine the PMP requires a minimum of 20 years of precipitation sequences. The results obtained from various PMP calculation methods are significantly different, as demonstrated by e.g. Suligowski (2010b). The PMP calculated by the statistical method may serve as a lower limit of the maximum precipitation.

**CONCLUSION**

Autocorrelation analysis of sequences and Fourier spectral analysis revealed a clear periodicity of the maximum precipitation. For precipitation with $t = 60$ min the maximum values occur roughly at 9-year intervals. The PMP values calculated using the statistical method for Cracow differ significantly from the ones calculated using the probability distribution and increase with increasing precipitation duration. The differences between the PMP and probable, and observed precipitation tend to decrease with increasing duration of rain. The $k_m$ values for Cracow can be described in the function of the precipitation duration by the exponential equation. The PMP with duration $t = 120$ min occurs with an average frequency of once every $10^6$ years. This storm return period is similar to the one presented by other authors. Given the obtained
results, the PMP cannot be a meaningful value in the calculation of flood protection systems. However, due to more frequent extreme weather events, this feature could be used as additional information in determining the flood hazard.

References


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